M500

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M500 is a student-operated and student-owned newsletter for Open University mathematics students and staff. It is designed to alleviate student academic isolation by providing a forun for public discussion of individuals' mathematical interests. Articles and solutions are not necessarily correct but invite criticism and argument.

MOUTHS is a list of names, addresses, telephones and courses of voluntary members, by means of which private contacts may be made by any who wish to share OU and general mathematical experiences or who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned, and do not necessarily represent those of either the editor or the Open University.

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Cover design is by Tony Brooks who writes:

The letters and numerals of the cover design like all such figures are only 'impossible' in the sense that the brain tries to interpret them as three-dimensional figures. This is because we are so used to seeing two-dimensional representations on paper of three-dimensional objects. As a result we tend to forget that a surface does have only two dimensions and that a diagram consists only of lines in those two dimensions. The third dimension is supplied to a diagram by our own interpretation and designs like the one on the cover show how easy it is to mislead the brain.

I have long had an interest in so-called 'impossible figures Those really interested in seeing what a real artist can do with them should see the later work of M.C. Escher (1898-1972) I can particularly recommend 'The graphic work of M.C. Escher', published by Pan/ Ballantine, price £1.25.

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GROUPS

An interest in the subject having been sparked off by the two units on Groups in M100, I got hold of a textbook only to find early on, or so it seems, that the subject falls into two parts: what one might call the theoretical (where this or that of varying degree of generality is proved or is required to be proved) and what one might call the practical (discover and exhibit such and such a group which is an example of this or that).

The more difficult part is the second. Or more precisely, perhaps, one finds oneself being asked to produce this or that example before one is really equipped to do so, unless it is with a great deal of labour.

A natural book to start with must surely be the one set for the M202 course. Turn to page 45 of Herstein's 'Topics in Algebra'. Look at problem 6: "Show that every subgroup of an Abelian group is normal". The second unit on groups in M100 is enough to make that problem trivial - one needs to know little more than the definitions of a group, subgroup, Abelian and normal subgroup for the problem to be seen as no more than a rearrangement of words.

However, look next at problem 7 on the same page. The asterisk at the side should warn even the arrogant but then, surely, having got to page 45 one ought to be well enough armed to solve the problem. It is this: "Is the converse of problem 6 true? If yes, prove it; if no give (a counter example) an example of a non-Abelian group all of whose subgroups are normal".

How does one set about it? Suspecting the existence of a counter example one starts constructing groups of low order (testing each one after it is constructed) with the idea of working upwards. But how does one do this? Setting down latin squares in a particular way (assuring oneself of three of the group axioms) is very time-consuming as it is. Worse still, one then has to test for associativity, and then find the subgroups (among the non-Abelian groups) and then test for normality. Crazy. bludgeony, hamfisted way must be wrong. But how, then?

A judicious hint from Margolis and there I was looking among groups of order 8. But if it had not been for Budgeon's 'The Fascination of Groups' (C.U.P.) - page 290 gives a classification of all groups up to order 12 - I might still appear to my fellow commuters to be attempting crosswords or code-cracking. Helpfully Budgeon points out that there are only two non-Abelian groups of order 8 and gives their defining relations. As bad luck would have it I picked the wrong one first - two more train journeys.

And there are other examples. There is of course Lagrange: order of a group is divisible by the order of any of its subgroups "Is the converse true? If so, prove it; if not..."(that is, given a composite number with a group of that order, must theref be subgroups with order equal to the factors of the order of group?) At least on this occasion Herstein tells you where to look for the answer (see bottom of p.36 and top of p.37), but even so the subsequent work was time-consuming.

Here is another example: "Two groups are isomorphic implies have the same number of elements of order 2, the same number elements of order 3, and so on. Is the converse true? If so prove it; if not..." An answer is given in the examples on p. 38 of the Dover paperback 'Introduction to the Theory of Groups Finite Order' by R.D. Carmichael. Here it is:

"Show that the permutations

(1,10,19) (2,11,20) (3,12,21)(4,13,22) (5,14,23) (6,15,24) (7,16,25) (8,17,26)(9,18,27)

and

(1,4,7)(2,5,8)(3,6,9)(10,15,17)(11,13,18)(12,14,16)(19,23,27)(20,24,25)(21,22,26)

generate a non-Abelian group of order 27. Show that the group generated by the following permutations is also of order 27 and that these two groups provide a counter-example of the sort required:

(1,2,3),(4,5,6),(7,8,9)".

(Ed: commas between last 3 perms intentional.)

No doubt it is easy, but just have a go at constructing the first group. For my part, I've come to the conclusion that I'm quite happy to accept the result.

If anyone knows of counter examples to the converse of other Group theorems I would be glad to hear of them: a knowledge of them could save a great deal of time. Hopefully, M202 is theoretical and not practical.

Bob Escolme, Merstham, RH1 3BL

what is maths?

If the Arts students will forgive the misquotation: "What is Maths said jesting Pontius and would not stay for answer". Linger awhile, because as maths students we should surely know what we are studying.

For instance, Russell Brass in M00/17 made his quote of the year when he said "M100 doesn't have a very high mathematical content". Whatever the validity of this as an objective appreciation, it certainly tells us something about Russell's expectations of M100. To a considerable extent I think I know what he means, since I encountered exactly the same reaction.

My own M100 study group must have been quite typical, heavily loaded with schoolteachers and a minority of other hopefuls: the young teachers glistening with A-levels and dewy fresh from their college experience, the more mature clutching their original editions of Euclid as a protective armour, and each with a private definition of what maths was all about.

With an exception the group reaction to M100 was one of uniform bewilderment. "This isn't mathematics", they said. "Give us an example and a page of problems and we'll do it, but what is this about - what are we supposed to be learning?" The young (of all ages) managed to leap the gap separating their expectations from M100 reality, but some with a more ossified superstructure dropped out into the intervening hiatus. In truth, I summed it up for many people when I said that M100 was the only intelligence test I knew that went on fo.r ten months.

If you think that this pre-concept of maths is atypical, then may I invite your attention to the comments of 'Glassblower'

(M500/16) when he questions whether M251 is a mathematics course at all. Now it is well known that if you can add 2 + 2 and get < 5, .then you. have all the 'mathematics' you need to tackle M251, because this course is not about 'numeracy but about communication; how to communicate with an idiot machine so that it doesn' t use its licence to produce garbage. (Thinks! Would that M251 had written M100 for us human idiot machines.)

So in this brief discourse we have already encountered three opinions of what maths is about. To Russell, 'Glasshlower' and my late M100 study group maths should be heavily numeral and turgid with manipulation; to M100 maths is the bright quiz-kid, the off-beat association, the kinky calculation; to M251 maths is about communication and paradoxically, it appears to matter little that the object of the communication need not be mathematical.

What is mathematics? Perhaps the experts can help us. In an M100 broadcast for Unit 17, Graham Flegg got together with Prof. Goodstein to discuss this topic, co-opting a philosophy lecturer, Susan Wilson, as their straight man. After 20 erudite minutes they defined maths, (I paraphrase), as being to do with mathematical objects, the word 'do' being shorthand for manipulate, construct, observe, or whatever takes your fancy. How we are intended to recognise the subset of objects defined by the adjectival recursion I leave, to your better imagination.

So is all lost for us poor students trying to get to grips with what it is that we are studying? Well, yes, it would appear so, but at personal level I choose to draw some significance from the official coupling of mathematics with philosophy, and see a morphism with the sex fiend who takes tranquillisers with his aphrodisiac. In both cases we are equipped and motivated to hunt out the object of our attention, be it mathematical or pulchritudinous; but if such objects do not exist, well who cares. We can treat it philosophically.

NB - For the indecently fussy the actual quotation is: . "What is Truth, etc. etc." from 'On Truth' by Francis Bacon, circa 1597.

Sinbad

unprovable facts

In M50C/9 Peter Weir mentioned a certain sequence which can defined recursively as follows:

S_1 is an arbitrary positive integer

$S_{u+1} = S_u/2$ if S_u is even . . . -x-

$S_{u+1} = (3S_u + 1)/2$ if S_u is odd. . . . -y-

He states that it is intuitively always true that there exists an integer U (which depends on S_1) such that $S_u = 1$. He has not proved it but managed to pick up a f ew results. I wonder if he can be persuaded to publish them. I have been giving the matter some thought and have one simple result which looks very pretty, even if it is not immediately useful.

If'we assume U_{S_1} always exists we can assign a label, n+1, to the class of those integers which initiate a sequence containing n elements obtained by an operation of the type -y-. That is, integers of class 1 have only operations of type -x- associated with them and are consequently of the form 2^a, $a \in \{0, 1, ...\}$.

Integers in class 2 satisfy $3[S_1/2^a] + 1 = 2^b$,

i.e. $S_1^{(2)} = {}^1/_3(2^{a+b} - 2^a)$.

The equation for class 3 is

$$3\left[\frac{3\left[\frac{S_1}{2^a}\right] + 1}{2^b}\right] + 1 = 2^c$$

or $S_1^{(3)} = 3^{-2} \,.\, 2^{a+b+c} - 3^{-2} \,.\, 2^{a+b} - 3^{-1} \,.\, 2^n$; and so on. In general the integer in the class n is of the form

$$S_1^{(n)} = 3^{1-n} . 2^{\sum_{j=1}^{n} a_j} - \left[\sum_{i=1}^{n-1} 3^{i-n} . 2^{\sum_{j=1}^{n-i} a_j}\right]$$

where $\sum_q^p = 0$ if $p < q$.

There was more, but it was on the blackboard and got wiped off so I shall give it a rest for now!

Eddie Kent

MATH-QUOTE - Ron Davidson

Shall I refuse my dinner because I do not fully understand the process of digestion?

O. Heaviside

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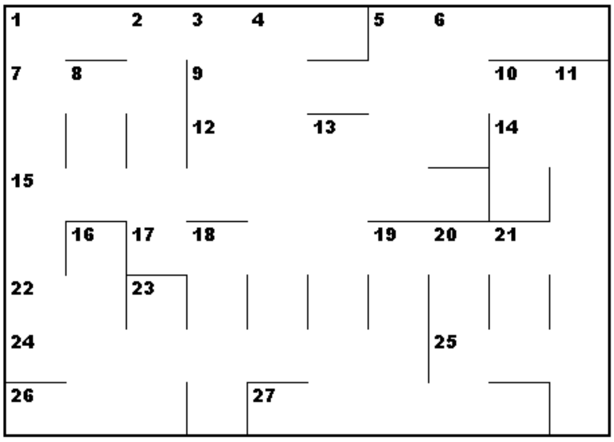
20/6

MATHEMATICAL CROSSWORD



Across

1. This sentence is primitive, almost indivisible (6)
5 How some computers run if asked to calculate 1/0 (4)
7. A gate with an output only when there is no input (3)
9. What boundaries do to interiors (7)
12. What happens to opinions in M500 (5)
14. An abbreviation for instance (2)
15. Just the function to leave the status quo (8)
17. Egyptian sun-god in Autumn will wet you (8)
22. What would we be doing without it? (2)
24. Italian mathematician who considered moving bodies (7)
25. "I see what I — " ←/→ "I — what I see", according to the Hatter (3)
26. Initially induced logistic loop may be bad (3)
27. A standard of perfection, two-sided in M202 (5)

Down

1. Change into lag, gives the solution to many school problems (7)
2. Number of elements in the 9 down of a group (5)
3. The average and lower lament (4)
4. At the start, but it might refer to elements with $t \neq 0$ (7)
5. It might be said by the net judge or the computer when told to assign a value (4, two words)
6. Highland gathering which maps $x + iy$ to $\sqrt{[x^2 + y^2]}$ (3)
8. It is strange that $f(x) = -f(-x)$ (3)
10. Any collection of objects, even clues (3)
11. Equality in Galois' time (7)
13. Having a short metal hoop, phone Marion shortly (6)
16. The opposite problem, almost Galois' last one (4)
18. Good idea X is horizontal line (4)
19. Woodcutter's cleaver, function of imaginary opponent in a legal battle (4)
20. A portion of surface or $\int_a^b f$ of R(f,a,b) when $f(x) \geq 0$ to some (4)
21. 80 yds of worsted or 300 of linen (3)
23. $\{(x,y,z) : x,y,z \in \mathbb{R}\}$ possibly, no more (3)

Michael Gregory, Farnham

(Michael very much wants comments on difficulty and interest, please)

POST MORTEMS

MST282 last year - I took it at the same time as ST285 - the latter I failed to understand until after Bummer School and then worked at it to the detriment of the maths. However, I survived both.

If anyone else is thinking of doing these two try the maths first and then the science the next year, especially since ST285 Unit 1 is a 'Double', four-week unit, on the Dynamics of Collisions which is just a repeat of MST282's units on momentum and energy.

Anyway, I'd like to express my thanks to the Bristol office for the one-day Sunday School which they laid on for MST282 (and all other Summer School-less maths courses) which did help clear away some of the uncertainties.

I agree with Sidney Silverstone (M500/19) that there were insuffıcent worked examples, and as he says, some of the problems were intractable and the text book based units were not as clear or as easy to understand as the purely Walton Hall written units. Most of us at Plymouth agreed that MST282 was very difficult. Perhaps extra worked examples could be provided if we who have finished the courses would be willing to pass on old TMAs and Specimen Exam papers with worked solutions. Mine are available.

1975 − well, MDT241 and T231. I trust these will be more straightforward.

Tony Mann, Plymouth PL9 9N

I have heard so many mathematics students complaining about the lack of Honours level courses, but I am extremely grateful to the OU for this very thing.

If there had been a choice I should never, never have tackled Partial Differential Equations or Quantum Mechanics, and should have missed the two courses that I have enjoyed more than any that I have ever done.

How on earth can you know which courses you would enjoy unless you know what the subject is about? The small amount of knowledge, gained from previous courses, about other subjects is not a very good indication, I find.

I do not quite know why I enjoyed PDE. Perhaps I am a masochist. The work-load is immense (no CMAs in 1975) and the subject matter I found very difficult. The first few units were the worst but things got a bit easier after that, and extremely

interesting

interesting. I would have liked to have spent twice as long on it, there were so many things that I missed.

Quantum Mechanics is fascinating, and not nearly as difficult or as time-consuming as PDE. The maths is about M201 standard and of similar type, and it is quite possible to do well with very little knowledge of Physics. I certainly found that the most difficult part and .decided to do very little work on the last few units which are on applications. I think it is an extremely well-run course and I would thoroughly recommend it as a possible 3rd-level maths course.

Jill MacKean, Hants S021 2DA

Last year's M100 course was my introduction to the OU as well as the maths (if you discount my schooldays 25 years ago). M100 was a struggle, particularly the calculus units which caused serious delay to my TMAs. I only completed the minimum number of TMAs and CMAs before I had to get down to intensive revision for the exam. Despite the fact that I had to leave the last 6 units until December (after the exam) I am pleased to say that I passed.

As for the course itself I can only offer praise. It made me work hard, but then it is a degree course. The calculus units were short of worked examples - it is no use setting lots of problems in integration and giving answers without actually showing the working. The handbook: 'Worked examples of Integration' would have been most helpful if the examples been easier ones. But perhaps the worst part was the insistence of the Maths Faculty on departing from Leibniz notation. By August I had given up the units in despair and turned to a basic book on Calculus in Leihniz notation. This was a tremendous; help and although I am no expert on integration I can at least handle the easy ones.

I wonder if any other newcomers to calculus found Leibniz much easier to understand. My theory, which I tend to apply to new methods of learning, is that the really bright ideas on new methods are developed by dedicated individuals whose sheer enthusiasm and will to achieve results often succeeds. But the flaws in their methods become apparent as soon as other teachers try to put the new methods into practice.

This is my (minimal) mathematical background with which I try to understand M500, yet somehow, after reading one year's output I don't feel that I understand more articles now than I did the beginning. Perhaps there will be a sudden burst of light and all will be revealed - or more probably M231 this year will enable me to advance a little. Never mind - keep up the good work with M500, because I always look forward to each issue,

Ted Haywood, Bedford

At the beginning of the OU academic year may I send a few words of encouragement to all 1975 M100 students. In particular I would like to pass on some of the lessons that I learnt from my M100 studies last year. M100 is difficult. All M100 students I've talked to agree about this. Fortunately, for all of us, the course is well-structured and the majority of units written concisely and clearly. Consequently our studies are made as easy as we can reasonably expect. The problems I found are in coping with the unfamiliar notation and in finding the time to study. The notation was particularly irksome when I did the calculus units. The time I spent on M100 was about 20 hours a week in the first couple of months. A lot of this time was spent on the TMA questions which I had great difficulty with.

For students new to functional notation and to studying, as I was, I believe that the same problems will arise. There's no easy answer but if I can offer advice it is to do two things. First, use all the help available, tutors, counsellors, other students (it's very helpful to talk maths with your colleagues) and, of course, MOUTHS. Second, ensure that you keep up with the units, certainly in preference to doing a 100% job on the TMAs.

Finally, don't allow yourself to get discouraged. All M100 students find the course difficult so you're all in the same boat. From my experience if you can keep going and finish the course you've a very good chance to pass. I wish you success and an enjoyable year.

John Carter, Wallington

Please appeal through M500 for any M201s (particularly in the South West) - especially clever ones - to contact me with a view to Self Help. Tutorials are few and far between, and 60 miles away, so I'm feeling isolated and desperate.

I hope you'll be having some nice easy 'Problems' soon to boost the morale of us poor mortals who have wandered in from the Science Faculty without realising what we've let ourselves in for.

Does anyone know a good quick guide to Leibniz for someone who can only just cope with 'OU-ese'?

Good luck to everyone for next October/November!

Wendy Cockburn, Saltash

Some comments on Datta Gumaste's problem in M500/18

CONSTRUCTION OF NEW ALGEBRAIC CONSTRUCTIONS FROM OLD ONES

1. The construction of a monoid from a semigroup is very straightforward with no need for ordered pairs, etc. A monoid is simply a semigroup with an identity element e, such that ae = ea for all members a of the semigroup. So given a semigroup S, we can simply form a monoid by taking a set {e} with just one element and defining a binary operation o, on the union of S and {e} by

   x o y = xy (the binary operation of S) if x,y ∈ s
   x o y = x if y = e
   x o y = y if x = e

2. The problem of constructing groups from monoids needs to be carefully stated before we can consider its solution. We are given, as an archetype of the kind of construction required, the construction of a field of fractions of an integral domain (a familiar example of this is the construction of the rational numbers from the integers). An important property of this construction is that the field we end up with contains the original integral domain (or, to be pedantic, a copy thereof). Since this seems to be a desirable property of a construction of this type (and also since dropping this condition would necessitate an excursion into category theory, which is not on our present itinerary), I shall address myself to the problem :

   Given a monoid M, can we construct a group G which contains (a copy of) M.

3. Now in a group we can cancel, ie we know that if ab = ad, then b = d, and if ab = db, then a = d. Therefore, if m is to be contained in G, it must also have this cancellation property. We shall call a monoid in which we can cancel a <u>cancellation monoid</u>.

4. To show that not every monoid is a cancellation monoid, and thus give an example of a monoid which cannot be a subset of (ie contained in) a group, consider the binary operation on the set {e,a} given by the following multiplication table:

	e	a
e	e	a
a	a	a

This is an associative binary operation with an identity e, but ea = aa and e ≠ a.

5. Thus M must be a cancellation monoid, but can we always construct a group containing a given cancellation monoid? I shall leave the proof of the following <u>theorem</u> as an exercise: If M is a cancellation monoid whose binary operation is commutative then we can construct a (commutative) group containing M. Hint: either use the construction of the field of fractions of an integral domain, neglecting the additive structure, or generalise the algebraic construction of the integers from the natural numbers.

6. Which leaves us with the noncommutative cancellation monoids - and this is where the problem becomes really interesting, nicht war?

Muhammad bin al-Rowl

SHAGGY MATHS

In M500/18 I liked your 'Arabic' proof. Thought it more elegant than Spivak. Also I liked Integer's 'Flowers'. It would have been enjoyed by Cassandra, late Daily Mirror columnist, who collected excruciating puns. Connoisseurs may like to add the following to their collections (pinched without permission from 'Cassandra at his finest and funniest' published 1967 by Daily Mirror Newspapers Ltd.)

To quote Cassandra: "The Black Art of fishing for the ultimate excruciating pun is still dredging deeper into the abyss to catch new horrors that live in eternal darkness. This is what we fished up yesterday. Get below decks. Close all hatches. Shut the water-tight doors. Submerge."

Ready?

The Polish Embassy were having a party on their National Day of rejoicing and hurriedly had to hire a pianist to play some suitable music - stuff by Chopin, with maybe a nostalgic piece by Paderewski just for the old folk in memory of days gone by. When the pianist, in tails and white tie, sat down at the piano he gave them everything from St. Louis Blues to The Young Ones. But no Warsaw Concerto. When they asked him why he said: "That's the way my musical cookies crumble." In deep anger the Poles threw him out onto the street. He hit the pavement screaming and cursing. A passer-by picked him up and asked why he was so incensed and so abusive at leaving the Embassy on his backside. He replied: "Forgive me - but forty square Poles do make one rude."

Bill Shannon, Leigh-on-Sea

TRIANGLE INEQUALITY

M500/18 contains an interesting proof of the triangle inequality. Here is yet another one which some may find interesting.

We have to prove that $|a + b| \le |a| + |b|$ (1)

First note that $|a| = \sqrt{a^2}$ for $a \in R$, and as usual we are interested in only the non-negative root.

Now simply relax and watch:

(1) is equivalent to $\sqrt{(a+b)^2} \le \sqrt{a^2} + \sqrt{b^2}$ ... (2)

(2) " $(\sqrt{(a+b)^2})^2 \le (\sqrt{a^2} + \sqrt{b^2})^2$ ... (3)

(3) " $(a+b)^2 \le a^2 + 2\sqrt{a^2}\sqrt{b^2} + b^2$ ... (4)

(4) " $a^2 + 2ab + b^2 \le a^2 + 2\sqrt{a^2}\sqrt{b^2} + b^2$ ... (5)

(5) " $ab \le \sqrt{a^2}\sqrt{b^2} = \sqrt{(ab)^2} = |ab|$ ... (6)

but (6) states that $\forall x, \; x \le |x|$ ... (7)

Now (7) is immediate from the very definition of $|x|$, for if $x \leq 0$ then $|x| = -x$ and if $x > 0$ then $|x| = x$

and hence $x \leq |x|$ . . . (8)

Now go hack to (1) from **(8)** and you will have proved that $|a + b| \leq |a| + |b|$.

Datta Gumaste, Erith

Another ancient manuscript on this subject has been found. It appears, from consideration of the style, to be part of a letter from Galileo.

"Do me the favour of conveying my greetings to the disciples of Signor Michael (del Spivak?) and tell them that as to the question about the triangle inequality perhaps even more surprising will this appear. I think you will agree that it is well put and that when we commence to deal with the matter of Examination then, by reason of its brevity and lucidity this demonstration will be remembered most willingly.

You must excuse my importunity if I persist in trying to persuade you of the truth of this proposition that

$$\big|\, |a| - b| \,\big| \leq |a - b|.$$

First take $x = a - b$. (In this way $x + b = a$). The demonstration you tell me you read about in MD/XVIII has assured you of truth of the proposition that $|x+b| \leq |x| + |b|$ and so you will assuredly agree that $|x+b| - |b| \leq |x|$. Thus, by substituting for x,

$$|a| - |b| \leq |a - b|.$$

I say that it is true that this contains something of the wonderful, but if we simply interchange the letters a and b something even more remarkable can be seen: $|b| - |a| \leq |b - a|$ $(= |a - b|)$. Now $\big||a| - |b|\big| = |a| - |b|$ or $-(|a| - |b|)$ from the definition of the modulus. Thus, in either case

$$\big|\, |a| - |b| \,\big| \leq |a - b|.$$

I have been too long and tedious with you; please pardon me and love me as your most devoted servitor - Leo."

Marion Stubbs

THE SECOND* LETTER EVER WRITTEN TO M500 BY M100/1975!

Many thanks for M500/19. I must admit, however, that on reading same I find myself in a labyrinth of strange symbols and incomprehensible logical progressions. No doubt all will be clear to me in a week or two, just as soon as I have digested Unit 1, M100, unless of course I prove to be a quite unsuitable candidate and not destined to become a mathematician after all. At the moment this seems more than likely, and I'm therefore reverting back to beer and skittles, and propose to cultivate my allotment most seriously this season, in the hope of coming across the square root of a Miner's, too!

I should be'happy to receive Edgar Swan's advice re the cultivation of carrots (M500/18).

Pred Popper (M100)

* – the first was from Leon Dunmore (M500/19). M100 PLEASE keep writing. In particular, shout loudly for help, in print or by telephone. Someone, somewhere is capable of sorting out your worries, even-if they occur every week - which they will.

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Because the Computer (all praise to the great one) has allowed me through last year's M202 I can feel arrogant enough to write to M500,

Posing as a GPO engineer, ie I use a pencil, rubber and slide rule, I tend to use maths on a day to day basis and have even been known to use a computer terminal in moments of stress, but I can say that most of the articles and problems appearing in M500 completely lose me. I know it's silly but what really frightens me is that some of these articles are written by size 14 middle-aged housewives of semi-det. suburbia who would be better employed washing, scrubbing and changing nappies, and I don't know whether to brush up on my maths or my washing, scrubbing, etc.

Best of luck to M500 in 1975 as, although I am moving on to the much sexier TM221 and T242, I will be reading and observing until my return in 1976 to M201.

Is a polygon circumscribed inside a synogon?

Lewis Bradley (1939 Bare Feet)

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I cast my vote for the title of our publication to 'M500'. Perhaps as a compromise to the 'Open Set' fiends we could call it 'Open Mouth'?!

Laurie Freeman

Voting as at 23.1.75 : M500 - 91; Open Set - 26; abstained - 31. M500 declared the winner and votes no longer being counted. MS

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20/14

Responding to the invitation for contributions to M500 is difficult, original conceptions being so elusive. Years age, for instance, the statement in Bell's 'Men of Mathematics' that conics cannot be constructed by the use of a straight edge and compass alone (the circle excepted) made me so cross that I immediately designed and constructed a simple compass that drew ellipses, straight lines and right angles at a flick of the fingers. Later, it was quite a shock to find that the master principle was known in 1932 and even, perhaps, much earlier. However, I was granted a patent for the straight lines and right angles bit and for some novelty in design.

Oh being introduced to the Newton−Raphson process for finding the roots of continuous functions (M100 Unit 14), I wondered if it always worked. I recall that the process involves drawing a geometrical tangent to a curve at, say, $f(u_1)$ as a first approximation, taking u_2 as the next approximation, where u_2 is the point at which the tangent intersects the x-axis, drawing tangent to the curve at $x=u_2$ and continuing the process for $u_3, u_4, \ldots$ when, normally, some u_k would converge to a value for x which gives $f(x) = 0$.

It occurred to me that if $f'(x) = f(x)/2x$ and that if a leftward half of the graph was below the x-axis and a rightward half above it, then we would find ourselves, as it were, on a merry-go-round, no matter where we began. Happily I was right and graciously given 10/10 (a rarity) for my answer to the relevant TMA question.

We have

$$\frac{f'(x)}{f(x)} = \frac{1}{2x}.$$

Therefore, integrating, $\ln(f(x)) = \ln(x)/2$. Therefore $f(x) = \sqrt{x}$. Hence

$$f(x) = \begin{cases} \sqrt{x}, x \in \mathbb{R}_0{}^{+} \\ -\sqrt{-x}, x \in \mathbb{R}^{-} \end{cases}$$

nullifies the Newton-Raphson process everywhere in its domain (except at $x=0$) and in this respect the function seems to be unique, possible variations may include those taking into account the additive constant of the integral calculus, or the multiplicative one of my logarithmic calculus. It has no practical importance, because we know what the root is, but I wonder if it is, indeed, unique. I just don't know.

In correspondence with Dr. Ketley he states that he was not attacking my logarithmic calculus, but I would like to be assured that 1974 readers of M500 agree. Otherwise my ear**lier** invitation stands, in spite of the fact that my painfully limited mathematical knowledge and taciturn nature may be fatal to my prestige if not to my cause.

Henry Jones, Knighton

SOLUTIONS

17.4 FISHERMEN (Already solved, but this contains some interesting original material)

The following method of solving equations in two unknowns, where the solutions are restricted to integers, is from 'A Treatise on Algebra', by Charles Smith, M.A., published in 1893.

It can be shown that all such equations are of the form ax + by = c or ax - by = c. These cannot be satisfied by integral values of x and y if a and b have any common factor which is not also a factor of c; and any factor common to all three can be divided out. So a and b must be prime to one another.

Let a/b be reduced to a continued fraction (see M100, Unit 34) and let p/q be the convergent immediately preceding a/b. (The convergent is the fraction obtained by stopping at any stage). It can be proved that aq - pb = ± 1.

Therefore $a(\pm cq) - b(\pm cp) = c$ ... (1)
and $a(\pm cq) + b(\pm cp) = c$ ... (2)

Hence from (1) x = cq, y = cp or x = -cq, y = cp is a solution of ax - by = c. And from (2) x = cq, y = -cp or x = -cq, y = cp is a solution of ax − by = c.

If x = r, y = s is one solution of ax − by = c, then all other solutions are given by x = r + mb, y = s + ma, where m is an integer. Similarly, for ax + by = c, x = r − mb, y = s + ma.

For Problem 17.4 I make it that 4x - 27y = 19, where x is the starting number and y is the final share. $\frac{4}{27} = \frac{1}{6+}\frac{1}{1+}\frac{1}{1+}$ so that the penultimate convergent is $\frac{1}{7}$. 4.7 – 27.1 = 1 and 4(7 x 19) – 27(1 x 19) = 19. Therefore one solution is x = 133, y = 19, and general solution is x = 133 + 27m, y = 19 + 4m. For m = −4, x = 25 and y = 3. So x = 25 is the minimum possible. (Is this the same as maximum permissible? There is an infinite number of positive integral solutions).

Incidentally, M100 shows an interesting continued fraction which gives the 'Golden Ratio '. Some others are:

$$\frac{1}{1+}\frac{1^2}{1+}\frac{2^2}{1+}\frac{3^2}{1+} \ldots = \log_e 2 \text{ (attributed to Euler)}$$

$$\frac{1}{1+}\frac{1}{1+}\frac{2}{2+}\frac{3}{3+} \ldots = 1 - e^{-1}$$

$$\frac{1}{-}\frac{r}{1-}\frac{r}{r+2-}\frac{r}{r+3-} \ldots = e^{r}$$

The 'Treatise' has quite a lot of interesting matter. MDT241 students might like to try this one:

A speaks the truth 3 times out of 4, and B 5 times out 6, and they agree in stating that a white ball has been drawn from a bag which was known to contain 1 white and 9 black balls. Find the chance that the white ball was really drawn. (The answer given is 5/8).

Tom Dale, Perth

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17.4 FISHERMEN Here is some more information about this problem.

A Cambridge legend says that no less a person than P.A.M. Dirac suggested the answer −2, and further reflections on this paradoxical result led him to the discovery of the positron. Reference: Elementary Particles, by A.A. Sokolov; trans. from Russian by W.E. Jones, Pergamon Press.

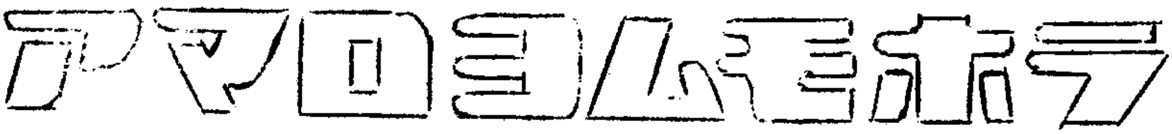
R.T. Finch

19.1 INSCRIPTIONS

$562_{12} = 794_{10}$ (Roger Claxton, Peter Needham, Eddie Kent)

19.2 TORELLI MURDER - Red (M. Stubbs, D. Gumaste)

17.3 ARRAY - Regret my solution in M500/19 NBG. It seems that 'array' does not mean a 'matrix' with rows, diagonals and columns as 'lines'. It means a subset of R consisting of points with integer co-ordinates. Full specification next time. Incidentally, Dr. Earl is now a fully-paid-up M500 subscriber, and is our first subscriber from 'Another University'. Ed.



20.1 SQUARE FACTORIAL - Hugh McIntyre

Prove, or disprove by counter-example,that there is no integer whose square is expressible as factorial n, for some n.

ie $m^2 \neq n!$ (m an integer) to be proved. (Solution in 12 lines already here, from Ian Dey and others at Walton Hall. Beat 12 lines if you can?)

20.2 BALLS - John Carter and Sinbad

Given: 12 balls, one (and only one) being lighter or heavier than the other 11. Using (or otherwise, if you stick to the basic rules) a counterbalance which allows you to determine whether the ball/balls on one scale are lighter, heavier or the same weight as the ball(s) on the other scale, find a scheme of weighings which ensures you find the unique ball in 3 or less weighings, and also tells you whether it is lighter or heavier than the others.

20.3 ALPHAMETICS - Harold Moulson

ALAS + LASS + NO + MORE = CASH. (5 solutions). The girl replied: SEND ÷ A = GIFT. (2 solutions).

20.4 DIVISIBLE INTEGERS - Datta Gumaste

Given: m is any positive integer
n is any positive integer > 3.

Prove (or disprove):

$mn - (m-1)^{n-1} + (m-l)^{n} - (m-2)^{n-2} + (n-3)$ is divisible by n-1. (For M202 students there is a time limit of 3 minutes.)

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ON M231

Many thanks indeed for so kindly sending me the copy of the M231 exam papers, i.e. all of the Specimen Exam and also Part II of the actual exam as well. I was not really sure if it looked hard or fairly straightforward - this is the sort of ignorance that keeps one going at the start of a new course, I guess. Nevertheless, I was filled with awe and admiration that you should have managed to make a Grade 2 out of that gobblygook. (None more so than me - fondly proclaiming to the world at large that (after M100 fog) I was incompetent at calculus - Ed.) I am certainly struck by the number of theorems, proofs and definitions one is supposed to know, and it is immensely helpful to me and others to be made aware of that right at the start of the course. Another important fact seems to be that you are not allowed to take a handbook into the exam, so that we had better be sure that we know our onions beforehand.

This is the sort of information we need, and I only hope that it is conveyed to students in the M231 Course Guide. I was very struck by Hugh Mclntyre's remark in M500/19 that he found M231 more difficult than the 1974 version of M202, which I had always thought of as the Everest of the Maths programme. Could we ask MOUTHS readers to rate courses in order of difficulty for other students? My very limited experience would place MDT241 as easier than M100, given that one comes to mathematics fairly fresh and raw, and I strongly suspect that the two really tough courses are M231 and M202, with M201 slightly behind. But of course I know nothing of mysterious areas like M321 or (for the first time this year) M331 or M332. I would very much like to see people sending you less tiresome problems, which always strikes me as a form of inverted snobbery, and more actual discussion and information about mathematics and the courses.

Why can't we get the OU staff to tell us more about the content of the new courses, for example, or what kind of courses they are planning? They seem inordinately, shy of passing on any information.

I would very much like to hear (by letter or telephone) from other M231/1975 students who would like to form up into an active self-help group at-a-distance for discussion of on-going course work.

Willem van der Eyken, London SE26 6RZ

EDITORIAL

As subscription renewals continue to arrive I do get quite a lot of (usually brief) remarks, some from 3rd level students, that M500 is above their heads. Now usually there are only two or three really mathematical articles, one of which usually wins page 1. If you read them at the speed that I type them (with two fingers) light can dawn. There is really no point in producing an M500 with no maths in it, and personally I find it stimulating (and depressing) to find out just how much there is still to be learnt. So the maths stays, as long as anyone writes any, whether or not anyone but the author understands it. Everything in this issue is EASY, at 2-finger-typing speed - unless one is just starting M100 - and that includes Eddie Kent's ferocious symbols! I hope some people who complain about the 'high' level will begin to write things which even M100 can understand. My own personal grouses concern the sheer length, and frequent tedium, of solutions to problems. I am sure that these could usually be condensed to about 4 lines. Those interested could contact the solvers for further details via MOUTHS. This is one of the purposes of MOUTHS. The Hoops Saga stands apart, since it is original research, and we are fortunate to be able to watch the creative process in action, and to file it for future comprehension.

But basically M500 is whatever YOU write, neither more nor less. If it is above your head, don't moan at me, but write something which IS your level, particularly M100. M500 does not claim nor even hope to be scholarly and erudite, so don't feel inferior or afraid to write. All I ask is for people to keep off the sheer corn - such as the Squaw on the Hippopotamus, which has already been consigned to the editorial WPB twice in two years.

Next, TM221 and SM351 rank as 'mathematics' - in fact anything now or in the future which has an M in its code counts as M500-fodder They count towards our miserable Kettle allocation of 10 maths credits, anyway, and even more so than MDT241 or MST282, if I understand the system correctly. So include these in the class of subjects which you feel inspired to write about.

I will be at the OUSA Conference at Liverpool this Easter, and hope some other MOUTHS will be there. If so, please let me know, and also sport a plain white circular badge saying M500/M0UTHS on it. Then we may be able to identify each other, and perhaps arrange a convivial meeting.

M500/21 is semi-empty. It also has no cover as yet, though it may be feasible to use the pretty diagrams which go with the ARRAY problem as a cover. So - as usual - please get writing! Contributions are needed as soon as possible - there is a lot of typing to do, and it is best to start very early.

The Weekend Work-in is reaching the final stages of planning. Booking forms will be automatically sent to those who have indicated interest already. Others, please ask. Don't leave it until 'later', please, as I have to know numbers etc. in order to finalise booking of accommodation and find the right number of staff, neither of which seems particularly easy.

M.

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