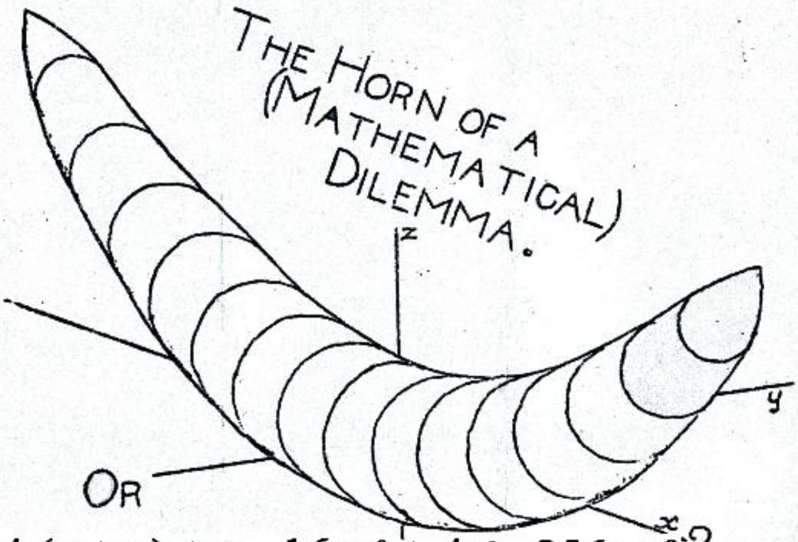
M500 21



$$64(z^2+y^2)+x^4-16zx^2+4x^2-256=0?$$

M500 is a student-operated and student-owned newsletter/magazine for Open University mathematics students and staff. It is desgned to alleviate student academic isolation hy providing a forum for public discussion of individuals' mathematical interests. Artilcles and solutione are not necessarily correct but invite criticism and argument.

Items for publication can be at any level, erudite or informal so long as they are likely to he of interest to OU students taking any OU course containing an M in its code. This includes not only the inter-faculty courses with codes beginning with M, such as MST282, but also those with an M in any position, e.g. TM221, SM351, AMST283 and the forthcoming AM289.

Students taking M100 and MST281 are particularly welcomed, and M500 is very willing to publish anything at all which they feel able to write!

MOUTHS is a list of names, addresses, telephones and courses of voluntary members, by means of which private contacts may be made by any who wish to share OU and general mathematical interests or who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned, and do not necessarily represent those of either the editor or the Open University.

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The cover design is by L.S. Johnson.

Freehand lettering throughout this issue was designed by Mr. William J. Smith, ARCA, Principal of Southampton College of Art.

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OU versus THE REST

HOW DOES AN OPEN UNIVERSITY DEGREE COMPARE WITH ONE FROM A CONVENTIONAL UNIVERSITY?

This often-raised question sometimes means "how will my degree count when I compete for jobs with conventional graduates?" It is too early yet to answer this question definitely as it will depend on the attitudes of employers, who may know little about the OU. Some may prefer mature students who have shown the determination needed to study for a degree at the same time as holding a full-time job, but others may be prejudiced against the unknown. In the long run the answer will depend on the experience of those who employ OU graduates.

It is however possible to be more precise in comparing the content of an OU degree course with that of a conventional course. First, let us estimate the amount of study time involved. A conventional university teaching year lasts about 25 weeks and an honours degree takes 3 years. So if students study from 30-50 hours a week the time taken is 2,250-3,750 hours. An OU honours degree requires 8 credits. The teaching year for one credit lasts about 34 weeks. So if students study for from 10-15 hours a week the total time taken is 2,720-4,080 hours. I have only guessed at the average time students study and I have ignored, for example, the vacation study that conventional students are expected to do. Nonetheless these rough figures do show that, at the very least, as much work goes into gaining an OU honours degree as in getting one from another university.

The OU does not offer honours degrees in mathematics alone, but just with the courses available in 1975 it is possible to gain 5½ credits in purely mathematical courses (full credits: M100, M201, M202; half-credits: M231, M251, M321, M331, M332). The course provision is best in the area of pure mathematics. M100, M202, M231, M331 and M332 are almost all pure mathematics, while M201 is about half pure mathematics, so there are about 4 pure mathematics credits in all (of course the division between pure and applied mathematics is a bit arbitrary). We can compare this with the courses available to the mathematics students in Leeds.

In Leeds the basic course unit is called a module. (1 module = 1 48-lecture course, but modules are often divided into smaller units. We often discuss the correct 'length for a module' and since this can also be a technical question in algebra it is an endless source for confusion and bad puns). Students taking an honours degree in mathematics have to take about 16 modules over three years, so roughly 2 Leeds modules = 1 OU credit. Leeds students are given a wide range of choice and so the 16 modules could contain from 4 to 11 pure mathematics modules. The other modules can be chosen from Applied Mathematics, Statistics and Computing Science. Thus in Pure Mathematics the OU course provision compares quite well with that in Leeds, but in the other mathematical areas the OU does not yet have much to offer.

What about the level of these courses? A comparison is not easy to make because of the different course structure. In Leeds students will often study the same subject in each year of the course going into greater depth in succeeding years. In the OU system a given subject is put into one credit course which is taken in just one year. For example, the M202 algebra contains some material Leeds students would meet in their first year - basic ideas about groups - other material they would the second year and other material only taken as third year options - Galois theory. My general impression,which it would take too much space to justify in detail, is that the pure mathematics OU courses taken as a whole are very similar in breadth and depth of coverage to the course a conventional university student would take if he or she devoted half the time to pure mathematics courses.

My conclusion is that in regard to pure mathematics the Open University course compares favourably with that of a conventional university. True OU students do not get the choice of courses that other students get, but I am far from convincel that a wide range of courses is necessarily desirable. It is more important to be offered well-planned and carefully prepared courses. It is possible to get a good education in pure mathematics from the OU and I think that anyone who gets an OU degree with good grades in the mathematics courses would be well qualified to take, for example, the M.Sc. course in pure mathematics that is provided in Leeds.

Course provision in applied mathematics, statistics and computing. is not yet very strong in the OU. This is a disadvantage for students who would like to concentrate in these areas, but for students whose main interest is in pure mathematics the opportunity to take courses outside mathematics seems to me ample compensation for the inability to take mathematics courses only.

Alan Slomson, Lecturer in Pure Mathematics, Univ. of Leeds, OU tutor for M100, M201, M202 in previous years, and this year M331 and M332

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THE RULES OF THE GAME

Richard Ahrens contributed a piece in M500/9 with this title. In it he raised some questions about the communication of mathemati which won't easily go away but can be pushed aside with varying degrees of success.

For instance, in the Logic units of M100 a pair of symbols are introduced which play a vital role in the whole of mathematics. These are $\forall$ and $\exists$, the universal and existential quantifiers. Since precision is a prerequisite of communication, even of mathematics, in a perfect world every assertion would he-accompanied by a quantifier; except in those cases where ambiguity is either impossible or intentional. However, as Richard points out, if an irksome ambiguity is present, especially in a CMA question, the safe course is to insert the strongest quantifier available - at least in imagination.

Let Bert Russell and Alf Whitehead (PRINCIPIA MATHEMATICA, introduction) say it: "All that is necessary... is the convention that, when the scope of an apparent variable is the whole of the asserted proposition in which it occurs, the fact will not be explicitly stated unless 'some' is involved instead of 'all'." Because this is a convention used at least implicitly by mathematicians and accords with general usage there should be a moment in each generation when it is no longer necessary to say it anymore. Perhaps it could be painted over the entrance to the OU or on page one of M100.

How many uses are there for the word 'define'? I can see three fairly easily; they are all used in PM so let us stay with it.
✳13.01 $x = y \,.\!=:\, (\varnothing) : \varnothing!\, x \,.\!\supset\!.\; \varnothing!\, y$ Df,
✳13.02 $x \neq y \,.\!=\!.\, \sim (x=y)$ Df.

Of these, ✳13.01 states that x and y arc to be called identical when every predicative function satisfied by x is also satisfied by y. This is a description of a use of a language such as might be found in a dictionary. The definition is written in the language of the definiens. The primary function of ✳13.02, however, is to embody a convenient abbreviation. But more basic than either of these is the type of negative definition of a primitive term such as that an atomic proposition is a proposition that contains no parts that are propositions. That is, although particular atomic propositions can be defined, eg "R(x) means that x has the predicate R", the class of atomic propositions cannot be defined as "all propositions of that type" since there is no all to be of. We have, therefore, to use a negative definition and hope that the law of the excluded middle holds until we are out of trouble. (Vain hope!)

Next Richard asks two, more impalpable questions. Why can we write “$2<x<6$” but not "$6 <x< 2$" for $x > 6$ or $x < 2$? This must be partly a matter of convention, allied to the plain-man's common sense (all honour to his highness). If there is a rule let us pretend that it is that $<$, which is a relation, is transitive. Then we can say $6 <x<2$ is true if and only if $6 < 2$. And is in, or and or both are out.

Finally, what is meant by "Let V be a vector space"? This reminds one of Descartes: "I imagine a triangle, although perhaps such a figure does not exist and never has existed anywhere in the world outside my thought. Nevertheless this figure has a certain nature, or form, or determinate essence which is immutable or eternal, which I have not invented and which in no way depends on my mind. This is evident from the fact that I can demonstrate various properties of this triangle, for example the fact that the sum of its interior angles is equal to two right angles, that the greatest angle is opposite the greatest side, and so forth. Whether I desire it or not, I recognise very clearly and convincingly that these properties are in the triangle although I have never thought of them before, and even if this is the first time I have imagined a triangle. Nevertheless no one can say that I have invented them or imagined them." Perhaps this vector space V is merely a particular and circumscribed bundle of axioms. But what is it that ties them together?

Eddie Kent (M332, AMST283)

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WOGGLE

M500 readers who have sons or friends who are Scouts or Cubs (or even sons or friends who are NOT Scouts or Cubs) may be interested in a brand-new little magazine called WOGGLE, edit annd produced by Robert Smith, aged 14, with some editorial advice from M500. WOGGLE 1 was published in February 1975, originally being intended as a typescript just for Robert's small Patrol but to editorial eyes it looked too good to have such a small circulation. After a little discussion it was decided to produce WOGGLE on a duplicator and to run 36 copies for his whole Troop.

Robert typed all of WOGGLE 1 himself and produced all the drawings - which, I may say, are better than the illustrations in M500. During the production run he became an expert on use of a duplicator without assistance. All the material came from books since the original idea was to give his Patrol some interesting items which he had read, but now WOGGLE has been published he would like to increase the number of readers and hopes very much that WOGGLE 2 will be filled with original contributions, letters and even some nice teasers for a Puzzle Corner. In short, that it could become a proper'magazine' for Scouts, or for boys who like the same sort of things even if not Scouts.

If 'Sons of M500' or their friends (even if not Scouts or Cubs would like to see a copy of WOGGLE 1, please send a STAMPED, self-addressed 9" x 4" envelope, together with another loose, unused 5p stamp, inside (to cover the cost of paper and stencils) and a little letter saying that they would like a copy of WOGGLE 1. Please also mention that they heard about WOGGLE 1 from M500. Send to:

Robert Smith, Southampton.

Please ask them to do this as soon as possible, because Robert has to run off another batch and needs to know in advance how many will be needed. WOGGLE is really well-produced and seems well worth supporting.

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HOOPS

Corrections to SUMMARY:

1. Relabel SAQ 6 as 'Definition 6' and treat it as a definition of 'special'.

2. Add Definition 7: If $a \in H$ define $R_a : H \rightarrow H$ and $L_a : H \rightarrow H$ by $R_a(x) = x \text{ o } a$, and $L_a(x) = a \text{ o } x$ that is, R_a multiplies by a on the right and L_a multiplies by a on the left. R_a and L_a are automorphisms of the hoop.

MINIMAL SUBHOOPS - Bob Margolis

Minimal subhoops are special (because otherwise they would have a proper subhoop - Thm 4 (Wilkie)).

Minimal subhoops are generated by two elements (otherwise any two elements would generate a proper subhoop).

Theorem 10: Minimal subhoops have prime power order.
The proof of this uses Richard Ahrens' construction associating Abelian groups and special hoops (Thm 8). We temporarily forget about subhoops.

Proof: Let (M,o) be a hoop without proper subhoops.
Then M arises from an Abelian group (M, +) and two automorphisms σ,τ of (M,+) with:
$\sigma\tau = \tau\sigma$, $\sigma(x) = \tau(x) = x \ \forall x \in M$
and $x \text{ o } y = \sigma(x) + \tau(y)$.

Suppose M does not have order p^a, p a prime.
Then $o(M) = p_1^{a1}, p_2^{a2} \ldots p_n^{an}$ with p_i primes and $a_i > 0$, $n \geq 2$.
Thus (M,+) will have a unique Sylow subgroup S_1 corresponding to p_1.
(Uniqueness follows because all subgroups of (M,+) are normal.)

Now $o(\sigma(8_1)) = o(\tau(s_1)) = o(S_1) = p_1^{a1}$ so both $\sigma(8_1)$ and $\tau(S_1)$ are Sylow subgroups belonging to p_1.
Thus $\sigma(8_1) = \tau(s_1) = S_1$.

We now have an Abelian group $(S_1,+)$, two commuting automorphisms σ, τ of S_1. $\sigma(y) + \tau(y) = y \ \forall y \in S_1$, and so (s_1, o): $y_1 \text{ o } y_2 = \sigma(y_1) + \tau(y_2)$ defines a proper subhoop of M.

Contradiction.

If we take this last remark and the fact that an Abelian group is the direct sum of its Sylow subgroups, we get:

Theorem 11: If H is a special hoop, then H is the direct product of subhoops of prime power order.

Proof: is by induction. If o(H) = p^a then the result is trivially true. Now suppose o(H) = n = $p_1^{a1} \dots p_k^{ak}$ and the result is true for hoops of order less than n. As before there is an'overlying' Ahelian group (H, +). Let S_i be the Sylow subgroup belonging to p_i (i = 1, ..., k). Then (H, +) = $S_1 + S_2 + \dots + S_k = S_1 + K$, say. Every element x of H can be uniquely written as

$$x = s + k \text{ for } s \in S_1, \quad k \in K.$$

Now by definition of + on H (see Defn. 7 and Thm 8)

$$x = s + k = \sigma^{-1}(s) \text{ o } \tau^{-1}(k)$$
$$= s' \text{ o } k' \, (s' \in S, k' \in K)$$

since, by the argument previously used σ, τ map $S_1 \to S_1$ and $K \to K$ and so do σ^{-1}, τ^{-1}. Thus, since s, k were unique and σ, τ are one-one s′, k′ are unique and every element of K can be expressed uniquely as x = s′ o k′. Also, if $x_1 = s_1 + k_1$, $x_2 = s_2 + k_2$

$x_1 \text{ o } x_2$	$= \sigma(s_1 + k_1) + \tau(s_2 + k_2)$	(defn of o in terms of +)
	$= \sigma(s_1) + \sigma(k_1) + \tau(s_2) + \tau(k_2)$	(H, +) Abelian
	$= (s_1 \text{ o } s_2) + (k_1 \text{ o } k_2)$	
	$= s + k$	(S_1, K subhoops)
	$= \sigma^{-1}(s) \text{ o } \tau^{-1}(k)$	
	$= s' \text{ o } k'$.	

i.e. S_1 o K is a subhoop. $S_1 \cap K = \{a\}$ where $a \in H$ is the element used to define $\sigma = L_a^{-1}$, $\tau = R_a^{-1}$.

Therefore

$$o(S_1 \text{ o } K) = o(S_1) \times o(K) = o(H)$$

so S_1 o K = H, and it is easy to show (SAQ11) that S_1 o K is isomorphic to $S_1 \times K$. By the induction hypothesis

$$K = S_2 \times (S_3 \times (S_4 \times \dots \times S_k))\dots)$$

Hence H = $S_1 \times K = S_1 \times (S_2 \times \dots \times S_k))\dots)$. Alternatively this result states that

$$H = S_1 \text{ o } (S_2 \text{ o } \dots S_k))\dots)$$

where S_i are the Sylow subgroup-derived subhoops.

Summary of results about minimal subhoops

Hoop morphisms give rise to a partition into cosets which is compatible with the hoop operation. Minimal subhoops have the Lagrange property. (Thm 6 (Sue Ahrens))
Subhoops of special hoops have the Lagrange property (Thm7 R. Ahrens)) Subhoops of special hoops give rise to quotient hoops.
Minimal subhoops are special. Minimal subhoops have prime power order.
Special hoops are the direct product of hoops of prime power order.

The search continues for non-special hoops. I would like to pose the following problems (- answers are earnestly and seriously wanted hence I'm allowing this extra space to Hoops in this issue - Ed.)

The way of approaching non-special hoops depends rather heavily on whether you believe they exist! If you do, then the task is to set about constructing one. Without wanting to be too discouraging, conversations with Chris Rowley, Alec Wilkie, and Richard Ahrens convince me that a non-special hoop will have to be rather large. The other way of tackling them is from a position of disbelief - assume there is one and try to derive a contradiction. There is a time-honoured way of beginning the process:

> "If there is a non-special finite hoop then there must be one of minimal order..."

In other words, if there are any at all, then there will be a non-special hoop H with the property that all its subhoops are special. Since (Thm 4) H will have a proper subhoop, it will have a maximal proper subhoop $M \subset H$ with no subhoops 'in between' M and H. M will be special and blind intuition suggests that the relationship between M and H may well be important in either constructing an example of H or getting a contradiction. Some obvious questions arise: does o(M) divide o(H)? Does M give rise to a quotient hoop?

For those who feel slightly more at home with groups rather than hoops how about pursuing the automorphism group of a hoop?

SAQ12 If A is the automorphism group of a hoop H, and L(R) is the subgroup of A generated by all left (right) multiplications then $L \lhd A$ $(R \lhd A)$,
NB: $L = \langle L : L_a : L_a\, x \to a \circ x \rangle$ etc.

Is it possible to tell special hoops from non-special hoops simply by looking at their automorphism groups?

Some other problems:

Given that 'special' for hoops is similar to 'Abelian' for groups suggests the following analogue of the centre of a group: $Z = \{ x \in H: (xa)/(bc) = (xb)/(ac) \text{ for all } a,b,c \in H\}$.

SAQ 13: Prove (a) Z is a subhoop of H.
(b) $(Zu)(Zv) = Z(uv)$ for all $u,v \in H$.
Hint: $Z(uv) \subset (Zu)(Zv)$ for any subset Z of H.

Is the quotient hoop H/Z special, non-special or what? Under what circumstances is Z non-trivial?

(The reason for these questions is that the 'right' answers could dispose of our minimal non-special hoop by the following scheme:

Prove: ① Z non-trivial; ② H/Z non-special; ③ Contradiction because $o(H/Z) < o(H)$ and H is minimal. Note that Z = H iff H is special).

Results to Bob Margolis, please.

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MAILBAG

My basic philosophy is that OU studies come first, followed closely by work, sex, beer, football, reading the paper, crosswords, picking my nose.

But being almost human (I'say almost as I get on well with computer terminals), I sometimes falter and OU work slides into oblivion, with a few beers, and a philosophical argument with my wife that a B.A. after my name would look silly anyway.

All appears to be lost, but one morning soon the postman calls and carefully sorted from the pile of bills it's "M500" to save the B.A.

On reading the articles written with such vigour, enthusiasm and guile, I take heart and mount the stairs, away to the study and am back on'course'. Seriously, M500 reminds me how much I enjoy maths and it always seems to come at the right time.

"What does the M of M500 stand for?” I asked my friends. They thought of: Miracle, Mate, Marion, Mycetes (the louder genus of South American monkeys), Muscle, Myth, Myriad, Myroscope, Mustard, Matrices (impossible I said - it's too popular), Murex (that was after watching 'Call My Bluff), Motto, "The feeding of the 10 times", Mumbo-Jumbo, Mystique, Mosaic (he was looking at the covers), Maze (so was he), Mark. Some fool even though of Mathematics (he is a bore).

Bob Dalziel (M201, MST282)

I would like to take this opportunity to tell you how much; enjoy reading M500 and having a go at the problems. I take great interest too in the comments of students and staff - many a time I was dying to take part in the fun, but my habitual inertia plus chronic lack of time, prevented me from doing so. I particularly enjoyed the Great Debate on Proper v. Improper Maths (surely a matter of relativity) and the lesser one on the sense (or lack it) of humour in mathematicians. I think that you are doing a great job, and enjoying it by the look of it. Many thanks for keeping on with this excellent work.

Minna Rapport (M231, ST285)

I thought that M500/20 was the most interesting I've seen yet. At long last people are taking part. My contribution is still in hand. I'm glad that Jill MacKean found Quantum Mechanics SM351 fascinating - so did I. I’ve wanted since leaving school many years ago to find out what it was all about. I would like to publicise as much as possible that if you can do M201 you can do SM351 without any trouble.

Ron Middleton (M231, M332, T341)

May I express my congratulations to you in producing such an excellent magazine/newsletter and for organising MOUTHS. I can honestly say that I found MOUTHS invaluable to me last year and I was fortunate enough to meet one of my fellow sufferers of M202 last year, with whom I had had many long discussions over the telephone, at Reading during my week of summer school for the course. One very minor quiblle I have is that under my entry in the current MOUTHS list you have omitted M201 from my previous courses.

It was sad to read of George Dingley's death, having met him whilst at summer school and found him to he an inspiration to us all in study and a great character to talk to.

Keith Charsley (M231, M251, T100)

Ed: M201 wasn't on your MOUTHS entry last year either! Incidentally - is M500 a newsletter or a magazine, folks?

I can understand the feelings of some students new to maths that unless you are at the M202 level of sophistication then you are not really part of the 'inner' maths club. I put myself in this category and last year didn't subscribe to M500 as I felt, quite incorrectly, that my attempt at MST281 somehow didn't entitle me to join the group. Even this year, having passed MST281 I felt that MDT241 and TM221 also were not quite mathematical enough. The editorial comment in M500/20 has dispelled these doubts and I am sorry I have missed out for a year. Please keep on pushing the message across.

Chris Green (MDT241, TM221, DS261, E262)

Ed: M500 is, categorically, mere packaging for MOUTHS. It brings MOUTHS to life by turning names into people. So of course anyone can join, as long as they have M codes somewhere - let the other faculties, or better still, the OU itself, start similar schemes for the rest! (I've got enough to do). And if you are a faceless MOUTH on a list, you turn yourself into a person by writing something - anything, within reason. OK? No. 1 started off with 2 pages, plus 6 MOUTHS, by the way.

Am I the remotest of the remote! I certainly feel it. I phoned my nearest MOUTHS neighbour, Riki Rickard, 30 miles away, to make contact, to find she is living in Aldershot. So with no BBC2 transmissions in this area and no study centre within commuter distance I need to be self-contained. Already I have come across problems in M100 Unit 1 which I am sure would make more sense with discussion.

Enjoying M500 and I'm sure it will fill a gap. Felt very pleased to be able to do one of the problems in M500/19, only to be shot down again when I found my 12-year old daughter did it twice as quickly. Hope one day it will all make sfense.

Mary Bibby (M100) An Grianan, North Connel

My particular interest (unfortunately not catered for directly in the Open University) is logic, about which I know a little having done philosophy earlier, and would like to know a lot more - particularly many-valued - prepositional calculus systems.

If any members are also interested I would be glad to be put in touch with them.

Andrew Carstairs (M202)

I enjoy the bits of M500 that I can understand and admire your stamina in coping with its production. However, some of the letters do reveal rather strange attitudes to university-type courses. I remember reading one letter whose writer complained that she 'couldn't do calculus'. (That was me - Ed.) Doesn't she realise that the fact that she 'can't' do calculus would probably mean that she couldn't go on and do an Hons. maths degree in a conventional university? (Yes - Ed.) At least in the OU she does have the option of a quick swap to some technology or arts course that doesn't require calculus and so could still obtain an Hons. B.A.

I am amazed that anyone has bothered to reproduce the interim data on the M231 survey, particularly the data obtained from tfe CURF which had less than a 50% response for the best return and only a 9% return on the worst. Additionally, these early returns are probably a very biased, self-selected group of the original sample. Somebody seems to need a course of MDT241.

I am continually amazed by the large number of mathematics teachers who are doing M100 and further maths courses. It makes it all the more difficult for the rest of us; apart from their background knowledge and experience and facility of manipulation they have the added advantage of actually using maths in their every day life. After a day spent with bacteria and other manifestations of the disease process in the human race, it's quite an effort to switch to nice, clean, abstract numbers and other mathematical manifestations. I am medically qualified m my last official maths teaching was O levels in 1953. I've been doing some medical statistics in a cook-book manner for years and I started on M100 in an effort to understand why I added A to B and divided by C, etc. Now I've become fascinated by the mora abstract side, so it's M202 this year (before it disappears) and, as a sop to my conscience and as preparation for the proposed mathematical statistics course, MDT241 in the hope that it should be mainly revision for me and not really new ground, Next year, if I survive 1975, it'll be the History of Mathematics, purely for enjoyment. I remember reading a letter in one of the M500's from another female medic., who very apologetically pointed out that her 'doctor' wasn't academically respectable. However, there are quite a few of us who, in addition to being bachelors of medicine and surgery and hence, by common practice addressed as 'doctor also have the 'academically respectable' doctorate of medicine. However, a few of my medical friends have a doctorate of philosophy, either instead of or as well as their medical one.

This letter has become long and rambling with no quotable bits for M500. But you can put in that I live only 10 minutes drive

From the A38 holiday route to the South West, so if anyone desperately needs to catch a maths radio/TV programme en route for their holidays, they're welcome to drop in (preferably after telephoning to warn me and make sure I'm at home).

Elizabeth White, MD.(M202, MDT241)

Ed: As I typed the last paragraph of this very quotable letter it occurred to me that maybe Elizabeth meant 'not to be quoted' Too late. I hope she was just being modest. If not for publication, please write PERSONAL in large letters all over things.

There are 19 copies still remaining of the M500/17 issue containing the M231 Survey provided by the IET. If you want one, please send the usual 9x1+ STAMPED addressed envelope with the usual 10p in small loose stamps inside, together with a note saying what it is for. Photostats of the survey can be provided for the 10p if there are more than 19 requests. I think the chief message of the survey is not that IET need a course of MDT241, but that students need to realise that if they don't answer questionnaires no-one else will either. We are all to blame in this respect.

SOLUTIONS

This month the mailbag seemed to be overflowing with solutions, and particularly with flowcharts for the BALLS problem, which obviously attracted a lot of interest. Unfortunately there is really no space for 3-page flowcharts. A new idea is now to be tried, which is to print the two shortest adequate (e & o.e.) solutions received, if there are more than two. I fancy the first (below) for the BALLS cheats a bit, but it showed a bit of lateral thinking. The flowcharts all revealed that it is not difficult to do by solid, slogging means. Leon Dunmore (M100) did it in 27 lines by excellent 'common-sense'.

<u>M500/20 MATHEMATICAL CROSSWORD</u> - Michael Gregory

Across: 1. Atomic 5. Amok 7. Nor 9. Enclose 12. Aired 14. EG 15. Identity 17. Rainfall 22. OU 24. Galileo 25. Eat 26. ILL 27. Ideal

Down: 1. Antilog 2. Order 3. Mean 4. Initial 5. A let 6. Mod 8. Odd 10. Set 11. Egality 13. Ring-ed. 16. Dual (& REF DUEL) 18. Axis 19. Proe (= cleaver, and F(= function) + ROE (legal character)) 20. Area 21. Lea 23. All.

20.1 SQUARE FACTORIAL: To show that $m^2 \neq n!$ (for $\forall m,n \in Z^+ - \{1\}$)

(a) Datta Gumaste

Suppose not. Then $m^2 = n!$ for some $m,n \in Z^+ - \{1\}$.

By the fundamental theorem of arithmetic, $m = p_1^{a1}p_2^{a2}... p_n^{an}$, each p_i prime. This implies:

$$m^2 = p_1^{a1}p_2^{a2}... p_n^{an} = n!$$

Therefore each p_i appears at least twice in the prime factorisation of n! Let r be the largest prime factor of n! Now consider r! In the prime factorisation of r!, r appears exactly once. Since each $p_i < n$, r! divides n! So n!/r! = n(n-1)...(r+1) = k, say. But each prime factor of k is less than r, as r is the largest prime factor of n! Hence r = some p_i appears exactly once in the prime factoristion of n! Contradiction. This assumes that $n \leq 2r$. (12 lines-if right!)

(b) Ian Dey and others at W.H. and Dr. Earl, also Dorothy Craggs

There is a theorem called Bertrand's Postulate, which says t for any integer n there is a prime number in the range

$$n/2 < p < n-2.$$

For such a p, n < 2p so that the only power of p dividing n! is the first power, p itself. Now suppose that $n! = m^2$ for some n, j = m for some n, then any prime divisor of n! is a prime divisor of m^2 and so of m. That is, the square of every prime divisor of n! is also a divisor of n! Bertrand's Postulate now gives a contradiction as it asserts that there is a prime divisor of n! whose square does not divide n! Obviously this same argument shows that

$$n! \neq m^k \text{ for any } k.$$

(c) No marks to all those who triumphantly sent in the counter-example 0! = 1, etc. instead of pointing out the pretty obvious error in the question!

20.2 BALLS 12 balls, one odd weight. 3 weighings to find odd one.

(a) Roger Bridgman

Define 'weighing' as any process by which pans are brought into balance, one balance per weighing.

Weighing 1: 6 balls on each pan. Remove balls in pairs, on from each pan, until balanced. Last pair removed (A) contains odd ball.

Weighing 2: Weigh one of A against known 'standard' ball from Weighing 1. j

(a): If pans balance, other ball of A is odd; go on to Weighing (3).

(b): If pans don't balance, you have found odd ball; comparison with standard ball shows whether heavy or light.

Weighing 3: Repeat Weighing 2 with other ball of A; case (b) must result and the problem is solved.

(b) Steve Murphy

Label the 12 balls with the numbers 1-12. Carry out the following:

	Left-Hand Pan	Right-Hand Pan	If LH Pan Heavy record	If RH Pan Heavy record	If Pans balance record
1st Weigh	6,7,9,11	5,8,10,12	+9	-9	0
2nd Wegh	2,4,5,11	3,6,7,12	+3	-3	0
3rd Weigh	4,5,7,8	1,2,10,11	+1	-1	0

The number of the 'odd ball out' is the modulus of the sum of the recorded numbers. Examining the result of the appropriate weighings shows whether it is heavier or lighter than the rest.

Suggested additions;

(1) Deduce a method by which the recorded numbers only are used to determine whether the ball is heavier or lighter.

(2) In n weighings it is possible to deal with up to ½(3n - 3) balls. True or false?

(c) Other solutions were received from John Carter, Sinbad, Keith Charsley, Leon Dunmore, Chris Green.

20.3 ALPHAMETICS - Harold Moulson

ALAS+LASS+NO+MORE=CASH = 1215+2155+86+3694 or 2124+1 2124+68+3809 or 1419+4199+75+2503 or 2124+1244+ 68+ 5807 or 5157+1577+38+2804.

SEND ÷ a = GIFT = 7852 ÷ 4 = 1963 or 6952 ÷ 4 = 1738

18.3 Arithmograms - Steve Murphy's solution to Hugh Mclntyre's question about pentagons (M500/19) is scheduled for M500/22.

19.2 TORELLI MURDER - Fred Popper requests details, esp. for M100: Here we go in condensed style: Label men L,R,D,S,B, statements 1,2,3. Make a 5 x 3. table, fill in steps of deduction viz:

B or D: ?TF) implies B or D: TTF) implies S: ?F? implies S:TFT
D or B: ?FT) D or B: TFT)

implies L: ??F implies L: TTF implies Red guilty, R: FTT. (Not his gun).

PROBLEMS

21.1 TWO BALLS - Graham Read

2 balls are joined by a non-elastic string. Place the string along a perfectly smooth table, one ball over the edge. Let it go and one ball drops, pulling the other ball along the table.

What does the motion look like after the second ball has left the table?

21.2 STEADY UP - L.S. Johnson

A normal four-legged table stands on an uneven floor. PROVE: There is at least one position for which all four legs are in contact with the floor.

<u>21.3 QUAD-ROOTS</u> - Bob Davies

Pub - in company with Mike Cook (M202 tutor), Mike Sharp (M332 student) "A quadratic can have more than four roots." "Uh?" "Of course. Consider (x-l)(x-2) = 0 (mod 6)" "Oh yes. Does that imply a graph like this: (Fig. 1)? " No, of course we can't have negatives."

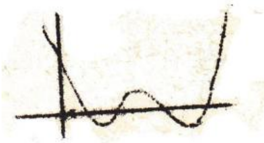


Fig. 1

Fig. 2

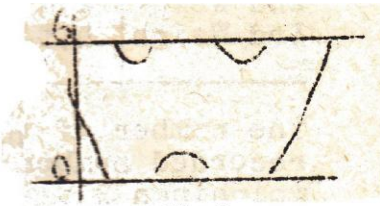


Fig. 3

"We should consider (Fig. 2) which leads to (Fig. 3). But that isn't very satisfactory, (a) the lines 6 and 0 ought to be coincident. Of course, that iinplies a cylinder, but (b) the domain is also limited and (c) we must be careful about extending our domain into the reals anyway."

"The last point presents no problem. We merely define 7½ = 1½ (mod 6) etc. The second part suggests a sphere - being wrapped round both ways."

"Ah, but what about the hole for the infinite points?" - long pause, "I know, consider the two axes as circumferences intersecting at 0. They intersect again round the back and it will be here that we puncture the sphere." "No, that's not good enough, as each axis represents the reals from 0 to 6. Hence, you are puncturing (3,3)." Pause. "But, still worse, it's nonsense to have the axes intersecting twice. How can we avoid it?" - " A TORUS!"

We haven't drawn the graph yet but it seems worth pursuing - what characteristics of the parabola will remain? Will there further joys considering other modular bases?

<u>21.4 ARRAY PROBLEM</u> (extended from 17.3) - Dr. John Earl, Univ. of Kent

Let $\lambda(n)$ denote the maximum number of points of an n x n square array which can be chosen so that no 3 of them are collinear. What can be said about $\lambda(n)$?

(1) Obviously $\lambda(n) \leq 2n$ (by considering just vertical or horizontal collineations). For n = 2,3,4,5,6,7,8,9,10,12, $\lambda(n) = 2n$. (See diagrams). Can equality occur for other n?

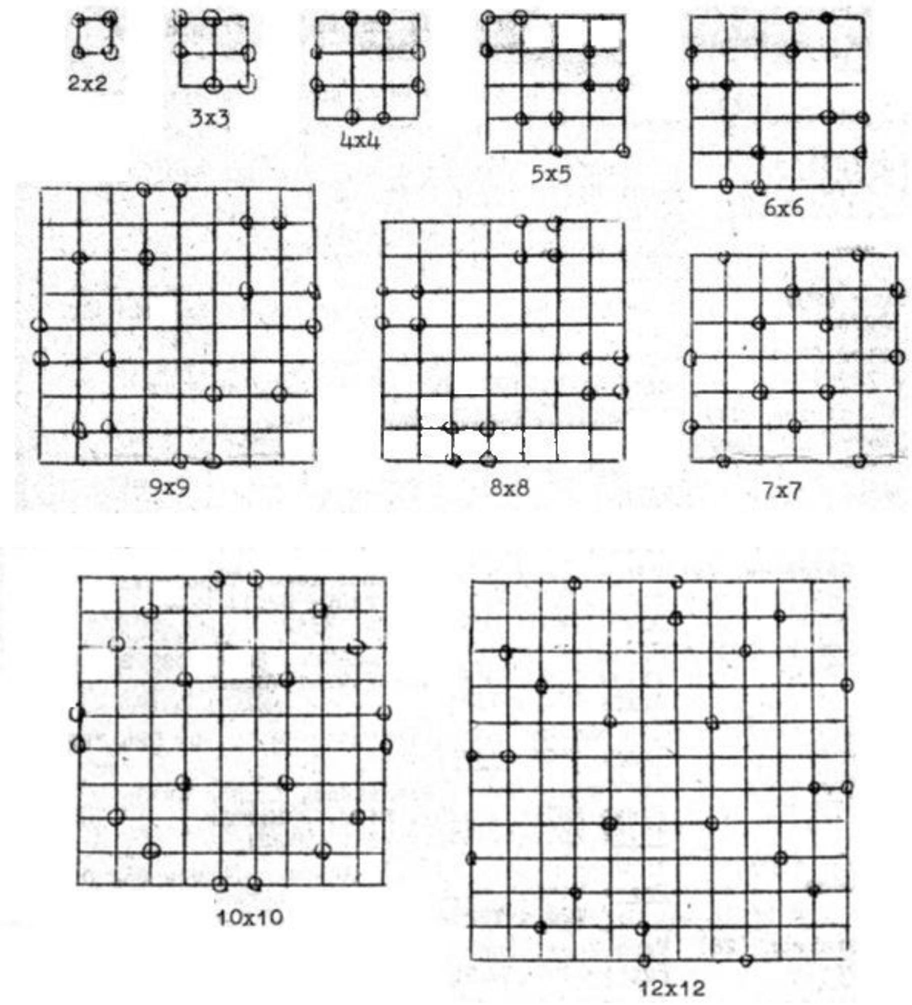
(2) Given $\varepsilon > 0$, there is an $n_0(\varepsilon)$ such that for $n \geq n_0(\varepsilon), \lambda(n) \geq (^3/_2 - \varepsilon)n$. More precisely, if n = 2p where p is a prime then $\lambda(n) \geq 3(p-l)$.

(3) Probabilistic arguments would indicate $\lambda(n) \leq (2\pi^2/3)^{1/3} n \cong 1.87n$

(4) $\lambda(n) \geq n$ for all n.

<u>References</u>: (2),(4): R.R. Hall, T.H. Jackson, A. Sudbery, K. Wild. Some Advances in the no-three-in-a-line Problem. J. Combinatorial Theory (soon).
(3): Kelly & Guy. Canadian Math. Bulletin, 11 (1968), page 527.

I would like to emphasise that I have played no part in developing these results. I merely passed on a limited form of the problem to the editor. Further correspondence and solutions would he better addressed to: Dr. A. Sudbery, Dept, of Mathematics, The University, Heslington, York. Dorothy Graggs has submitted two solutions of the 11 x 11 case (to be published in M500/22).



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21.5 FIND THE NEXT TERMS (by N.J.A. Sloane - extract from Jnl. of Recreational Mathematics, vol.7 no. 2 -1974. Baywood Pub.Co.Inc)

Find the next two terms and a rule for generating the sequences:

(7) 1,3,4,7,11,18,29,47,76,123,...
(8) 1,2,3,7,43,1807,3263443,...
(13) 1,2,3,4,9,27,512,134217728,...

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EDITORIAL

Thanks to all, the duplicator purchased during Spring 1974 belongs totally to M500. The target figure of £97 was reached on 21/1/75, and now we have an additional £31 towards a new electric typewriter complete with assorted golfball typeface* (probably costing over £300!) There is no hurry for this, but any new editor should be given the tools for the job.

Requests for the sample (M500/18 is being sent - 260 copies made for the purpose) continue to arrive daily. The response is not quite the same as for 1974, which is statistically intriguing. M251, M321 , M331 and TM221 have only just received the Stop Press publicity. By 27 Feb 1975 I had sent out 169 samples and recieved 51 new subscriptions in return; in 1974 the comparable cumulative1 total was 29th January 1974 - 212 samples, 50 new subscriptions. We ended 1974 with 488 samples and 243 new subs as a result. Incidentally, the number of people who enclose any sort of note with their s.a.e. + 10p can be counred on one hand!

Envelopes have doubled in price now, and 9 x 4 have gone unobtainable. I have 1000 9+ x 4+ which weigh just too much for 2 oz postage added to 10 sheets of A4 paper, so I fear that we must lose 1 sheet per issue while they last, and hope 9 x 4 come back again afterwards. The funds can stand the increased price, I am glad to say.

M500/22 IS VIRTUALLY EMPTY, apart from a few short pieces. HELP - as usual! We have covers for two issues. We need a lot of anything else, but maximum is still 600 words, even in famine.

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CORRECTION TO M500/20

Page 11, line -5: Equation (2) should be $(\sqrt{(a+b)^2})^2$

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