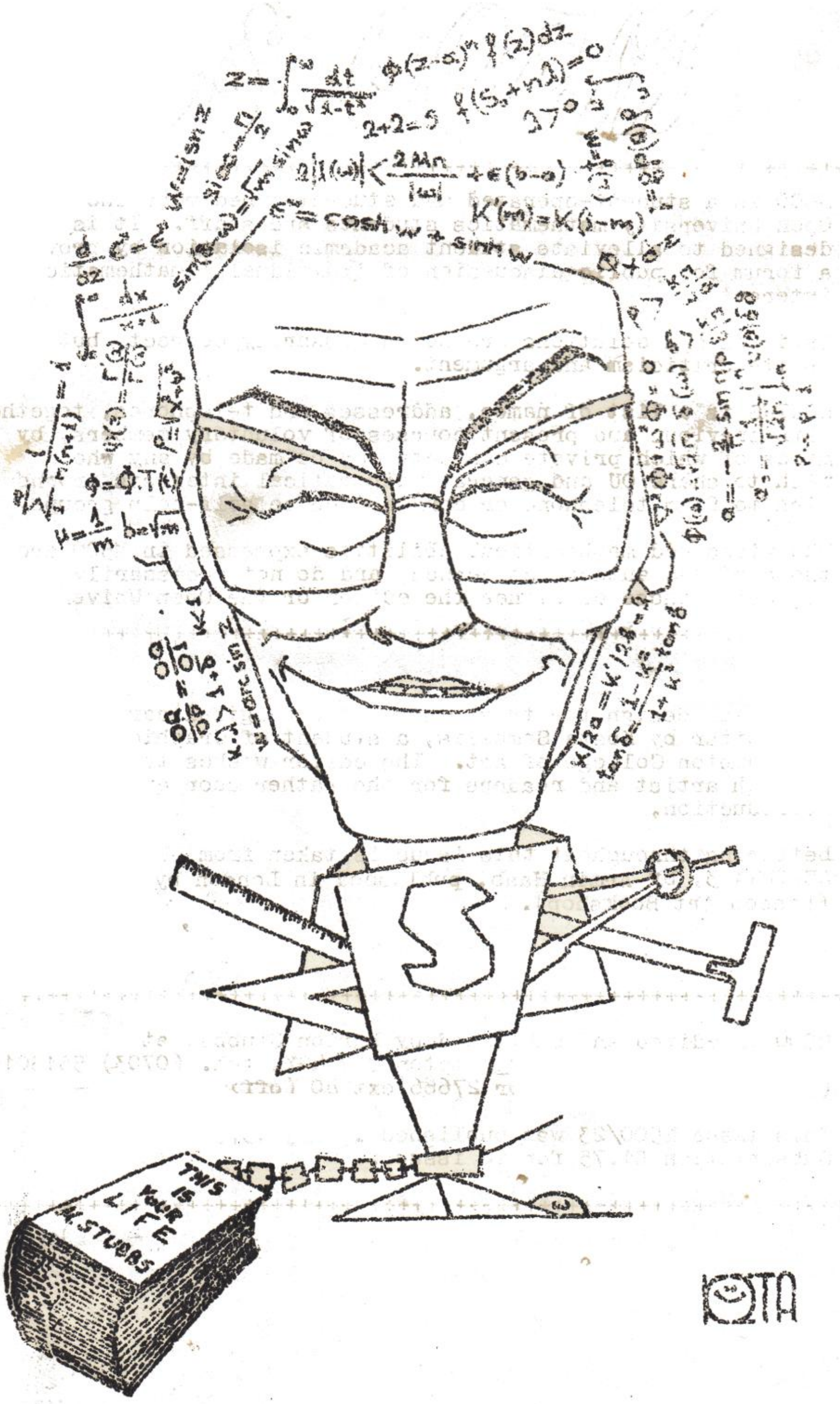

THIS IS YOUR LIFE M. STUBBS
KOTA

M500
23

M500 is a student-operated and student-owned magazine for Open University mathematics students and staff. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct, but invite criticism and argument.

MOUTHS is a list of names, addresses and telephones, together with previous and present courses of voluntary members, by means of which private contacts may be made by any who wish to share OU and general mathematical interests or who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the. authors concerned and do not necessarily represent those of either the editor or the Open University.

The cover design for this issue is an original caricature of the editor by Kosta Samoilis, a student of Graphic Design at Southampton College of Art. The editor wishes to apologise to both artist and readers for the rather poor quality of reproduction.

Lettering throughout this issue is taken from LETTERA 1 and LETTERA 3, by Armin Haab, published in London by Alec Tiranti Ltd. (London Art Bookshop).

M500 is edited and published by Marion Stubbs.

This issue M500/23 was published in May 1975.
Subscription £1.75 for 10 issues.

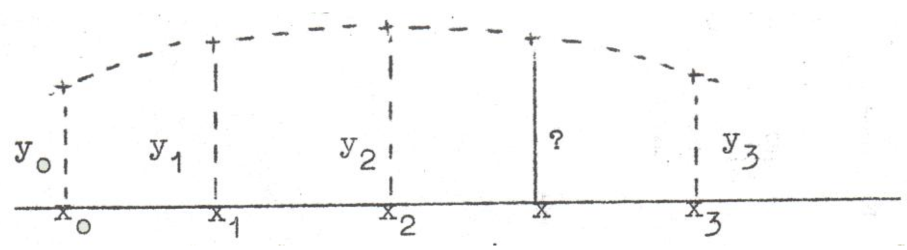
poly interpolation

I was interested to see the use of the Lagrange formula:

$$\sum_{k=0}^{n} y_k \prod_{\substack{i=0 \\ i \neq k}}^{n} \left(\frac{x - x_i}{x_k - x_i}\right) \qquad (1)$$

by two M500 readers to give an answer to the 1,2,4,8,l6,? question (M500/19). When the formula appeared in M231 02 (Spivak p. 47 problem 6), and when it occurred to me that M100 students would also be looking at interpolation polynomials in M100 04, I thought it might be interesting to tie these together and write a short account of polynomial interpolation for M500.

The basic problem is: given (n+1) data pairs (x_k, y_k), k = O(1)n; find the value of the unique n^{th} degree polynomial $p_n(x)$ such that $p_n(x_k) = y_k$, k = O(1Jn, at some point x in the interval $[\min_k\{x_k\}, \max_k\{x_k\}]$.



Lagrange's formula (1) is clearly the answer (try substituting $x = x_k$), but it can be rearranged into many different and useful forms.

<u>Newton-type formulae</u>

Let $p_n(x) = a_0 + (x - x_0)a_1 + (x - x_0)(x - x_1)a_2 + \dots + (x - x_0) \dots (x - x_{n-1})a_n$ (2)

then $y_0 = a_0$

$y_1 = a_0 + (x_1 - x_0)a_1$

....

$y_n = a_0 + (x_n - x_0)a_1 + \dots + (x_n - x_0) \dots (x_n - x_{n-1})a_n$.

We could solve these for the a_k, but if we compare (2) with (1) and equate coefficients of x^n we se that

$a_n = \sum_{k=0}^{n}(yk/\prod_{\substack{1=0\\ i\neq k}}^{n} x_k - x_i)$ = y[x_0, x_1, ..., xn], say, to indicate it's dependence on x_0 ... x_n. It can be shown that

$$y[x0, x1, ..., xn] = \frac{y[x_1,x_2,...,x_n] - y[x_0,x_1,...,x_{n-1}]}{x_n - x_0} \quad (3)$$

and the y[...] are here called <u>divided differences</u>. If we let $y[x_k] \equiv y_k$ we can calculate a table of such differences using (3):-

x_0	$y[x_0]$			
		$y[x_0, x_1]$		
x1	$y[x_1]$		$y[x_0, x_1, x_2]$	
		$y[x_1, x_2]$		$y[x_0, x_1, x_2, x_3]$
x2	$y[x_2]$			
		$y[x_2, x_3$		
x3	$y[x_3]$			

etc.,

e.g.

1	1			
		$\frac{2-1}{2-1} = 1$		
2	2		½	
		$\frac{4-2}{3-2} = 2$		$^1/_6$
3	4		1	
		$\frac{8-4}{4-3} = 4$		
4	8			

Then $p_3(x)$ = 1 + (x-1)1 + (x-l)(x-2)½ + (x-1)(x-2)(x-3)(1/6). (We can reproduce $p_3(5) = 15$, cf. M500/19/10).

If the domain values $\{x_k\}$ are ordered and equally spaced with spacing h then the divided differences are

$$y[x_0, x_1, ..., x_j] = \frac{\Delta_h{}^j y(X_0)}{h^j} = \frac{\Delta_h{}^j y(y_0)}{h^j}, \text{ say}$$

and writing $x = x_0 + sh$ in (2) gives

$$p_n(s) = y_o + s\,\Delta_h\, y_0 + \frac{s(s-1)}{2!}\Delta_h^2 y_0 + ... + \frac{s(s-1)...(s-n+1)}{n!}\Delta_h^n y_0$$

the finite difference form of M100 04.

In this case

In this case we usually construct a <u>finite</u> difference table:-

x_0	y_0			
		Δy_0		
x_1	y_1		$\Delta^2 y_0$	
		Δy_1		$\Delta^3 y_0$
			$\Delta^2 y_1$	
x_2	y_2			
		Δy_2		
x_3	y_3			
etc.				

e.g.

1	1			
		1		
2	2		1	
		2		1
3	4		2	
		4		
4	8			

Then $p_3(s) = 1+s(1) + \frac{s(s-1)}{2!}(1) + \frac{s(s-1)(s-2)}{3!}(1)$. (Again, $p_3(x=5) = p_3(s=4) = 15$).

Peter Hartley (STAFF-MOUTHS)

Ed: Sorry - this article continues for 2 more pages, and I have had to split it into two parts. The second part, entitled 'Neville–Aitken type Methods' will follow in M500/24. Peter has also contributed a follow-up paper on Interpolation and Least Squares, probably in M500/25.

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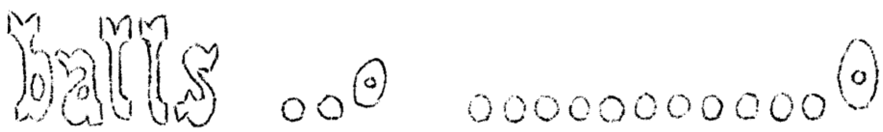
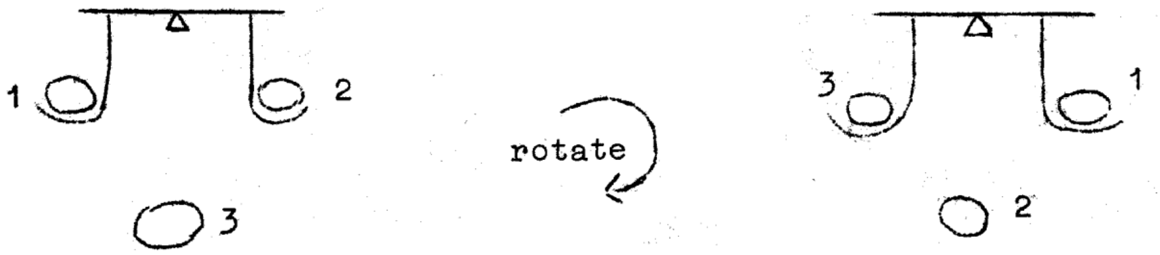


In M500/21 Steve Murphy asks if it is possible to find, in n weighings on a balance, one odd ball from ½(3n–3) balls, and to say if the odd ball is heavy or light.

The following solution is not my own but belongs to a friend I knew in Sydney - Alan Macdonald. Alan, (he is dead now), was a professional snooker player in Sydney, with no formal education past primary school. He once attempted to enrol at the University of New South Wales but we could not persuade the University authorities to admit him since he lacked any formal qualifications. He certainly did not lack intelligence as his beautiful solution to the weighing problem demonstrates.

Lemma 1: Suppose we have 3 sets of k balls each and one ball is odd. Put 2 of the sets on the balance and note position of the balance. Now rotate the 3 sets as shown and note new position of the balance.

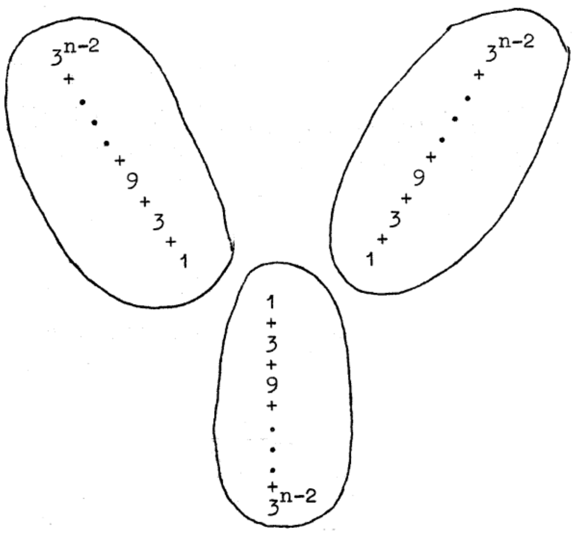


It is now possible to state which of the 3 sets contains the odd ball and also if it is heavy or light. This is just like solving the 3 ball problem with 2 weighings.

Lemma 2: Suppose we have 3^m balls, one of which is known to be heavy, then we can find the odd ball in m weighings.

Simply divide into 3 sets of 3^{m-1} balls each and attempt to balance any two of the sets. We will now know which set of 3^{m-1} contains the ball and we have m−1 weighings left to find it. After m weighings we will be down to one ball. Of course the same thing is true if the odd ball is known to be light.

Now for the solution to the whole problem. Divide ½(3n− 3) balls in the following pattern:



Put 2 of these 3 large sets on the balance and note position—Weighing No. 1. Now rotate the small sets containing 3^{n-2} balls each as in lemma 1—Weighing No. 2. If balance does not change position, rotate sets of size 3^{n-3} —Weighing No. 3. Continue until balance changes. If this happens at weighing no. r we can now say by lemma 1 that the odd ball belongs to a certain subset of 3^{n-r} balls and also if it is heavy or light. Lemma 2 tells us that n−r more weighings are enough to find the odd ball.

Richard Ahrens (STAFF-MOUTHS)

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SEQUENCES

Ref: Tony Brooks' letter about sequences, M500/19 p. 9.

I can sympathise with Tony Brooks' feelings about intelligence tests. Time was when I specialised in alternative answers to them. It always seemed to me that they were pretty good at indicating whether or not a person was thick, but not much else. As the interviewer could have seen this for himself without resorting to artifice, the whole business tended to leave me unimpressed.

As to sequences: in any IQ test there are not likely to be alternative answers, but M500 is not in the business of testing intelligence (whatever this might fee), so the chopper.is not going to come down if we write {2,4,6,8,3,...} instead of some other person's {2,4,6,7,...}.

The mapping: Rule→Sequence is (or should be) one-one, but the inverse is not so simple. This makes life interesting if we don't take it too seriously. Part of the appeal of Complete-the-Sequence problems lies in the prospect of astounding the compiler with alternative "solutions". There is no question of finding The Rule - only a rule.

Suppose we have a rule: $u_{n-1} = 2u_n$, or some such. The sequence we get is dependent only on the number we start with, the domain we are in, and where we choose to stop. For any sufficiently restrictive set of conditions there is precisely one sequence to be got. The rule may be considered as a symbolic representation for the sequence. (Who wants to write out the even integers in full?) If, now, we attempt to start with a sequence (without invoking a rule), we write something like { 2,4,6,8,...}, meaning that this is to be taken as the even integers. As Tony Brooks has demonstrated, it isn't very good notation, but it's probably as good as we're going to get. The point is, we cannot guarantee to get back to our original rule from it. A rule which yields {2,4,6,8,} followed by any old thing churned out by the rule is perfectly respectable as a solution to the problem. We need not justify it beyond demonstrating that it generates the required bit of the sequence.

So Tony Brooks' last sequence (1,**2**,4,8,18,52,206,1080,6994 followed by 4996, 801,602**,** **25,**8l,8,4, 2,1) fails as a solution. It certainly starts with the given sequence. However, no rule comes with it.

Obviously, the longer the given sequence, the less chance of finding a solution different from that intended by the compiler.

Hugh Mclntyre

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Ref: Michael Gregory's plea for interesting curves (M500/19 p.2)

Anyone with a month to spare might care to plot x^x for negative x, especially for non-integers.

H.McI.

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MAILBAG

As a recent subscriber to M500 I cannot but be impressed by its standard and content. I am amazed, not only by the effort which must go into the development and solutions of the problems, but also by the seemingly endless maths courses which the average contributor has passed. Having scraped through A100 and withdrawn half-way through M100 last year, I am filled with admiration.

I am looking forward to any mathematical meetings which are arranged and the Birmingham weekend in September. Meanwhile I would like to recommend any local maths or non-maths people to come to an inter-disciplinary lecture on mathematical models at Kingston Study Centre, Carbury Park Road, at 7.30 pm on Thursday 22nd May. It is to be given by Professor Crank of Brunel University, who has proved an excellent speaker on previous occasions. Admission is free and there is a bar on the premises.

(Mrs) Chris Stockbridge

Ed: What mathematical meetings??? News welcomed. As to the B'ham '75 Weekend, I'm inclined to say there won't be one in 1976 if you don't buck up a bit. 41 concrete bookings so far, backed by cheques–some post-dated as far ahead as July, which is OK by me, but just not enough. You can't provide 12 staff for 41 students. Come on, lads. Fortunately I hear that OUSA has at least 20 more for me, otherwise my deposit of £50 paid out of my own pocket, mad fool that I am, would be in danger. I've recently discovered that our date clashes with the Jewish New Year, for which profound apologies, but blame that London Day School on Sept. 13th which left us no choice.

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I was at first surprised that Peter Hartley hasn't received any calls as a STAFF-MOUTH. On closer examination, however, I saw that he is tutoring M100, M201 and M231, which are, I think, the easier courses. I personally have about half-a-dozen calls a month, mostly from M100 and M202 students in the South-East. The concensus of the enquirers seem to indicate that where they are bogged down, they do not want to know why this is so, but merely to talk about the particular reading passages or SAQ to clear the air a bit. I would very much like to have more M500/M0UTHS readers 'phoning in, as it gives me a good opportunity to freshen up on what I've done–at the moment I seem to have a select circle of 'phoners.

Hugh Tassell

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It was with great relief that I read your justification for inclusion of HOOPS articles (ie that it is original research, to be filed for future comprehension–M500/20). I became a subscriber some time before the beginning of this saga, but have not ceased to be bewildered by it all until now.

My first (very first, momentary) impression was that of an error having occurred in the very efficient typing machine (permit the thought). This was followed quickly by the feeling that perhaps it was all a 'Con' , a very clever parody on the Groups we were all so involved in some time ago–quite an appealing idea this, in view of the childhood connotations of carefree days with our hoops and little pieces of wood to bowl them with!

However, the steady and frequent recurrence of the topic began gradually to undermine my innermost convictions that I might be making some headway in this mathematical world of abundance and diversity into which I have plunged myself. Here was something being freely discussed and presented as though it were on everybody's tongue these days, and I had never even heard the word in such a context!

You may say why on earth didn't I write before and air my misgivings since this is what M500 is all about. Well, I am wondering whether the reluctance to admit to shortcomings is a function (the exponential kind?) of experience, or learning.

I can recall many parries made in the past, by lecturers, graduates, etc.:

"Well, of course, that is more of a puzzle than an A-level question." (No solution offered by Head of Maths Dept. at a College

or, "I never did like Group Theory." (This to ward off any particularisations which might be made by me in conversation arising out of an enquiry re my current studies–again a College lecturer.)

"Well, shall we make a start on what I have prepared and we'll come back to that later if we have time." (We didn't have time to pursue this enquiry made before class!)

or - one of my favourites, of OU origin: "FIGURES FRIGHTEN ME TO DEATH!" (or putting oneself 'off the spot' when in confrontation with mental arithmetic, etc!)

I am thinking of arming myself with such replies, ready for the time I need to defend my position on the B.A. pedestal. But seriously, it does seem rather sad that concepts which one grasps so well and becomes so involved with can slip away so easily in quite a short time. Obviously, the thing to do is to specialise! Anyway, I shall return to past copies and re-read the HOOPS articles with new hope.

I like the fact that some of M500 is above my head–like playing table tennis against a more able opponent. I am pleased to see the very healthy-looking MOUTHS list and am very happy to subscribe.

Maureen Childs

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Although, as yet, I don't understand a line of the mathematics in M500, perhaps intermittent exposure to it will engender some familiarity with the symbols–rather like learning Arabic.

It is a most enjoyable, lively production; I did not realise that mathematicians had such a good sense of humour.

It is about 40 years since I did any mathematics, so I would like to remain an unseen, unheard non-M0UTH at least until a pass in MDT241 gives me some assurance in answering the telephone to anyone numerate. So perhaps, with luck, next year I shall be able to apply for the full treatment.

As you will see, any sort of contribution is quite beyond me, but would you accept an answer to the question: "What is Maths?", ie:

'Maths is a figure of speech.'

I'm sure someone has already thought that one up; someone always has. Keep up the good work. M500 is a delightful change.

"MOKE" (MDT2U1, following D-courses)

Ed: We now have 305 subscribers, of which 29 are non-M0UTHS, and of the 29 at least 8 were staff, at the last count. Most non-MOUTHS write little notes like the above, and seem exceptionally timid. Would anyone care to work out the probability of being selected at random from the MOUTHS list by a ferocious, numerate student demanding an answer to a question? Incidentally, TM221 please don't forget to ring me! My calls all seem to be editorial ones, but I like computing, and can answer at least some questions on it, inc. M100 & M251.

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I think that M500/21, which saw it come of age, was quite the best yet, and if it is any sign of the developing maturity of M500, it promises well. What an interesting piece of comparison by Alan Slomson; could I ask him where in the areas of pure or applied mathematics he thinks the OU ought to provide more courses–it is obvious that the field of statistics needs developing, and I believe that a course team is being brought together to do this. Incidentally, it would be interesting to hear of such developments from the OU itself?

I was lather horrified to see, in Dr. Elizabeth White's letter, a suggestion that M202 was to disappear! Is this really true? I was rather hoping to do it in 1976, so reassure me that it will still be there if I survive my current bombardment of calculus in M231.

Willem van der Eyken

Professor Pengelly writes:

"I would like 'to take this opportunity to make the Faculty policy on M202 absolutely clear. This course will certainly not disappear after 1976. In fact the current Faculty view is that it should be allowed to run for "its natural lifetime". This means that the course will almost certainly run up to and including 1978 and possibly even in 1979, provided that student numbers do not become too small. In order to combat this latter danger, I have asked Ian Dey and John Mason to prepare some promotional material on M202 for use in this year's Summer School exhibition."

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I am writing with rather an unusual request. A publisher has commissioned us to provide puzzles for a new puzzle magazine; this will "be very different from GAMES AND PUZZLES–simple puzzles, about 50 per issue, but all on a theme. We are looking for possible contributors, able to provided given agreed number of puzzles on a given theme to a time limit, I am wondering whether M500 readers know of anyone who might be interested in contributing?

Payment would be £1,75 per puzzle, or £2 per puzzle if no editing to speak of was required at this end. The first four subjects, just to give you an idea, are Sport, Holidays, Pop Music/TV and Crime. If you know anyone who wants to get to work straight away, we can take puzzles for the last two of these provided they arrive before May 5th.

If anyone can suggest any names, I would be delighted to hear from you.

I continue to find more of interest in M500 than I have time to study as closely as I would like. Keep up the good work.

David Wells (Puzzles Editor), GAMES & PUZZLES

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PEEVE

Why do mathematical course writers liberally and illogically besprinkle .their work unnecessarily with Greek letters? I am tempted to ask this question after reading a proof in M332 which used 8 Greek letters, k Roman lower-case and 2 Roman upper-case letters. I would not mind if there were some logical consistency in their use, but on the page in question, 2 Greek letters represented real variables, 3 were real function identifiers and 2 were complex function identifiers. And of the 4 Roman lower-case, 2 represented real variables and 2 were complex variables. The rationale of this usage baffles me. Neither does it assist learning when I get halfway through a proof to find that I cannot remember whether 'h' is real or complex and have to back-track to find out. Would it not have been possible to use the convention that Roman = real and Greek = complex (for example)?

By analogy with computer programming (not so dissimilar), it is common practice for experienced programmers to choose meaningful identifiers for theip variables, as this makes, programming easier and minimises careless errors. In my view it would make life easier for students if mathematicians did the same.

I know that to some extent tradition and wide-spread existing usage have to be taken into account, but my impression from M100 days is that the OU is not afraid to throw tradition overboard if it prevents efficient learning.

✳ Mathematics has been described as the 'Queen of Sciences'—supremely logical—capable, even, of proving irrefutably that something exists without anyone knowing a single example of that 'something'. It seems to me, then, that it would be no bad thing to remove the spots from the 'Queen's' face by being logically consistent in the use of notation.

Would any professional care to comment on the desirability and/or practicability of some convention as indicated above?

✳ Note: Queen not King. MGPs can foam at the mouth here.

Bill Shannon

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I STARTED LOSING MY HAIR AT THE AGE OF SIXTEEN

An American professor of statistics was getting over to the first year course that a statistical correlation between two series of events does not necessarily establish a causative link.

"Just because smoking of cigarettes and lung cancer are correlated", said the professor, "does not of itself prove that smoking causes cancer. Of course, the medical profession are agreed that there is indeed a direct causative link between the two, but the statistical correlation itself does not prove it."

"You may have noticed", he continued, "that the front seats of the vaudeville, are always filled with bald-headed men. Now, if you believe that smoking causes cancer because the two are correlated you would conclude that looking at girls in tights makes your hair drop out. In fact, it is the over-activity of a certain hormone which both makes your hair drop out and makes you want to look at girls in tights."

Would an attractive M500 female like to meet up with a bald headed M500 male?

Bob Escolme

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MATH-STRACT - Hugh Mclntyre

Here is an extract from a poem by Francis Thomson—"The Hound, of Heaven". It describes perfectly, I think, a particular stage in the mathematical learning process:.

Yet ever and anon a trumpet sounds
From the hid battlements of eternity:
Those shaken mists a space unsettle, then
Round the half-glimpsed turrets slowly wash again.

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OPUS

The working programs "below have now been produced, and TM221 students who have not yet tried them are invited to experiment:

1. MINILOADER to load the LOADER (or to load any other suitable program) direct from the keyboard. 27 instructions by M. Stubbs, 21 instructions claimed by "Ernie"-met-at-Liverpool, whose address is not known. Several miniloaders of 15 words are claimed by Walton Hall, and one has been supplied to M500, but it does not load directly and only from the keyboard. The miniest Miniloader which uses only the keyboard will be published in M500, so please send them in. The 15-word 'Loader C' from Richard Maddison will also be published. It follows the basic 15 words with 52 words of mixed keyboard and switches, so that it is educational if not practical.

2. DIGITAL 1-SECOND STOP WATCH. Programmed by M. Stubbs and B. Chinchen independently, but is now flowcharted in Radio 7, so not worth mentioning! 56 instructions, accuracy about ± 2 seconds in 10 minutes, ie 0.003 secs per sec.

3. ALPHABETICAL DISPLAY ON M1 AND M∅. Done by "Ernie" with special designs for letters involving diagonals. Best to use lower case, not caps.

more opus

1. Locations 010_8 and 011_8 hold a double length integer, the most significant part being in 010 and the least significant in 011, in two's complement form, ie bits 7 to 0 of 011have values $2^7 \ldots 2^0$, and bits 6 to 0 of 010 have values $2^{14} \ldots 2^8$, and bit 7 of 010 has value -2^{15}. Similarly, locations 012 and 013 hold another double length integer.

   (a) Code a subroutine to add the two double length integers and store the result in (say) 014 and 015.

   (b) Code another subroutine to form the two's complement (ie negative) of a double length integer.

   More fancy versions of this can be invented–eg using registers as pointers to the even numbered locations.

2. Take two 8-bit numbers whose product is known to be less than 8 bits. Code a program or subroutine to multiply the two numbers and store the product.

3. Take two 8-bit numbers whose product requires more than 8 bits. Code a program or subroutine to multiply the two numbers and store the product as a double length 16-bit number.

4. More difficult, and preferably needs knowledge of material up to about Unit 10 of TM221. I intended it for Unit 15.

The OPUS is to be used to simulate a minicomputer terminal.
The keyboard simulates the terminal keyboard.
The mintrons simulate the terminal printer.
The data switches simulate the characters transmitted as output from a main computer to a terminal.
The data lights simulate the characters transmitted to the main computer from the terminal.

The main computer is the master in a master–slave relationship, controlling communications along the line between the master main computer and the slave terminal. If it transmits the control character (a particular ASCII pattern is set on the data switches) that selects the minitron/printer then all (data) characters from then till an end-of-message character are copied from the line (data switches) to the minitron/printer. If it transmits the control character that polls the keyboard (another ASCII pattern on the data switches) then both the keyboard-ready light comes on if the human operator has indicated that he/she wants to input a keyed message (for example by operating a control console switch in a particular way), and an ACK acknowledge ASCII character is transmitted back (ie displayed on the data lights each for say 0.1 sec).

If on the other hand the character to poll the keyboard is transmitted by the master main computer (set on the data switches) but the human has not indicated he/she has an input message pending, then a NAK negative acknowledge character is transmitted back (ie displayed on the data lights for 0.1 secs).

If the above is too complicated to get into OPUS store then maybe a simpler version is needed. I haven't tried coding the version above, though I did try coding a similar problem for one of my earlier instruction sets for OPUS, which was not the instruction set that I finally recommended or was used on OPUS.

In ASCII, ACK is 006_8 and NAK is 025_g. There is no single character for "end-of-message" in ASCII, and the use of an end-of-message character in the above problem is an over-simplification. I suggest you use 004_8.

Richard Maddison

Maths Faculty, Walton Hall.

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PROBLEMS

23. 1 "FOR M100 READERS" - Tony Williams

If you have reached Unit 12 ana have usecf the computer you may like to try the following, which was inspired by an assignment question in last year's M100:

Let p be the function p: $x \to \frac{x}{\ln x}$ $(x \in R^+, x \neq 1)$

1. Derive an expression for p′(x), the first derivative.
2. Using this expression write a program to compute values for the following:

   (i) (b-a)p′(a) (ii) $\frac{b}{\ln b} - \frac{a}{\ln a}$

   where a,b are large and positive (take as > 10) and a < b.
3. Do your results in (2). show that (b-a)p′(a) is less than, equal to, or greater than $\frac{b}{\ln b} - \frac{a}{\ln a}$? (Try various values, eg a=100, b=1000; a=10000, b=10001)
4. Show that, theoretically, there is only one answer to (3). Does the computer agree?

   (No solution supplied).

23.2 PULLING THE CHAIN - L.S. Johnson

A flexible chain, length L, lies coiled on a smooth table. One end is pulled horizontally at a constant speed V. Total mass of chain M, hence final kinetic energy T = ½MV^2. Now, neglecting all friction effects, the force required equals rate of change of momentum, say $P = \frac{d(mv)}{dt} = v\frac{d(px)}{dt}$ where p = the linear density and x the length in motion. Therefore P = Vp dx/dt = V^2p. The distance through which this force is applied is L. Work done = $LV^2p = MV^2$, twice the K.E. To where has the balance gone?

23.3 HOW LONG- IS A PIECE OF STRING?- Dick Terry

A rope passed over a pulley has a weight at one end and a monkey at the other. There is the same length of rope on either side of the pulley, and equilibrium ie maintained. The rope weighs 4oz per foot.

The age of the monkey plus the age of the monkey's mother is 4 years, and the monkey weighs as many pounds as his mother is years old. The monkey's mother is twice as old as the. monkey was when the monkey's mother was half as old as the monkey will be when the monkey is three times as old as the monkey's mother was *when the monkey's mother was three times as old as the monkey.*

The weight of the rope plus the weight at the end is half as much again as the difference in weight between twice the weight of the weight, and the weight of the monkey.

How.long is the rope?

... If you detect any ambiguity, take either alternative. (Filched from an ancient electronics magazine).

23.4 "IT IS OBVIOUS!!" - Datta Gumaste

Given: u_m = {a: a < m and a is relatively prime to m; both a and m are positive integers}

(u_m, ⊗) is a group where ⊗ is "multiplication modulo m". M, n are relatively prime.

Prove or disprove: u_{mn} is isomorphic to u_m x u_n where u_m x u_n is the direct product of u_m and u_n ... ... (A)

If (A) is true, show that φ(mn) = φ(m)φ(n) where φ is Euler-φ function, and m, n are relatively ptime ... ... (B)

PROVE (B) even if (A) is false.

If (B) is true, you will have proved that

Euler φ function is a morphism ... ... (C)

If (C) is true, it is immediate that Euler was a Morphisian! ... ... (D)

DO NOT PROVE (D). It is obvious!!

23.5 DUEL STRATEGY - Richard Ahrens (STAFF-MOUTHS)

Three men, A,B and C, decide to fight a pistol duel along the following .lines. They will first draw lots to determine who fires first, second and third. After positioning themselves at the vertices of an equilateral triangle, they will fire single shots in turn and continue in the same cyclic order until two of them have been hit. The man whose turn it is to fire may aim wherever he pleases. Once a man has been hit, whether killed or not, he takes no further part in the duel. All three men know that A always hits whatever he aims at, B is 80% accurate, and C is 50% accurate.

Assuming that all three adopt the best strategy, and that no-one is hit by a shot not aimed at him, who has'the' best chance to escape unscathed? What is the exact probability of escape of each of the three men?

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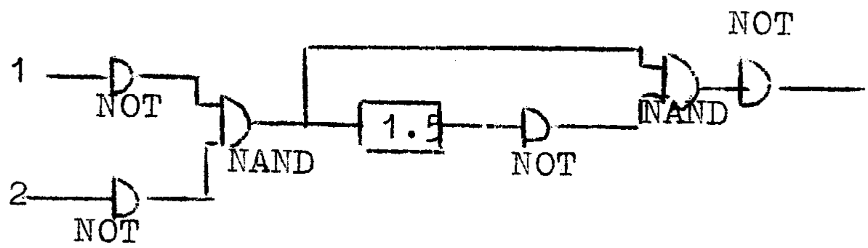
SOLUTIONS

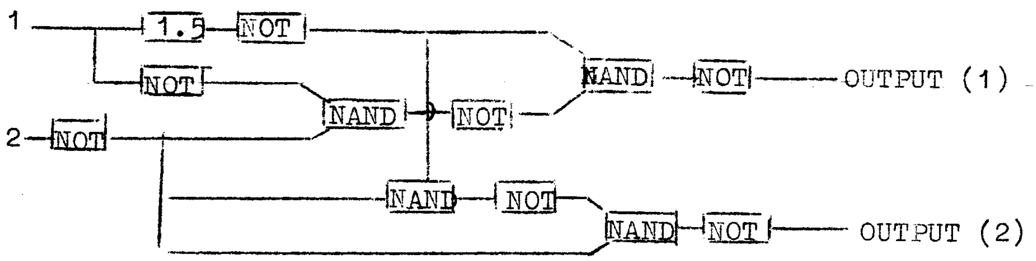
22.1 BLACK BOX - the following are all solutions received by 24/4/75.

(1) - Michael Gregory



which is fairly easy.

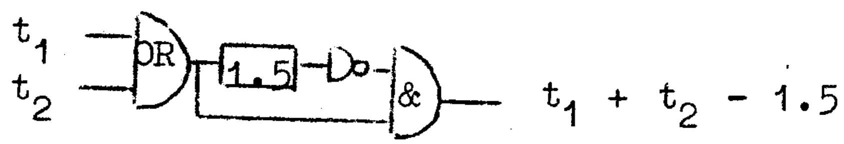
(2) - Hugh McIntyre



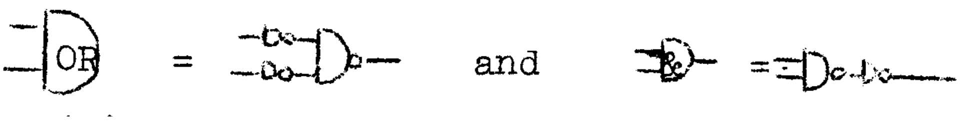
The question does ask "how would you construct the "black "box?" What interests me more is "Why?", ie how did the problem arise?

(3) - John Parker

(1) This is easily achieved by allowing the output to be t_1 or t_2 inhibited for the first 1.5 time units.

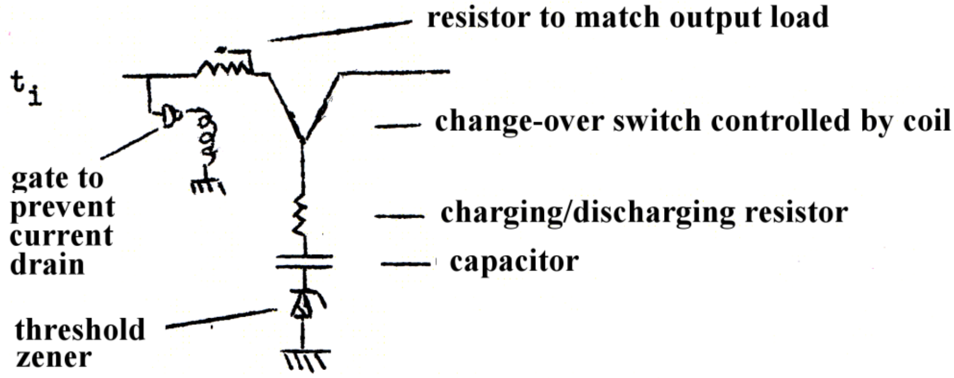


where



Output (2) by the same argument should be t_1 followed by or preceded by 1.5 inhibited by t_2. So we. need output to commence with the onset of t_2 that t_2 can appropriately inhibit output. Hence we need a box $\boxed{t_1}$, where a pulse of duration t_1 is output when a signal is input.

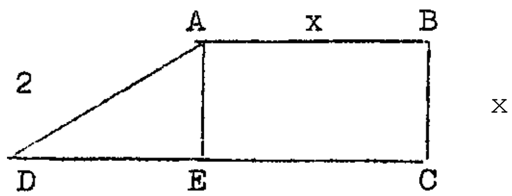
Delaying t_1 or/and t_2 for n x 1.5 ($n \in \omega$) can never solve the problem, for it can never cause the leading edges of t_1 and t_2 to coincide, unless t_1 happens to be 1.5. Such a box could possibly be constructed thus:



Of course, voltage levels and component values would need to be chosen carefully to be compatible with the logic circuit. With such a box it is fairly easy to construct output (2)

<u>22.2 QUADRILATERAL</u> - <u>all</u> solutions received by 24/4/75

<u>(1) - Jerry Humphries</u>



CD $\not<$ CE otherwise area ADCD $< x^2 < 2^2 < 4$.
Area as drawn = $4 = x^2 + \sqrt{4 - x2}\ (x/2)$. Therefore $x = 2$ or $\sqrt{3.2}$

<u>(2) - Steve Murphy</u>

(No diagram supplied).
Put AB = h, AD = h+l (l <u>might</u> be negative). Then $h^2 + ½hl = 4$ and $h^2 + l^2 = 4$, so that l = 0 or l = ½h. The former gives l = 2 (square) and the latter $l = 4/\sqrt{5}$ (a trapezium, in fact).

<u>22.3 SKETCH GRAPH</u>

Two or three solutions received, one of which suggested, perhaps humourously, that it should be used as a future cover for M500. Good idea. Watch with bated breath for it– probably M500/24. Have you read page 18 of this issue, requesting special items for 24?

22.4 FIND THE NEXT TERMS - all solutions received by 24/4/75

(9) 1,4,11,20,31,44, 61,100,121,144, 171,...

Ans: 220, 251. Rule: Natural squares, Base 8. (J.Humphries)

(10) 1,1,1,2,1,2,1,3,2,2,1,4,1,2,2,5,...

Ans: 1,2. Rule: Put the sequenoe in one-one correspondence s with the natural numbers. Put a 1 where there is a prime and fill the gaps 2,2,3, 2,2,4, 2,2,5 and so on. (Jerry Humphries' friend - Steve Hobbs).

Ans: 2,2, Rule: $u_{4n} = (n+1)$ $(n \in Z^+)$ and

$$\frac{u_{4n-1}}{u_{4n-3} + u_{4n-2}} = n+1 \quad \text{subject to}$$

$u_r > 0$ and $u_{4n-3} + u_{4n-2} + u_{4n-1}$ being a minimum. Thus $2 + 2^2 = 6$ gives the terms above. (Steve Murphy)

(12) 4,6,7,9,10,11,12,14, 15,16,17,18,19,20,22,23,24,...
Ans: 25,26,27,28,29,30,31,32, 33, 35,36,37,38,39,40 Rule: Natural numbers after removing the Fibonacci sequence generated by 1,2. (J. Humphries, H. Mclntyre, S. Murphy)

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M500/24

Very little is in hand, apart from 3 or 4 mathematical articles and a few short pieces. We are down to the last 4 problems in the file, some of which (from our prolific problem-setters) have been waiting since 1973. The time seems good to ask for immediate shorrt comments from students and staff subscribers for a special CON RES issue No. 24. This may or may not reach you by May 31st, but could be useful in case of hesitation before Sept. The idea has been inspired by the following suggestion from Prof. Pengelly–but note that it is only a tentative idea:

"We might consider promoting an M500 project. In particular, a project concerned with providing registration advice to students. What I have in mind is a special issue of M500 that the Faculty would distribute to all students which would include material written by students, part-time staff and central staff on the courses currently available. The material could discuss the courses themselves, useful patterns of courses, and could give views on why it is useful to take particular course patterns, what students have got out of the various courses and the relations between the courses. If this sort of project is to have any real impact, then it would be necessary to produce it as M500 is at the moment, under an independent editor, so that it is not seen as simply yet another piece of faculty publicity, even if that's wnat it would be."

Ed. says-- not so - it would simply be a piece of M500 publicity! 200 words maximum per person, please. COUNT THEM! Thanks - M.

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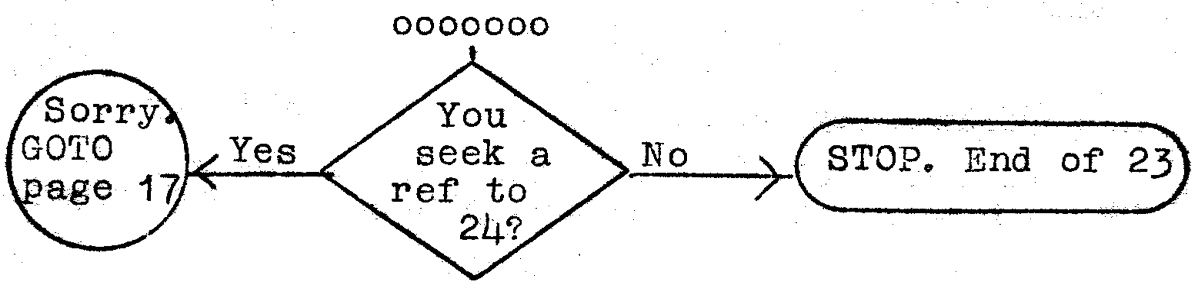
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Yes
Sorry. GOTO page 17
No
STOP. End of 23