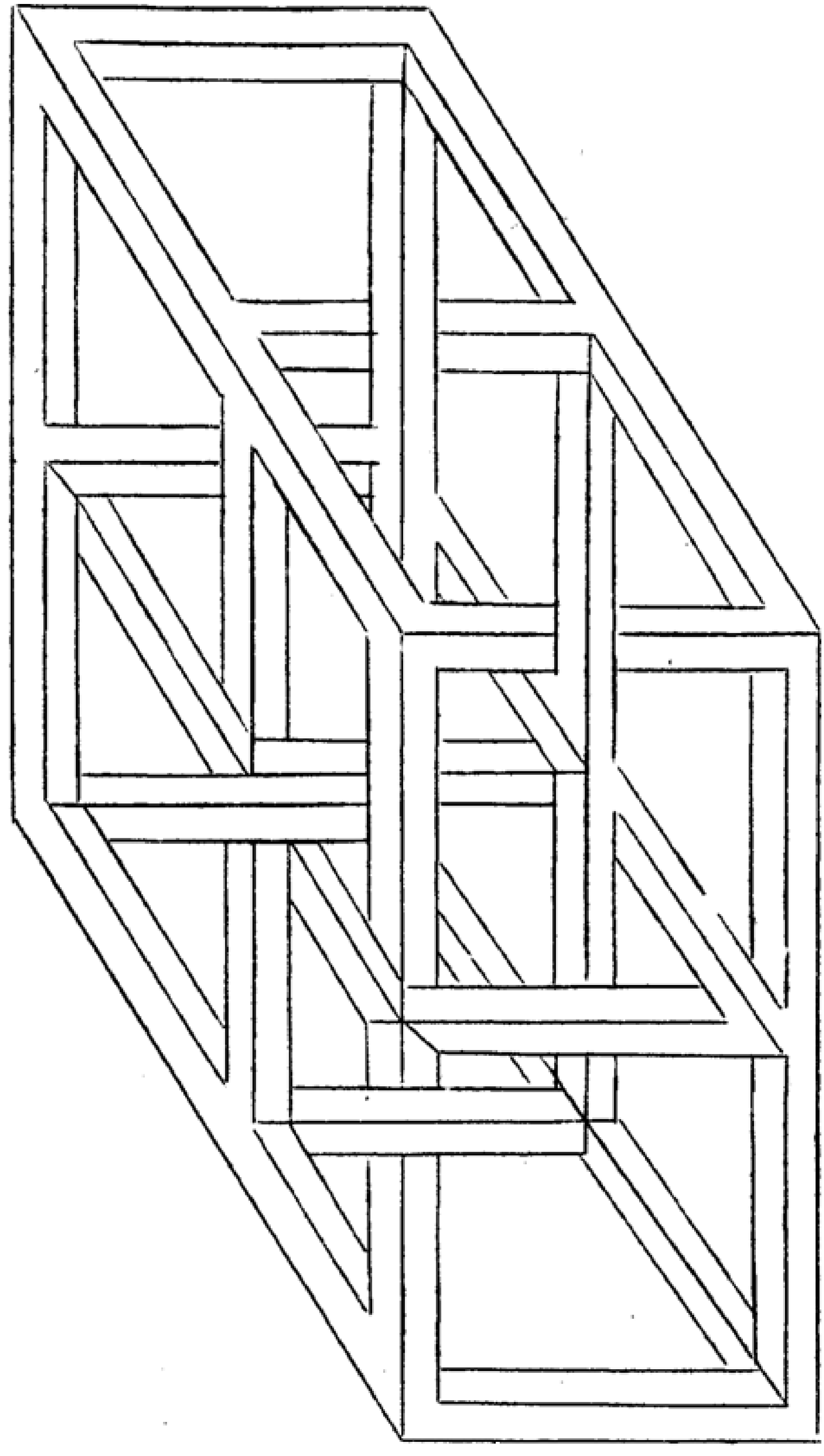

M500 24

M500 is a student-operated and student-owned magazine for Open University mathematics students and staff. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct, but invite criticism and comment.

MOUTHS is a list of names, addresses and telephones, together with previous and present courses of voluntary members, by means of which private contacts may be made by any who wish to share OU and general mathematical interests or who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned, and do not necessarily represent those of either the editor or the Open University.

The cover design for this issue is by Tony Brooks whose comment on this and similar 'impossible' figures is that they are only impossible in the sense that the brain tries to interpret them as three-dimensional figures. This is because we are so used to seeing two-dimensional representations on paper of three-dimensional objects. As a result, we tend to forget that a surface does have only two dimensions and that a diagram consists only of lines in those two dimensions. The third dimension is supplied to a diagram by our own interpretation and designs like the one on the cover show how easy it is to mislead the brain.

M500 is edited and published by Marion Stubbs

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Membership: M500 - 316; MOUTHS - 294 members

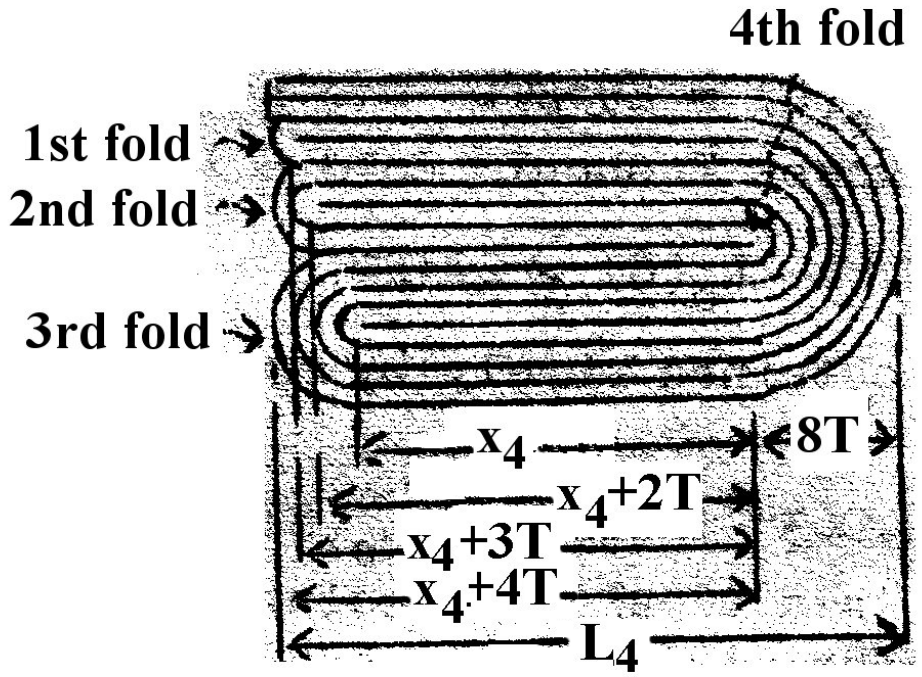
PAPER FOLDING

Can a sheet of paper be folded 8 times in half? In practice, probably not. Distorted - yes. Folded - no. The following is a mathematical investigation of the problem.

We establish 3 rules at the outset: Rule 1 - folding shall be uni-directional (an expedient, since we have not looked at alternate idirection fo1ding); Rule 2. - we can fold (bend) paper to zero inside radius; Rule 3 - we ignore friction.

Start with length L_0 and thickness T, and fold once, in half.

Repeat n times. The sketch below shows the situation after 4 folds. Rule 2 gives the ompressed profile - Rule 3 guarantees that the original edges stay level with each other.



For n= 4 we have (writing x for x_4):

$L_0 = 8x+4(x+2T)+2(x+3T)+2(x+4T)+\frac{\pi T}{2}(4+3.3+2.5+2.7+9+11+13+15)$,

$= 16x + \frac{T}{22} + \frac{85\pi T}{2}$, and

$x = L_0/16 – 11T/8 – 85\pi T/32$.

But $L_4 = x + 12T$, so $L_4 = L_0/16 + \frac{T}{32}\ (4 - \pi)\ 85$.

By studying L_n for successive folds we get the following relations:

$L_n = \frac{L_0}{16} + \frac{T}{2^{n+1}}\ (4 - \pi)\ K_n$ - - - - - - - - - - - - - - - - - (1) (n >1)

where

$K_n = 2n - \sum_{a=1}^{n-1}(n-a)\ 2^{2a} = 4K_{n-1} + 1$ - - - - - - - - - - - - - - (2) ($K_1 = 1$)

$$x_n = L_n - 3.2^{n-2}T \quad - \quad (3)$$

$$x_n = \frac{L_0}{2^n} + \frac{T}{2^{n+1}}\ [(4 - \pi)K_n - 3.2^{2n-1}] \quad - - - - - - - - - - - - - - - - \quad (4)$$

Now, if $x_0 = 0$ we cannot form the $(n+1)^{th}$ fold, so we let $x_n = 0$, and denote by $L_0(n)$ the initial length which yields $x_n = 0$.

Using (4) we get

$$L_0(n) = T(3.2^{2(n-2)} - \tfrac{1}{2}(4 - \pi)K_n) \quad - - - - - - - - - - - - - - - - - - - \quad (5)$$

If we have already performed the 8th fold, $x_8 \geq 0$, so let $x_8 = 0$, and from (5) we get $L_0(8) = 2912T$. The required condition, that a paper of thickness T should fold 8 times, is thus:

$$L_0(8) \geq 2912T.$$

The general case is ohtained merely by writing ≥ for = in (5).

Newsprint is about 0.003" thick, so $L_0(8)$ for this ≥ 8.736" (222mm). Obviously this idealistic solution could not be folded in practice with values close to the equality, but this is more a consequence of the physical structure of paper than (we hope) of the mathematics.

.Hugh Mclntyre

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SPLINE FUNCTIONS

With referance to the article (by Peter Hartley in M500/23) and planned articles (M50O/24 and 25) on polynomial interpolation, I would like to say a few words on another very powerful method, namely; that of spline functions. However, until I can ‘move about’ with confidence in the necessary normed spaces (roll on the time when I can commence M331) I am obliged to stay at a low level and consequently will not mention convergence.

Basically a spine function s(x) is a piecewise polynomial function where pieces of the polynomial are joined together so as to possess as much overall smoothness as possible without the function being globally a polynomial. Thus, instead of fitting one polynomial through n knots (a 'spliner's' term for data point) we are fitting (n-1) polynomials through the n knots with a necessary loss of continuity at the knots.

For instance with a cubic (or cubic natural) spline s(x) would be in C^2 between the knots, but at each knot only continuity of slope would be possessed (ie s(x) in C^1.

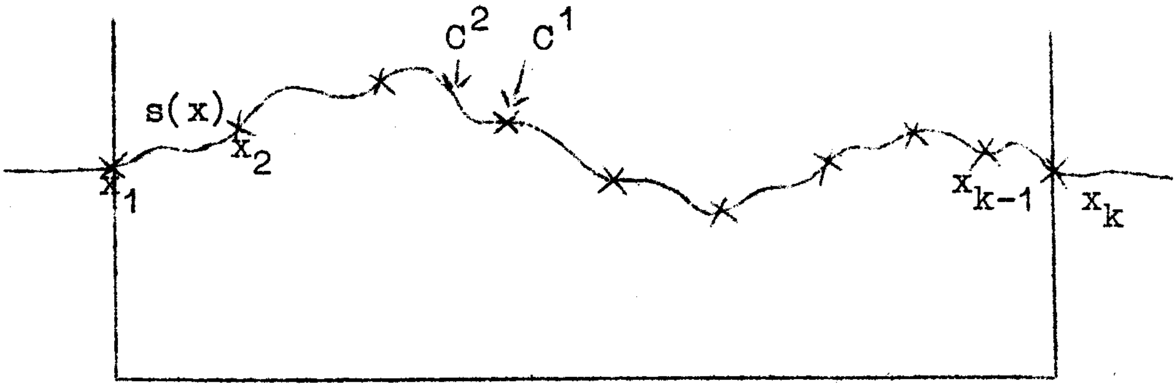
Definition:

Given a strictly increasing sequence of real numbers $x_1, x_2, \ldots, x_k$ ($x_1 < x_2 < \ldots < x_k$) a spline function s(x) of degree n with knots $x_1, x_2, \ldots, x_k$ is a function defined on the entire real line having the two properties:

(i) in each interval (x_i, x_{i+1}) for i = 0, 1, ..., k where $k_0 = -\infty$, $k_{k+1} = \infty$, s(x) is a polynomial of degree n.

(ii) s(x) and its derivatives of orders 1, 2, ..., n−1 are continuous everywhere, $s(x) \in C^{n-1}$.

One particular sub-class of splines is of particular interest in interpolation. This is the natural spline function. Other sub-classes are b-splines and l-splines. One advantage in using, say, a cubic natural spline (a natural spline differs from a spline inasmuch as in the intervals $[x_0, x_1]$ and $[x_k, x_{k+1}]$ the polynomial s(x) is linear, ie $s''(x_{i+1})$, $s''(x_i)$, $s''(X_{i+1})$ is tridiagonal which facilitates a very elegant solution using a slightly modified form of LU decomposition.



The main advantage in using a spline stems from the disadvantages of numerous and severe undulations when fitting a polynomial of high degree to a large number of data points as in Lagrangian-type techniques (although interpolating splines are really only suitable for computer usage).

Splines are also used in two point boundary value problems such as

$$y''(x) + p(x)y'(x) + q(x)y(x) = r(x), \qquad y(a) = \alpha,\ y(b) = \beta,$$

where s(x) is forced to satisfy the differential equation at the knots $x_1, x_2, \ldots, x_k$.

Similarly, splines have been used in the solution to the one-dimensional heat conduction equations

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

where in the space domain a combination of second derivatives of splines are used to approximate $\partial^2 u/\partial x^2$. The t-derivative is replaced by a normal central difference scheme.

References: Ahlberg, Nilson and Walsh: Theory.of splines and their applications. (The Spliners'"Bible") or a more readable text––Greville, T.N.E. Spline functions. Academic Press.

Brian Heward

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poly interpolation

(Continued from M500/23. The basic problem is: given (n+1) data pairs (xkyk), k = O(1)n, find the value of the unique n^{th} degree polynomial $P_n(x)$ such that $P_n(x_k) = y_k$, k = O(1)n, at some point x in the interval [$\min_k \{x_k\}$, $\max_k \{\{x_k\}$].)

Neville-Aitken type Methods

Denote by $p_{ij}(x)$ the polynomial interpolating the data (x_k, y_k), k = i(1)j; p_{ij} is of degree j– 1. Then

$$P_{i,j}(x) = \frac{(x-x_i)p_{i+1,j}(x) - (x-x_j)p_{i,j-1}(x)}{x_j - x_i} \qquad (1)$$

Proof: Let $x = x_i$, x_j and x_k ($i+1 \le k \le j-1$) in turn, where i = O(1)n, $j = \overline{i+1}$(l)n and $P_{kk}(x) \equiv y_k$.

This scheme is usually applied for a given value of x, producing a table:

$x - x_0$	y_0				
		p_{01}			
$x - x_1$	y_1		p_{02}		
		p_{12}		p_{03}	
$x - x_2$	y_2		p_{13}		p_{04}
		p_{23}		p_{14}	
$x - x_3$	y_3		p_{24}		
		p_{34}			
$x - x_4$	y_4				

etc.

the left-hand column being preferred to the x_k themselves because of the form of (1).

Eg: data as before, x = 5, and the sequence to be fitted is 1,2,4,8,....

4	1				
		5			
3	2		11		
		8		15	← value of cubic, as before.
2	4		14		
		12			
1	8				

This method is very popular because the 3rd column are all linear interpolates (using successive pairs of data points), the 4th column quadratic interpolates, etc., and in a more practical (!) problem these can indicate the success or otherwise of the interpolation process. Here is a more typical problem:

Estimate p(3) from this data:

t	-3	-2	0	2	5	10
p(t)	0.407	0.423	0.454	0.485	0.530	0.602

<u>3 - t</u>	<u>p(t)</u>					
6	0.407					
		0.5030				
5	0.423		0.4980			
		0.5005		0.5010		
3	0.454		0.5005		0.5005	
		0.5005		0.5003		0.5004
1	0.485		0.5002		0.5003	
		0.5000		0.5002		
–2	0.530		0.5002			
		0.5012				
–7	0.602					

ie it is very likely that p(3) = 0.500 correct to 3D.

Interested readers might like to consult "Theory and applications of Numerical Analysis" by Phillips and Taylor (Academic Press, 1973) Chapter 4.

Peter Hartley (STAFF-MOUTHS)

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ESSAYS IN MATHS COURSES

Stop Press 3 for MST282 gives details of changes to the format of the exam paper. I was shocked to see that no less than 15% of the marks will he given to an essay question.

I thought immediately of two reasons for this - firstly that this was an attempt to reduce the high marks that embarrass the Maths Faculty, and secondly that essays are a sop to that infamous Kettle plan.

Essays are not new in Maths courses. In M100 1973 a TMA question involved an essay on mode vs. arithmetic mean - to get full marks merely involved disguising a simple list of pros and cons as an essay. But 15% is beyond a joke.

Let mathematicians be literate as well as numerate, by all means. But do not erect barriers in the way of specialists. Have mathematical courses and courses to improve communication and literacy, but not all combined.

Is this evidence of a trend in the OU? Will we be asked to write on such topics as "Social aspects of the Lebesgue Integral"?

This article is intended to provoke discussion. Do you think essays should be set? I do. But do you think that they should count for 15% of the marks? I don't.

Peter Weir

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STATISTICAL DIFFICULTIES

A friend has approached me with the problem of testing a given set of data to determine whether the individual results are related in any way, and further to tell whether that relationship has a greater than chance probability of occurring.

The data consists of a series of experimental results each consisting of approximately 50 values. These values are not necessarily all different, the entire population will not necessarily be accounted for in any one reading and there is only a qualitative relationship between the data items. Thus, a data item is a description of an occurrence rather than a quantitative or relational value. One can assume that there are approximately 50 variables in the population.

Given this information, what is the best way of testing the data for a relationship? Or has the experiment paid insufficient attention to the evaluation of the results? I have scoured bookshelves but qualitative analysis does not seem to be a popular subject. I would be very grateful for any references or help that M500 readers can give.

Roger Claxton

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THE RULES OF THE GAME

In M500/21 Eddie Kent provides answers to most of the particular questions I asked in my article: "The Rules of the Game" (M5OO/9). I must.say, however, that he has not given quite the answers I expected. Eddie has responded like a mathematician (no offence intended):– If something is ambiguous all you need do is say what you mean more carefully and elaborately. There are two things wrong with this approach, in my opinion. First, if you adopt a convention that is too clumsy people will not use it consistently no matter how precise it is and second, you run into the danger that people will not understand the explanation of the convention. In either case you are probably worse off than when you started.

As I said in my first article, the OU is in the business of communicating mathematically with its students and I don't think the problems that arise are going to have mathematical solutions. Professional mathematicians do not often misunderstand the mathematical papers that they read, despite the-very low quality of the writing. This is because the mathematician has spent years getting used to the oddities and conventions of his subject. On the other hand, it is clear that OU students often have great difficulty reading and understanding the material they are given. The present M100 does assume that students are capable of reading the sort of English that mathematicians write. (By English I mean everything in the texts; even though some sentences consist entirely of mathematical symbols they are (or should be) grammatical English sentences). Should the team that is about to rewrite M100 recognise this problem and consciously devote part of its effort to teaching people to read and use Mathematical English? This is a problem that has been largely ignored by writers of mathematical texts, but the reason that these writers could afford to ignore it is that students usually have a tutor or lecturer to act as interpreter. The OU can not assume that its students have easy access.to an interpreter and so I believe we must face the problem of teaching people to read.

If you disagree with me so far please write and say so. If you agree I can ask another question: How can the problem be tackled? Many of our students already find Unit 1 of M100 repellant. They came to the OU expecting to learn some new mathematical techniques which would enable them to solve problems that they could not do before. They are unwilling to wade through a lot of wordy material that is quite unlike their expectations. It is not obvious to them, as it presumably was to the course team, that we must spend some time making sure that we are speaking the same language so that we can communicate effectively. If we expand the introductory part of M100 to include material on reading mathematics must we run the risk of repelling more of our students for reasons like the above?

I would be very interested to have the views of M500 readers oh the above questions and I also think that M500 could be of help to the team rewriting M100 by pointing out examples where mathematicians use English in an unexpected way. It is often easier for a student to spot these oddities than a mathematician who has learned to live with them. I append a few examples but I think I would classify these as plain bad English rather than samples of mathematical conventions:

1. <u>Ml00 Unit 3</u>

> A function f with domain A and a binary operation o on A are compatible if whenever $f(a_1) = f(a_2)$ and etc. ...
>
> (Very odd use of the word "whenever")

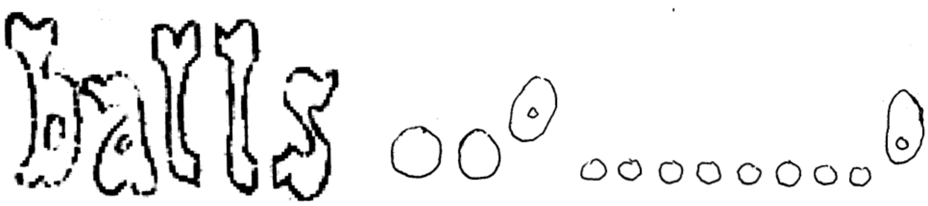
2. <u>M100 TMA 01 1975</u>

> How many different functions $g: B \to B$ are there?
>
> (How many different people Fred are there in your living room?)

3. <u>Herstein p. 39</u>

> Now if x is any element of HK, $x^{-1} = hk \in HK$ and so $x = (x^{-1})^{-1} = (hk)^{-1} = k^{-1}h^{-1} \in KH$, and so $HK \subset KH$.
>
> (This is a sentence construction, beloved of algebraists, that I find very clumsy.)

Richard Ahrens (STAFF-MOUTHS)



<u>Plagiarised from Dan Pedoe: "The Gentle Art of Mathematics", who in turn plagiarised it from F.J. Dyson and R.C. Lyness (Math. Gazette,Vol. 30. Oct. 1946).</u>

Richard Ahrens' article on Balls (M500/23) has answered one of the points raised in M500/21 regarding whether one can find, in n weighings on a balance, one odd ball from $\frac{1}{2}(3n-3)$ balls, but still leaves open the question of recorded numbers to determine whether the ball is heavier or lighter.

Take the $\frac{1}{2}(3n-3)$ balls and number thern from 1 to $\frac{1}{2}(3n-3)$

labelling each ball by its number written in ternary notation, ie using the numerals

000...01, ..., 111...10.
\ n /

Now add a second label to each ball, again a ternary numeral, obtained from the first label by changing 0's into 2's and vice versa. For example, if n=k then ball numbered 15 will have labels 0120 and 2102. Labels come in two sorts; if the first change of digit in a label is 0 to 1, 1 to 2 or 2 to 0 we call it a clockwise label, otherwise it is anticlockwise. The label 0120 is a clockwise label and 2102 is anticlockwise. The fact each ball has one clockwise label and one anticlockwise one follows from the way we constructed the second label from the first.

We are now ready to start our weighings and record the results. For the k-th weighing place in the left hand pan all the balls which have a 0 in the k-th place of their clockwise label and in the right hand pan those having a 2 in the k-th place of their clockwise label. If the left hand pah goes down put $x_k = 0$; if the right hand pan goes down put $x_k = 2$, and let $x_k=1$ if the pans balance. On completing the weighings we have a label $x_1x_2...x_n$ which identifies the ball which is different in weight from the others. Not only that but if the label is a clockwise one the ball is overweight and if anticlockwise it is underweight.

Bob Coates, Staff Tutor in Mathematics

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OBITUARY - DR. STANLEY GILL

Dr. Stanley Gill died on April 6th 1975 at the age of 49. He was a pioneer in the early development of the digital computer. He went up to St. Johns, Cambridge, and did his National Service with the Mathematical Division of the National Physical Laboratory, where he contributed to the development of the Pilot AGE. He returned as a research student to Cambridge and became involved with EDSAC in the Mathematical Laboratory.

He was best known for his work on the Runge-Kutta-Gill method of solving differential equations, and also for methods of diagnosing program errors (see his 'The Preparation of Programs for an Electronic Digital Computer', 1951.)

He was elected a research Fellow of St. Johns in 1952 and spent some time in America. He was President of the British Computer Society, 1967-68, and consultant to the Minister of Technology. In the 60s he established himself as a link between industry and academia and published many papers on advanced systems programming.

Eddie Kent

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Monday's Child

Time can be measured by watching the movements of heavenly bodies because to us their motions are uncharging cycles. Two systems are in common use: one based on the lunar cycle, the other on the sun. The sun gives better results for normal dating purposes.

One difficulty is that the day does not divide the year in any simple way. The year contains about 365.2142216 days and many people have tried to devise systems that use these incommensurable units in simple and practical ways.

Romulus introduced a year of 300 days which had ten months. His successor, Numa, brought the number of months up to 12 and there matters stayed until the introduction, by Julius Caesar, of the year with 365.25 days. He disposed of the odd quarter by making every fourth year a leap year. This worked well enough until the 16th century and enjoyed world wide use.

Since the Julian year was too long, by 1582 there was an accumulated error of 10 days. Pope Gregory XIII disposed of that and decreed that in future the only 00 years that would be leap years were those whose century number is a multiple of 4. Some of us might see the first leap year to round off a century since before Sir Isaac Newton invented his abominable mechanicks.

And now:- Gauss's formula for finding the day of the week corresponding to a given date.

$$W = D + M + C + Y \pmod 7$$

where W is the day of the week (Sunday = 1), D is the day of the month, M, G and Y are as in the following tables. If the month is January or February diminish the year number by one before looking up Y.

Monday's child is fair of face,
Tuesday's child is full of grace,
Wednesday's child is full of woe,
Thursday's child has far to go,
Friday's child is loving and giving,
Saturday's child works hard for its living,
And the child that is born on the Sabbath day
Is fair and wise and good and gay.

A.E. Bray, Traditions of Devon (1838), ii, 288

MONTH	M
January	1
February	4
March	3
April	6
May	1
June	4
July	6
August	2
September	5
October	0
November	3
December	5

FIRST TWO DIGITS OF THE YEAR			C
Gregorian			
15	19	23	1
16	20	24	0
17	21	25	5
16	22	26	3
Julian			
00	07	14	5
01	08	15	4
02	09	16	3
03	10	17	2
04	11	18	1
05	12	19	0
06	13	20	6

LAST TWO DIGITS OF THE YEAR									Y
00	06		17	23	28	34		45	0
01	07	12	18		29	35	40	46	1
02		13	19	24	30		41	47	2
03	08	14		25	31	36	42		3
	09	15	20	26		37	43	48	4
04	10		21	27	32	38		49	5
05	11	16	22		33	39	44	50	6
51	56	62		73	79	84	90		0
	57	63	68	74		85	91	96	1
52	58		69	75	80	86		97	2
53	59	64	70		81	87	82	98	3
54		65	71	76	82		93	99	4
55	60	66		77	83	88	94		5
	61	67	72	78		89	95		6

Eddie Kent

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MATH-QUOTES - Ron Davidson

"All number are made up of some multitude of units, so that it is manifest that their formation is subject to no limit."

Diophantus of Alexandria, c. AD 250

"In the first place, arithmetic of of two kinds, one of which is popular and the other philosophical."

Socrates (you might have guessed.)

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OPUS

Loader C

This is an alternative to 'Loader A' , the loader given in section 4.2 of TM221 Home Experiment Book Part B.

Use the switches as usual to put in the following program and data.   All digits are octal.

address	contents
002	107
003	000
004	063
005	373
006	072
007	376
010	367
011	213
012	323
013	241
014	375
015	050
016	004

When you have checked that you have entered the program correctly and that that the lights L show 017 with Display Up put 014 in P and switch to run. L should show 107.

In the next part ' ab.c' means set S to ab0 (top five bits only) and key c on the keyboard.   Also, (def)' means that def should then appear on L, since the effect of setting a pattern on S such as ab0 followed by a depression of a keyboard key is to deposit the pattern abc in location def and to put def on L. Just before the depression L shows one less than def. If you make a mistake see below. Work down the columns.

05.2	(110)	01.6	(125)	11.4	(142)	15.3	(157)
17.4	(111)	37.3	(126)	23.4	(143)	22.6	(160)
26.2	(112)	31.6	(127)	07.1	(144)	26.4	(161)
27.3		07.6	(130)	37.3		34.0	
07.3		00.0		06.1		31.4	
37.6		14.0		37.3		31.4	
36.3		00.4		32.1	(150)	31.4	
07.1	(117)	05.0		23.6		21.4	
37.1		11.3		22.6		27.4	
04.0		13.4		05.0		07.1	(170)
14.3		00.3		14.6		37.0	
05.6		25.3	(140)	27.6		05.0	
04.5	(124)	05.0	(141)	00.6	(156)	14.4	(173)

If you made a mistake, use the switches to set the contents of location 2 to one less than the address of the location at which you wish to restart, then set the starting address on P and run, as above.   For example, suppose you put 076 in 130 correctly but forgot to clear S and just keyed 0, thus putting 070 in 131, you could do:

Stop
Set S to 002, Deposit Up, Enter
Set S to 130, Deposit Down, Enter
Set S to 014, Deposit Up, Enter (ie: put 014 in P)
Run.
Carry on to 00.0 by setting S to 000 and keying 0 to put 000 in 131.

The above loader, locations 110 to 173? is called Loader B, and is operated in the same way as'Loader A'of TM221 HEB/B, section 4.2, when loading other programs or checking contents of selected locations. Loader C consists of the complete set of instructions entered into locations 002 to 016 plus Loader B, as shown above. Comments would be welcomed.

Richard Maddison, Maths Faculty

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MINILOADER TO LOAD THE TM221 HEB/B LOADER SOLELY FROM THE KEYBOARD

Enter the following in the usual way using switches S. Work down the clumns.

010	053	020	023	030	312	040	254
011	376	921	050	031	312	041	050
012	071	022	014	032	312	042	010
013	373	023	212	033	253	002	000
014	061	024	272	034	050	004	1st address of program
015	373	025	323	035	012		to be loaded = 110 if
016	321	026	050	036	132		program is the Loader
017	050	027	036	037	004		

Note that this Miniloader may be used to load any program which does not use the above locations. Check that you have entered it correctly, then set P to the contents you have put in 004. Run.

Depress keys on the keyboard continuously, not using % or #. For example, if you wish to enter 'Loader A' press keys as follows: 0 then 5 then 2 then 1 then 7 then 4, and so on. If you make a mistake, STOP. Enter the required location into 004, using the switches. Bet P to 010 and run. Then enter the contents of the selected location from the keyboard, as above.

If you wish to use the Miniloader to load a program which does use locations 010 to 042 you can alter the Miniloader location numbers.accordingly, but you must also alter the JUMP instructions to match. This involves the contents of 020, 022, 027, 035 and 042.

The Miniloader is-primitive, but effective for some purposes. You have to keep reverting to switches at breaks in continuity of addresses

Marion Stubbs

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M100 SELF-HELP GROUPS

I am writing to ask if any other M100 students are running an M100 Self-Help Group, successfully or otherwise.

Here in Cheltenham we are struggling along with one. We have an official M100 tutorial every other week and our SHG tries to fill in the gap in between. Our tutor/counsellor is convinced that the whole thing will fall to pieces as soon as the summer weather comes. 3 or 4 of us are determined it will not.

The greatest problem is trying to win over those students who are falling behind. If they do not come to the official tutorials it is hard to contact them, given the OU policy regarding names and addresses. It is frustrating at times when there are a few of us on the course ready to help any others who are also behind, but we cannot contact them. Obviously there are those who prefer or have to work alone and one must not interfere with their wishes.

I know some students live far out from the Study Centre and fear that as the SHG is not official, we may not meet regularly and they may travel in and find no-one there. To counter this we make sure someone always rolls along to run the SHG.

At the moment we are in danger of becoming a 'clique' of four or five regulars. We find it mutually beneficial to meet regularly and moan collectively and even do some maths occasionally. But we would like to see some of the stragglers coming along.

I am wondering if any other M100 students are trying to run a SHG and what sort of response they have had.

Dave Windsor

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EDITORIAL

I apologise for the lateness of this issue, caused mainly by interim production of the M500/24 Special Issue on Courses, suggested in M500/23. This was completed (and approved by a special Student/ Staff Editorial Board convened for the occasion) and thanks are due to students who responded so rapidly to my appeal. Unfortunately, we have encounted a printing dispute at Walton Hall, and at the time of writing, the outlook is very depressing. I have hastily produced this issue - also called M500/24, to add to confusion - but to crown everything, the typewriter broke down in the middle. Some pages have literally been typed with one RH finger while the LH pushed the carriage along one letter at a time! Not my month? Sorryl

M500/25 is decidedly thin, with items on file only from Peter Hartley, Roger Claxton, Eddie Kent and Steve Murphy. The barrel of PROBLEMS is also very low indeed–but please try to find some from unusual sources, not from eg Scientific American!

STOP PRESS: Latest news of M500/24 Special Issue is that it may be ready by mid-July. This same dispute (Printing v. Reprography) also makes your courre mailings late sometimes, it seems, so we should, perhaps, extend sympathy rather than criticism towards our academics whose freedom of communication is restricted by events beyond their control.

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SOLUTIONS

23.3 HOW LONG IS A PIECE OF STRING?

This problem was solved by Chris Green, Jerry Humphries and Mervyn Savage, each using a different set of equations. It seems best to give the short solution given by the original 'ancient electronics magazine', and readers who are still stuck should contact Chris, Jerry or Mervyn for details, please.

It can be calculated that the monkey is 1½ years old and the monkey's mother 2½ years old. So the monkey weighs 2½ lb and the rope 1½ lb. This makes the rope 5 feet long. There is a slight ambiguity at the end of the second paragraph of the puzzle: "...when the monkey’s mother was three times asold as the monkey." If this is taken to refer to the monkey's present age, then the rope is six and two-sevenths of a foot long.

Mervyn capped his solution with another monkey puzzle, also involving a desert island, five men and a quantity of disposable coconuts. However, this is merely a variation on the Three Fishermen problem which has already appeared in M500. Interested students may refer to 'Mathematics in the Modern World: readings from Scientific American', Chapter 14: Theory of Numbers, where a solution to this type of problem is given.

23.5 DUEL STRATEGY

(a) Roger Claxton

My calculations give C (50% accurate) the best chance of escape. The probabilities are:

A (100% accurate): 0.24166...,

B (80% accurate): 0.3111...,

C (50% accurate): 0.44722...

There must be a moral (or blunder) in it somewhere. Justification available if required (at present a rather spidery looking tree diagram!)

(b) Jerry Humphries (72)

This is an .old problem. I enclose a photostat of a solution from 'More Mathematical Puzzles and Diversions from ‘Scientific American'.

... the poorest shot, Jones, has the best chance to survive. Smith, who never misses, has the second best chance. Because Jones’ two opponents will aim at each other when their turn comes, Jones' best strategy is to fire into the air until one opponent is dead. He will then get the first shot at the survivor, which gives him a strong advantage. Survival chances:

Smith (never misses): 3/10
Brown (80% accurate): 8/45 = 0.1777...
Jones (50% accurate): 1– (Smith + Brown) = 47/90 = .5222...

Another solution is quoted, when real-life strategy is used:

Fifty-fifty Jones, against his best interests, will blaze away when able at the opponent he imagines to be most dangerous. Even so, he still has the best chance of survival, 44.722 %. Brown and Smith find their chances reversed. Eighty-twenty Brown's chances are 31.111% and sure-shot Smith comes in last with 24.l67%.

Perhaps, as MMP&D suggests, there is. a moral somewhere.

<u>22.1 BLACK BOX - Roger Claxton</u>

I don't think Hugh Mclntyre's solution (M500/23) works. A Boolean evaluation for Output (2) with $1.5 < t_1$ (sufficient for a counter-example) gives:

$$\overline{\overline{\overline{\overline{\overline{1.5}_{t_1} \wedge \overline{t_2}}} \wedge \overline{t_2}}} = \overline{\overline{\overline{1.5}_{t_1} \wedge \overline{t_2} \wedge \overline{t_2}}} = (\overline{1.5}_{t_1} \wedge \overline{t_2}) \wedge \overline{t_2} = (\overline{1.5}_{t_1} \wedge \overline{t_2})$$

which is not the required value. Output (1) also falls down. Compare Michael Gregory's and Hugh's expressions:

$$M: \overline{\overline{\overline{\overline{t_1} \wedge \overline{t_2}} \wedge \overline{1.5}_{t_1}}} = \overline{\overline{t_1} \wedge \overline{t_2}} \wedge \overline{1.5}_{t_1} = (t_1 \vee t_2) \wedge \overline{1.5}_{t_1}$$

$$H: \overline{\overline{\overline{\overline{\overline{t_1} \wedge \overline{t_2}}} \wedge \overline{1.5}_{t_1}}} = \overline{\overline{\overline{t_1} \wedge \overline{t_2}}} \wedge \overline{1.5}_{t_1} = (\overline{t_1} \wedge \overline{t_2}) \wedge \overline{1.5}_{t_1} = \overline{t_1 \vee t_2} \wedge \overline{1.5}_{t_1}$$

Sorry not to be more constructive. I am certainly not getting anywhere with Output (2).

(Ed: Roger asked me to check this. Regret I do not begin to understand this problem or its solutions. Please use MOUTHS to find suitable experts to check anything doubtful.)

<u>23.2 PULLING THE CHAIN</u>: No solutions offered. L.S. Johnson who proposed it has a solution, if required.

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<u>ERRATUM - PROBLEM 24.3, page 17, following</u>

Delete the '<u>Either</u>' part of what is to be proved. Substitute:

...Prove: <u>Either</u> There exists a matrix P in C_{22} such that for all A in the image of f, $P^{-1}AP$ is a diagonal matrix, <u>or</u> ...

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PROBLEMS

24.1 FOUR FOURS - Ann Jamieson

(a) Some time ago I read that all integers from 1 to 22 could he expressed in exactly four number 4's together with any mathematical symbols such as +, −, x, powers, logs, roots, etc. Example: 1 = (4+4)/(4+4), 2 = (4/4)+(4/4) or (4x4)/(4+4).

I have managed to do this for all the integers except 19. Could some reader either give me the solution or tell me it is impossible and so put me out of my misery.

(b) Ed: It is possible to do this using one single formula for all integers up to 100 (see GAMES & PUZZLES No. 24). Problem 24.1(b) is to find the formula (without looking at G & P No. 24!)

24. 2 QUADRATIC COEFFICIENTS

Let a be a positive integer and let b,c be integers.

Suppose that $ax^2 + bx + c$ has two distinct roots in the range $0 < x < 1$. Show that $a \geq 5$ and find such a quadratic with a = 5. (Taken from MATHEMATICAL SPECTRUM, vol.7 no.3 - subscription 70p p.a. from Univ. of Sheffield)

24.3 COMPLEX HOMOMORPHISM - Bob Margolis (STAFF-mouths)

Materials: G (a finite group)
C_{22}–set of all non-singular 2 x 2 matrices with complex entries
f: $G \rightarrow C_{22}$–a group homomorphism

Prove: Either f(g) is of the form

$$\begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \qquad \text{for every } g \in G$$

or the order of G is divisible by 2 and (more work) order of $G \geq 4$.

The only solution known to me at the moment involves a general theorem, requiring a lot of work. This is a special case which ought to be easier.

24.4 FIND THE NEXT TERMS

Find the next two terms and a rule for generating each of the following sequences. (N.J.A. Sloane, JOURNAL OF RECREATIONAL MATHEMATICS, vol.7 no.2, Baywood Pub Co Inc)

(14) 1,1,2,1,2,2,3,1,2,2,3,2,3,3,4,1,...

(15) 1,2,3,4,5,6,7,6,6,10,11 ,12,13,14,15,8,17,12,19,20,21,22,23,18,10,...

(16) 1,8,11,69,88,96,101,111,181,609,619,...

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