

M500 28

M500 is a student-owned and student-operated magazine for Open University mathematics students and staff. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct, but invite criticism and comment.

MOUTHS is a list of names, addresses and telephones, together with previous and present courses of voluntary members, by means of which private contacts may be made by any who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned and do not necessarily represent those of either the editor or the Open University.

The cover this month is our "Merry Christmas" star. Lovingly traced by Marion Stubbs from the glossy 4-colour design on an advertising brochure for CONTACT by Arthur J Pulos (published by Information Canada.)

M500 is published by Marion Stubbs.

Contributions for M500 should be sent to the editor - Eddie Kent.

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Material not covered by the above to Marion.

Please note that anything sent to any of the above will be regarded as potential material for publication unless marked PERSONAL.

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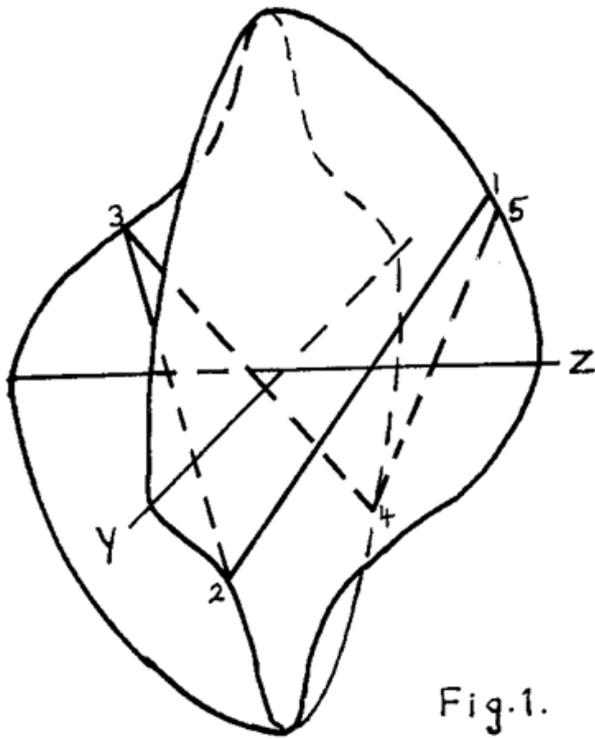


Fig. 1.

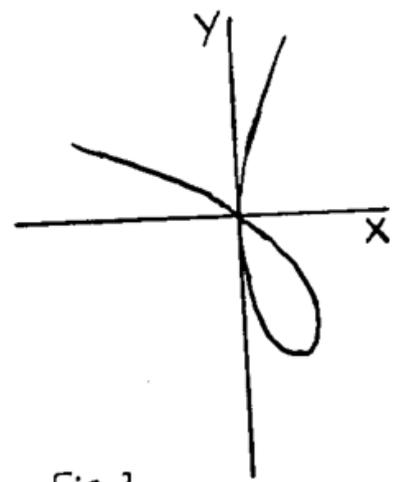


Fig. 2.



Fig. 3a.

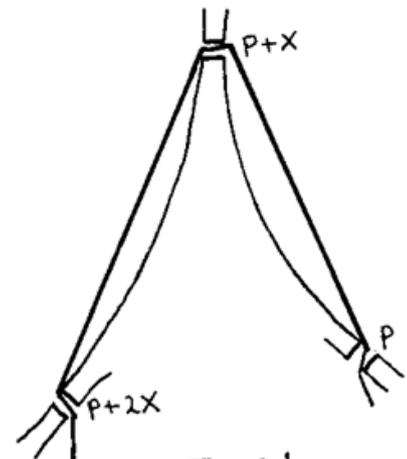


Fig. 3b.

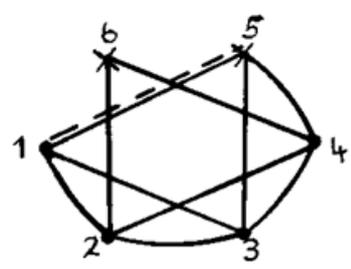


Fig. 4. $n=6, x=2, q=2$.
 circuit: $1-3^{\wedge}4-6-2^{\wedge}3-5-1^{\wedge}2-4^{\wedge}5---$

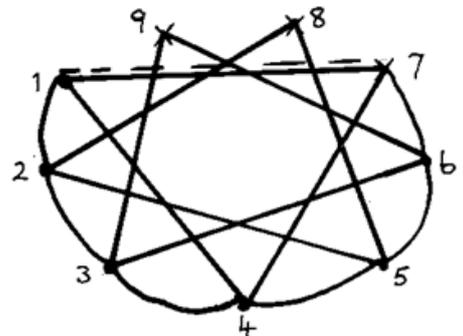


Fig. 5. $n=9, x=3, q=3$.
 circuit: $1-4^{\wedge}5-8-2^{\wedge}3-6^{\wedge}1^{\wedge}2-5^{\wedge}6-9-3^{\wedge}4-7-$

CONSTRUCTIONS

Michael Gregory

In 1966 my wife and I went to an exhibition of Naum Gabo's work at the Tate Gallery. Apart from a rotating painting in various shades of blue, the item which interested me most was 'Linear Construction No. 2'. This consisted of two sheets of perspex with notches all round and fitted together at right angles. It was about 3ft across and hung from the ceiling. (Figure 1.) A nylon monofilament was passed round and round (one circuit: 1-2-3-4-5 is shown) so that interwoven ruled surfaces were formed. I was fascinated.

Assuming algebraic equations – $z = f(x)$, $y = g(x)$ – I tried to find an equation for the three-dimensional curve formed by the intersection of the ruled surfaces (ie, formed by moving a straight line) in one sector. I used up a lot of paper, discovered quite a lot of 3-D coordinate geometry and calculus. This led me to look at Cayley's ruled cubic:

$$x^2 + xyz - y^3 = 0 \quad 1$$

which has an interesting section, at $z = 1$ (figure 2). The 'ruled' character can be seen by rearranging equation (1), giving

$$xyz - y^3 = -x^2$$

therefore $y(xz - y^2) = -x$. Introducing a parameter k we obtain

$$y = -kx \quad \text{and} \quad xz - y^2 = x/k.$$

Each value of k specifies one position of the generating straight line. I found W L Edge's "The Theory of Ruled Surfaces" difficult, but I think the experience of Nering and Herstein may make another attempt worth while.

I reverted to more practical lines of attack, and made a model of Gabo's construction, about one quarter of the original size. The main problem was controlling 60 yards of nylon monofilament in a fairly small flat.

My more recent constructions have been original and made from a single sheet of perspex cut in an interesting shape with holes (200 to 400; 1/32" or 3/64") drilled round the perimeter, with considerable loss of drill bits. The next job is to bend the perspex into an interesting shape in the oven. Oven gloves are essential; and advice from our two young children adds interest to the process. One has to avoid any sharp bends which are liable to cause cracks on cooling. After cutting a sufficient length of monofilament to complete the construction, and sometimes dyeing the thread, the tedious job of threading begins. To save time in future I plan to use slots rather than holes

where there is no chance of the thread slipping out. At first I wondered whether the construction would collapse or the thread break, but after 4-5 years they are still in good order.

When threading, I decide on a step length x which is constant for the construction. For a hole p , (holes are numbered consecutively) two possible types of threading are illustrated in figures 3a and 3b. With (a) each hole has two threads; (b) works well when there is a twisted portion of perspex. The $(p+x)$ -hole has only one thread through it. If n is the total number of holes, x the step length as above, and q the number of holes connected as in (b) then I number the holes so that

1 to $(n-q)$ are as in (a)
 $(n-q+1)$ is special) shown as X's in
 $(n-q+2)$ to n are as in (b)) figures 4 and 5.

Two examples of 'acceptable' circuits are illustrated in figures 4 and 5. To make a complete circuit some compromise for the last lap may be necessary.

For a particular value of n , I have developed a fairly efficient tabular method for finding some acceptable values of x and q . This does not give all the solutions, nor tell you whether there is an acceptable circuit for particular values of n , x and q . One can check this by listing the complete circuit, a lengthy process for n greater than 100. It is not difficult to devise an algorithm for calculating the sequence of thread positions for any n , x , q . We have

$$p \rightarrow \begin{cases} p+x \rightarrow p+x+1, & \text{when } p \leq n-q-x \text{ or } p > n-x \\ p+x, & \text{when } n-q-x < p \leq n-x \end{cases}$$

where the RHS of the mappings are residues (mod n).

The mapping $p \rightarrow p+x$ with no extension to $p+x+1$ occurs at irregular intervals in a case such as that of figure 5. An equation for the k th term as a function of n , x , q would be of interest, but does it exist? An incomplete circuit arises if (i) the $(n-q+1)$ th hole is used twice before $2n-q$ applications of the mapping; eg $n=8$, $x=q=2$, or (ii) p cycles in a proper subset of $\{1, 2, \dots, n\}$, eg $n=6$, $x=2$, $q=0$.

Comments, discussion and dissertations will be welcome. I should like to thank John Gray for the new printing of the diagrams.

The fine structure constant is 137. It is said that when Wolfgang Pauli died and met God he asked "Why 137?". God handed him a bundle of papers. After studying them Pauli said, "Das ist falsch."

++++

π (IN THE SKY)

(This letter was received at the editorial office in mysterious circumstances, to the accompaniment of a clap of thunder. It purports to be from John Machin.)

My dear Marion Stubbs,

I would like to reply to some points raised by you in M500, issue 27.

In the first place π is exactly $16 \arctan 1/5 - 4 \arctan 1/239$ which is not a series but a number, although calculating by this means can only give an approximation unless you have infinite patience.

Secondly I was able to use \arctan because my good friend Brook Taylor assures me that

$$\arctan x = \sum_{k=0}^{n-1} \frac{(-1)^k x^{2k+1}}{2k+1} + e_{2n}(x)$$

where

$$|e_{2n}(x)| \leq \frac{x^{2n+1}}{2n+1} \text{ if } 0 \leq x \leq 1.$$

Now $T_{11}(\arctan 1/5)$ gives $3.158328934 < 16 \arctan 1/5 < 3.158328972$ and $T_3(\arctan 1/239)$ gives

$$-0.016736309 < -4 \arctan 1/239 < -0.016736300.$$

Combining these gives π , correct to 7 places, as 3.1415926.

Why not use $\pi = 4 \arctan 1$? you ask. Try it! From Mr Taylor's formula $\arctan 1 = 1 - 1/3 + 1/5 - \dots$ and this leads, after 18 calculations, to

$$3.119 < 4T_{18}(\arctan 1) < 3.147$$

which is accurate to one decimal place. $22/7$ is better.

What is π ? It is usually defined as the area of the unit disc, which is the set of all points inside or on the boundary of a disc of radius 1.

This disc is congruent to the region between the graphs of

$$g_1(x) = \sqrt{1-x^2} \quad \text{and} \quad f(x) = -g_1(x) \quad \text{for} \quad -1 \leq x \leq 1.$$

Each of these is continuous on $[-1, 1]$ and thus integrable thereon* so that the area between their graphs is measurable and given by

$$A_1 = \int_{-1}^1 g_1(x) - f(x) dx = \int_{-1}^1 2g_1(x) dx = 2 \int_{-1}^1 \sqrt{1-x^2} dx^{**}$$

On the other hand, setting $g_r(x) = \sqrt{r^2 - x^2}$, $-r \leq x \leq r$, we obtain the area of the disc of radius r ,

$$A_r = 2 \int_{-r}^r g_r(x) dx = 2r \int_{-1}^1 g_r(rx) dx = 2r \int_{-1}^1 \sqrt{(r^2 - (rx)^2)} dx = 2r^2 \int_{-1}^1 \sqrt{(1 - x^2)} dx$$

$$= r^2 A_1;$$

so that $A_r = \pi r^2$.

I have the honour, &c

JM (transmitted through EK)

* See Spival Theorem 13.3. ** This is the definition of π given by Spivak.

ADVICE FOR NEW STUDENTS

Sam Banks

Being a new MOUTH myself I've read the request for constructive advice from the first F-year MOUTH, Joyce Moore. I would suggest that she consult the most up-to-date O-level and CSE texts for a start, SMP (Schools Mathematics Project) are very good but there are plenty of others. The local librarian or warden of a teachers' centre will help.

Topics covered clearly and simply with good graded exercises include Sets, Mappings, (definitions not always the same as the OU's but no harm will ensue) Relationships, Logic, Linear Algebra (including matrices), Flow Charts, Symmetry – leading to Groups, Probability and Statistics (some of the CSE books are very good), etc. The BBC pamphlets for secondary school mathematics are excellent.

And for taking to bed – who better than W W Sawyer for sheer delight. Your Penguin bookseller will effect an introduction.

USE OF POCKET CALCULATORS

Brian Woodgate

'Examination Arrangements 1975' gives 12 courses that allow the use of pocket calculators (including MST281) and in fact states that they may be used instead of slide rules in all but three exams. Could someone please explain this policy with regard to MDT241, the only maths course in the three?

Also, why do some allow notes in printed material and some do not?

1. +
 524288. =
 .00000190734863 *
 1.0000019073486 +
 1.0000019073486 x
 1.0000019073486 x
 1.0000038147008 x
 1.0000076294161 x
 1.0000152588904 x
 1.0000305180136 x
 1.0000610369585 x
 1.0001220776425 x
 1.0002441701879 x
 1.0004883999948 x
 1.0009770385241 x
 1.0019550316524 x
 1.0039138854535 x
 1.0078430894063 x
 1.015747692864 x
 1.0317433755585 x
 1.0644943930088 x
 1.1331483127471 x
 1.2840250986815 x
 1.648720454044 x
 2.718279135583 x

e

Peter Weir

e on a calculator ($\lim_{n \rightarrow \infty} (1 + 1/n)^n$). On the left is a calculation done on a printing calculator from work. It's quite expensive but doesn't have a store. The calculation starts off with $1 + 1/n$, $2^{19} = 524288$ calculated previously. Result to 5 decimal places.

I have played with calculating e this way on a computer. It's not a very fast way. Result with $n = 2^{65} = 36\,893\,488\,147\,419\,103\,232$ is $e = 2.718\,281\,828\,459\,045\,235\,3\dots$ after which it is not reliable. The formula seems to give e correct to the same number of decimal places as the number of digits in n, (I would hate to do the error analysis!) It takes some time - I used 500 digit accuracy, something you can't do without having the whole (32K) machine to yourself.

A F T E R M A T H

Chris Sexton

Throw 91 maths students together for a weekend at Aston and what else is there to do but maths? Nothing. Maybe I was spoiled by going to Stirling a few weeks earlier but, deary me, if I were sweet seventeen Aston would be way, way down on my UCCA form.

But after all, the whole idea was to do maths, so where better than on the campus with minimum distractions? The one distraction which kept me awake all Friday night, namely ring-road traffic careering past, was easily overcome by simply not

attempting to sleep the following night. The fact that teaching ended at 10pm and pubs closed at 10.30pm didn't altogether defeat us.... we have ways..... The Midland Youth Orchestra rehearsing in the same teaching block was rather pleasant during differential equations. The vertical distances took some getting used to and I was disappointed to find my options limited by a window which opened only 3 inches.

However the experience in toto was superb. Throw 91 random people together and they'd spend all weekend searching for common factors; throw 91 people together with one rather obvious common factor, OU maths, and they're never short of conversation. So the social side took care of itself.

The teaching side was really excellent. I arrived feeling like a patchwork quilt which had been lying around in bits all year, I left feeling that at last I was being pieced together. The organisation was fantastic - not a hitch. I don't know how Marion did it. Many thanks to her and all the staff involved and here's to the next time.

"Come home, Joe, all is forgiven" (Confucius)

SEQUENCES

Hugh McIntyre

Could I reassure Tony Brooks that I had no intention of implying in M500/23 that he was incapable of producing a rule for his sequence. What I said was that he had not, in fact, done so. There is, he will admit (?) a point beyond which arbitrary rules have zero value. For instance, Give the next term in the sequence 1, 2, ... and a rule. My choice is the well known integer 457461463467479487491499503. (See M500/26, page 9.) Why? I fancy it is why. It has the distinction of being the only 27-digit integer with that particular sequence of digits. Where, Tony, is the point? That I have done it is indisputable. That I could produce an infinity of such rules is beyond doubt (like to bet?) What has it to do with mathematics?

I leave Tony and the rest of M500 readers to decide on the mathematical merits of the following. It is applicable to any sequence of finite length, and purports to give a unique extension of the sequence. Given the sequence as follows $U_1, U_2, U_3, \dots, U_n$. Then the next term is given by the rule:

$$U_{n+1} = aU_n + bU_{n-1} + cU_{n-2} + \dots + bU_3 + aU_2 + U_1,$$

where the coefficients a, b, c, \dots are taken from Pascal's Triangle by the rules:

a is the coefficient of x^n and of x in the expansion of $(x+1)^{n+1}$,

b is that of x^{n-1} and of x^2 in same, etc; and

The signs of the coefficients are governed by the rule:

in mU_K , m is positive if K is n, otherwise m is alternately - & + with $n-1$ and $n-2$, etc.

This Universal Rule gives Respectability to the most Unlikely Sequences, viz: 1, 2, 4, 3, 4, ... continues with 21. Indeed it Confirms the Respectability of Socially accepted Sequences like 1, 2, 3, 4, ..., n, for this continues by application of the Rule with $n+1$, $n+2$, etc

DING!

A triangle is a square that's lost a corner. Unless it's a set-square in which case they're all there or you'd take it back.

Squares were once all the rage for holding markets in, but triangles gained ground once Mr Pythagoras told everyone that triangles had squares all round them. Before that only the eternal ones were known to playwrights.

Three flat sides and three corners is what triangles have. Whereas squares have blunt corners you must handle squashed triangles very carefully as the corners become extremely sharp, as in axes.

Students use them for calculating things like slopes but not everyone likes them; musicians even hit them for pleasure.

Lytton Jarman

SPECIAL ISSUES

You will remember that there were two M500/24s [not here, EK], one very professional looking and late. This one, which had the distinguished appellation "Special", contained pieces from anyone who cared to write, on the subject "OU courses I have known."

Well, could we do it again, please I

About 200 words wanted; the idea being to give current students some idea of course options and patterns and help in considering their future with the University. So write and tell us why you did what you did and what you would rather have done if only someone had told you. ie, what would you have liked someone to tell you.

As with 24-Special it will be distributed from Walton Hall to all M-students. But this time Professor Pengelly would like to have it out in time for Conditional Registration in May. This means that everything ought to be in the M500 front parlour by the end of January.

We would like the further adventures of our gallant pioneers from 24-Special together with reports from the new generation who have struggled through post-foundation OU mathematics. And of course reports from those who meant to do something last time but.

Please send contributions (for the Special only) to Marion Stubbs. As last time she will convene a staff-student editorial board.

D E A R E D

From Miek Warden - Although I am down for S100 in 1976 I feel a great need to renew my subscription for M500 for various reasons.

In spite of the MWWI I may not have passed and so I must be kept informed of the next Maths weekend to find out what I forgot to study this year and to have a good excuse for a marvellous weekend.

Then I can't bear the thought of saying goodbye to all of you to get myself lost in pursuit of Jules Verne, to be mutilated by chemical experiments, or to be stung (do they?) by the flies I shall be expected to breed, without the life-buoy in sight which will drag me back to the sane and sound world of mathematics,

I suppose there is always the geological society, but it can't be the same!

From Don Wright - I found the free copy of M500/24 the most useful data on OU courses that I have seen in five years. If you produce stuff like this then I obviously can't afford not to be a subscriber.

SUMMERTIME BLUES

Pat Pammenter

This is a cry from the heart - that is if budding mathematicians have hearts and not computers.

Has anyone ever thought of a rehabilitation centre for post Summer School students? Perhaps other students haven't felt as I did after my week's intensive study at Reading this summer. After all, women suffering from post-baby blues are cared for—post-operative blues are acknowledged as in need of care, so how about post-summer school blues?

After my week at Reading - exhilarating, deflating, confidence giving, shattering, exhausting, enlightening, mystifying, ..., n (this sequence has no limit) I felt sadly in need of care on my arrival home. With maths formulas coming out of my ears, maths gags and jokes coming out of my mouth, I had to return rapidly to being a wife and mother with washing, cooking, shopping, housework to be attacked swiftly and with gusto. The transition from student to mum is a traumatic one. One misses the company, the different way of life, even the pressure of lectures and the effort of being in the right place at the right time—but above all one misses the opportunity to talk maths. It's like being deposited in a foreign land with no-one speaking your language.

I can recommend Summer School at Reading as a good slimming course. In spite of the excellent meals and perhaps because of the constant trek from Hall of Residence to Lecture rooms and/or theatre, I was able to lose one pound in weight. Had I not eaten so well I'm sure I could have achieved a greater weight loss.

Of course, other students may have felt differently or not so disturbed by Summer School but if there is anyone who feels as I have done, perhaps they have a solution or a formula for me to follow next time. If so I'd be glad to hear it. I know the feeling of depression and desolation doesn't last but how does one get through that first week back in a less exhausting way emotionally?

Incidentally my serial number is E0345293 which certainly isn't the highest but comes in the category asked for. Also, for Joyce Moore whose plea was in M500 26, I was in the same boat this year and feel that if I managed to keep going I'm sure she can. My School Cert days are 36 years away!

One, two, plenty. - Tasmanian method of counting.

(Pinched from W W Sawyer because people seem to have stopped sending in usable quotes!)

M 5 0 0—THE PRINTING

Roger Bridgman

Marion did a great job getting M500 into print at all, but now that it seems to be accepted as a benign growth upon the immaculate body of the Maths Faculty, I think it's time we went litho. Not too professional though please; we don't want to polish all the friendliness out of it. Something looking like Manifold would be fine.

—THE STRUCTURE

Jim Marchant

Marion Stubbs is very worried by the present structure of M500 because of demands from the Inland Revenue. Fortunately with the skill of a Maths graduate Marion was able to prove last year that her profit was in fact a loss. However, in future years, tax may be payable. If we were an Association or Society this problem would be overcome provided we had a written constitution.

At the Aston weekend the idea was discussed of putting M500 on a formal basis by having a written constitution. This was acceptable to the majority present providing it was kept as simple as possible.

Since a constitution is written for people and M500 is a magazine then this would be a difficult problem. Could we call ourselves "The Open University Mathematical Association" or something similar? Would this be acceptable to the membership and to the Mathematics Faculty at Walton Hall?

If this were accepted then M500 would be our magazine and MOUTHS a subset of members who wished to communicate.

Ed - True, the existence of M500 depends on the existence of the OU - but unlike the other societies our membership is not restrictive (in fact there are two nonOU members and we would like more) so it would seem anomalous to use "OU" in our main title, don't you think. I like "The M500 Society", what about you?

"exact, accurate, definite, precise, well-defined, just, right, correct, strict, severe; close; literal; rigid, rigorous; scrupulous; punctual, mathematical, scientific; faithful, constant unerring; particular, punctilious, meticulous, nice, delicate,"

—Roget, Truth

CHEZ ANGELIQUE!

John Jaworski

The lucky ones at Summer School this year seem to have been Stirling-goers. As well as everything like scenery, beer until 11.30 and the only four consecutive days without rain in Scotland since Robert the Bruce, they got CHEZ ANGELIQUE.

CHEZ ANGELIQUE was about problem-solving, but don't switch off, because it had just a bit more flair in it than Polya. Something clicked, and what happened was a number of weeks of a mathematical night-spot.

People came, they saw and they stayed: they forsook the bars, they stood outside on tiptoes to try and see in. For hours on the last night of all, they came and begged for more.

What happened? Well, Richard Ahrens did a strip-tease; most people got worried by the 40 faithless wives, everyone got worried when John Jaworski tried to give away £1 each night, when John Mason fell over on his way back from Chez A. and when Alan Slomson couldn't make up his mind about which of the 10 beautiful girls to marry. Angela Dean and John Kassab (it began life as Club Kassab) gave the whole affair more than a name – a racy style all its own. Fred Holroyd's drunken parties (a million, Fred?) will be remembered by anyone lucky enough to get an invitation.

There is a theorem: ANGELIQUE's THEOREM

The world is divided into two classes: those who have already ordered a copy of Chez Angelique – the Bumper Late-Night Problem Book; and those who didn't know that it was on sale.

For the few unfortunates in the second class: there is to be a limited printing of a collection of problems from the halcyon days. It will cost you £1 (post free) and we don't mind much whether you order it from John Mason, Mathematics Faculty, OU, or from M500.

PSST! Feelthy pictures? Photo-strip by Richard – our own gatefold.

From Margaret Corbett - I am attending a 12-week half day release course for the MRCP. I guess I had better complete this attempt at Membership – success or failure – before thinking about OU courses again (and then will find I've forgotten my maths!) but would be interested if there are any snippets of information someone can send me. Like – I didn't have a 1975 Handbook or any information about the courses for 1976; so don't throw away your old Handbook when you get a new one, send it to me instead – I promise to return the postage! Out of date info is better than none.

MORE LETTERS

From B J Frear - I was interested to read the 'tailpiece' regarding Dice Star Trek. I shall be pleased if you will send me a copy. I must thank you for all the work you put into producing M500, reading 27 has certainly brightened up a dull day today - even after finding that the subs have gone up to £3.

From Geoff Bennett - I have never played \$STTR1 but you have wet my appetite by the description of Dice Star Trek: please send me one! I am opting for M501 (ie retiring) next year (due to impending promotion and pressure of work) but hope to return to real Ms in 1977.

From C Auger - I should be most interested to have a copy of 'Dice Star Trek'. I should like to take this opportunity of thanking you for all your efforts in organising the Maths Work-in. I found it a most stimulating and helpful experience - especially the Analysis group.

From Mike Geraghty - Please send me Dice Star Trek. Having spent the year on the star ship Galois, lost in Metric Space, being attacked by a tribe of Primitive Functions and almost losing my prime factor in a field of polynomials I am hoping sanity will return in the land of +, -, x etc.

From Norman Cunningham - Please send me a copy of 'Dice Star Trek'. Many thanks for all you are doing; the Birmingham weekend couldn't have been bettered. I'd go on two - Easter and late summer next year. 201 exam was a complete disaster, completely beyond those I know who took it. But here's hoping!

From Geoff Ward - Please send me one of those Star Trek games of yours. At last I will be able to use my pocket calculator for something. In the two years I've had it its only value to me has been doing probabilities the long way in M100. Does a faulty '0' key make things more interesting? (nonMOUTH)

From Sue White - Glad to hear your printing is being mechanised at last, for your arm's sake! Have just sent off my cheque for next year. The increase still leaves M500 as good value for money. Yes, please, I should like a copy of Dice Star Trek, if only to keep hubby amused while I get down to catching up on a year's sewing before next year's courses start!

From Michael Masters - I was pleased to hear of your new game Dice Star Trek and I enclose my cheque to pay for a copy. I hope that I get on better than I did with \$STTR1. Unfortunately I never got the hang of the navigation and so the Enterprise veered from one end of the galaxy to the other and only once by chance nearly rammed a Klingon!

And the rest of this sentence umop-epxsdn sx [that last bit was 'is upside-down'-EK]

SOLUTIONS

27.2 TWENTY QUESTIONS

The highest number determinable in 20 questions is (if 0 is excluded) 1 048 576; by 'binary chop'. Bill Shannon, Peter Weir, Brian Woodgate, Michael Masters, Peter Needham.

27.3 FIND THE NEXT TERMS

a) 1,2,4,9,11,29 - 31,32; a path on the cover of M201 units; Jeremy Humphries, Peter Weir, Brian Needham. Brian Woodgate suggests - 32,34 arranged in this matrix where column 2 minus column 1 gives the row number and column 3 minus column 2 equals 2.

$$\begin{array}{ccc} 1 & 2 & 4 \\ 9 & 11 & 13 \\ 29 & 32 & 34 \end{array}$$

b) $\frac{1}{4}, \frac{1}{2}, 1, 3, 6, 12, 24, 30$. This was the intention of Tom Dale: our old sterling coins in pence; also derived by Peter Weir. Other suggestions were - 36,72 from Brian Woodgate: "Taking differences a_{n+1}/a_n we get 2,2,3,2,2,3,2, ...;" Michael Masters: - 25,50 "take the sequence as a fibonacci series but first adding $\frac{1}{4}$ for the first two, then $1\frac{1}{4}$, then $2\frac{1}{4}$ etc.," and Peter Needham, - 24,48: $(2+F(u_r))u_r=u_{r+1}$, $u_1=\frac{1}{4}$; where $F(x)=0$, $x \neq 1$; $F(x)=x$, $x=1$.

Thus encouraged I venture to propose the following unique 5th degree polynomial – of the kind recently discussed in M500 –

$$p(x) = \frac{17}{480}x^5 - \frac{29}{48}x^4 + \frac{379}{96}x^3 - \frac{559}{48}x^2 + \frac{931}{60}x - 7.$$

It is easy to see that $p(1) = \frac{1}{4}$, $p(2) = \frac{1}{2}$, and so on. The next two terms, $p(7)$ and $p(8)$, are $29\frac{1}{4}$, 79. EK

27.1 PERRY

No-one has come up with any suggestion for this so we will leave it; for a bit. 3 points: P5 (on the cover of 27) should be P4; a is the distance OT and c is OS; the curve goes through T, P₁, P₂,

25.2 CUBIC HYPERCUBE; another solution, from the onlie begetter of the problem –Roger Webster, University of Sheffield.

The product of four consecutive positive integers cannot be a perfect cube.

Let n be a positive integer. Then we show that the product $n(n+1)(n+2)(n+3)$ cannot be a perfect cube. This is obvious when $n=1$ and we assume that $n>1$. One of the integers $(n+1)$ and $(n+2)$ must be even and the other odd; say p is even and q odd. Now q has no non-trivial common factors with any of n, p and $(n+3)$, whence, if $npq(n+3)$ is a perfect cube, so too must both q and $np(n+3)$ be. But $np(n+3)$, which equals either $n(n+1)(n+3)$ or $n(n+2)(n+3)$, cannot be a perfect cube as, for $n>1$, it lies strictly between the consecutive perfect cubes $(n+3)^3$ and $(n+2)^3$. This establishes the desired result.

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P R O B L E M S

28.1 HINTS TO SURVEYORS; Henry Jones

How do you divide a triangular building site into two equal areas by a fence at right angles to the frontage?

28.2 NEW YEAR RESOLUTIONS; Tony Forbes

Around this time of the year I usually start thinking about New Year Resolutions. In the past I found that I enthusiastically begin the year with a dozen only to break them all by February. However, this year I am trying a different approach. These are my 12 New Year Resolutions; which ones will I keep?

- 1) To keep at least 6 resolutions but not resolution 7;
- 2) To keep at least 4 resolutions but not resolution 5;
- 3) To keep exactly 2 resolutions;
- 4) Not to keep resolution 5;
- 4) To keep exactly 4 resolutions;
- 6) Not to keep resolution 7;
- 7) To keep exactly 6 resolutions;
- 8) To keep at least 2 resolutions but not resolution 3;
- 9) To keep exactly 8 resolutions;
- 10) Not to keep any odd-numbered resolutions;
- 11) To keep at least 8 resolutions but not resolution 9;
- 12) Not to keep any even-numbered resolutions.

28.3 ROLL MB OVER; Roger Bridgman

Eight cubes are marked with X and O on opposite faces and placed in a box in configuration A. Get them into configuration B without lifting or sliding any.

X X X		O O O
X X	(A)	O O (B)
X X X		O O O

Ed - Please use the following notation: L,R,U,D mean, respectively, roll a cube to the left, to the right, up or down. You will notice that all first moves are equivalent to LUR so please start with that. John Mason would like to see a proved minimal solution.

28.4 INTO THE UNKNOWN; Bill Shannon

Ship A with a maximum speed of 30 knots is chasing ship B whose maximum speed is 15 knots. Neither ship has radar. Ship B enters an extensive fog bank and from then on is invisible to ship A. On the assumption that ship B, on entering the fog bank, changes course and maintains its new direction unchanged at maximum speed, what plan should ship A follow to guarantee that ship B will be intercepted?

28.5 UNDER-PRIMES; Hugh McIntyre

Prove that the number of odd primes less than x is itself less than $(x+2)/3$. Can this be generalised?

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EDITORIAL - Eddie Kent.

Well, this is the last of the hand-driven issues. Sad really. But there's a bright new year ahead and I am sure everyone will join with us in hoping we all enjoy it.

Peter Hartley (staffMOUTHS) writes to thank Willem van der Eyken for his letter in 27 and commiserating with Ann Jamieson. He says he will be ex-staff come January; I trust he will not desert us. By the way Peter, you are not the first to call me Eddietor; I don't even suppose you will be the last.

Lots of people have replied to Machin-pi; I will try to get them in one a month. For the rest, well, we're a bit short. Now would be a good time to write something for M500 before M-other starts again. How did the exams go, by the way?

Could those who write complaining of having to send several letters to M500 now instead of the original one only, please read the reverse-of-cover spiel, lines -7 to -4.

Polyhedron models, says Marion, make excellent Christmas decorations, so get the kids to try this one:

THE RHOMBITRUNCATED ICOSIDODECAHEDROIT

by Marion Stubbs

Cut templates from card for a decagon, a hexagon and a square, with all sides 1 unit long. Place the template on the material to be used, such as computer-waste punched cards or even wallpaper leftovers, but make sure you use stiff material for the decagons, or they will sag. Prick through each vertex with a sharp point. Join up the pricks, cut out, leaving about $\frac{1}{4}$ " all round the pencil lines for gluing, score the pencil lines and fold the tabs inwards.

Cut 12 decagons, 20 hexagons and 30 squares. Surround each decagon with alternate squares and hexagons. Glue together - UHU is very suitable.

Decorate as desired. Meanwhile keep children occupied constructing their own tetrahedra, octahedra and dodecahedra (4 and 8 equilateral triangles, or 12 pentagons respectively) and supply them with large quantities of old magazines, scraps of fabrics, poster paint, etc, for decoration of their finished models. *

Polyhedron models make ideal Christmas decorations, 3-D birthday cards, and a never-failing topic of conversation.

* Sketches of the Platonic solids are in M100 Unit 30 TV notes.

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ARCHIVES: I have found 15 copies of M500/8 (October 1973), 16 imperfect copies of M500/2 (March 1973) and 5 copies of M500/3 (April 1973). Send 9x4 stamped envelope + 10p loose stamps to Marion Stubbs. Envelope and stamps will be returned with M500/29 if you are too late. First come first served. They are sweet!

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PUBLISHORIAL - Marion Stubbs

6 December 1975

I asked for one page for new MOUTHS and now find I need 2 pages for everything I want to say. Actually, there is little point in printing new MOUTHS until the GRAND OPENING MOUTHS LIST 1976, due January or February. There are 3 corrections to old ones, viz: Tony Brooks is now in Belgium, Brian Alden is at a new address, and Simon Kaminsky is re-corrected. Rest follows in 1976.

MATHEMATICS WEEKEND WORK-IN 1976

41 students and 4 staff had signified interest by Dec. 1st and £5 voluntary deposits gambled to the tune of £80. Of the 41 students, 21 had been at Aston 1975 and vote: 10 = OK; 10 = Very Suitable; 1 = Never Again. Aston and Lanchester Poly are running neck-and-neck regarding prices = approx. £14 each for accommodation PLUS classrooms @ £3 per morn, aft or eve EACH. Aston has better Conditions = deadline 2 weeks before the event, Lanchester (at Coventry) = 6 weeks before. Both require 10% deposit now = about £160 for 100 places, more for more. I hope members still filling in forms will have plenty of faith and will fork out their voluntary fivers to meet this. Total price to students will be £25 as anticipated, with possible partial refund (perhaps £1-£2) afterwards if more than 100 students come or if tuition costs and expenses are less than the estimated maximum. We have to allow at least £1000 straight off for classrooms and tutors costs. One student demands a bar in Residence as a condition. OK, lad - you organise it, you get the licence, you find a barman or DIY?? Lads of similar mind please contact Joe Harrison! Probable date will be September 3-4-5. Northern students think Brum is South, Southerners think it is North. It is, factually, 3 hrs by rail from Southampton, 5 hrs from Newcastle, 1¾ hrs from London and Preston. Weekend Returns are 'cheap'. Lanchester at Coventry is further South. Aston is winning, on my estimation, but carry on voting and please hurry. Other conferences are already booking. 1975 stats show that the bulk arrived between 5pm and 7pm on Friday and left before 3pm Sunday. 1976 programme will therefore be the same = 6pm social gathering, 8pm start tuition, depart after Sunday lunch. Bridge addicts contact Richard Shreeve.

SUBSCRIPTION RENEWALS: These are pouring in at the same rate as in November 1975. By 1st December 80 had paid up, including many of the new members 'adjusting', for which we are grateful. Donations for Equipment Fund seem fantastic to me, averaging £1, ranging from 23p to £5. E.F. now stands at £150. (£700 needed!!!) Post-dated cheques are OK, if renewers are genuinely short. Please send to Peter Weir, not to me. I'm swamped with MWWI '76, data from Peter and orders for DST (and soon for Chez Angelique?). 60 copies of the First Edition of Dice Star Trek remain. Sesame has 40p price wrongly. Grateful thanks to M500 who twigged the error and sent 50p correctly. Prof. Pengelly says DST is middle school level. Our youngest known addict is 14. Parents complain that sons no longer talk but go off to rooms playing DST for hours. Peter Weir came back for 8 more copies. Enough recommendation? Enough from me? Happy Christmas—Marion.

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