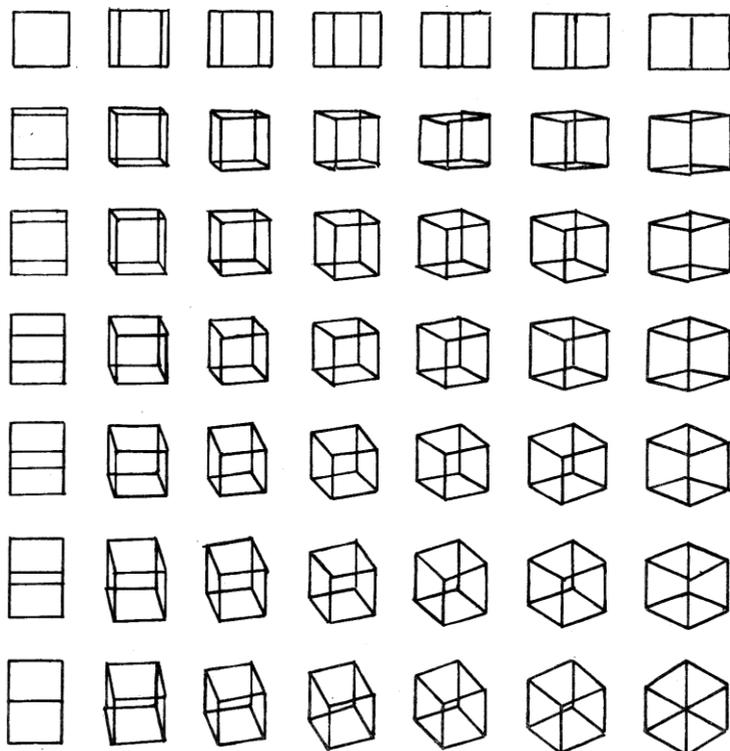


# M50029



M500 is a student-owned and student-operated magazine for Open University mathematics students and staff. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct, but invite criticism and comment.

MOUTHS is a list of names, addresses and telephones, together with previous and present courses of voluntary members, by means of which private contacts may be made by any who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned, and do not necessarily represent those of either the editor or the Open University.

The cover design is taken, regrettably without permission, from "The language of pattern; an enquiry inspired by Islamic decoration" by Keith Albarn and others, published by Thames & Hudson, 1974. This book is recommended to all interested in Vedic mathematics and geometrical designs. The drawing was the first-ever attempt at pen-and-ink tracing By M.S., who wishes, out of misery, to remain anonymous.

M500 is published by Marion Stubbs

Contributions for M500 should be sent to the editor - Eddie Kent

Subscriptions, address changes, MOUTHS data, and membership details of any kind to the membership secretary - Peter Weir

Material not covered by the above to Marion

PLEASE NOTE THAT ANYTHING SENT TO ANY OF THE ABOVE WILL BE REGARDED AS POTENTIAL MATERIAL FOR PUBLICATION UNLESS MARKED "PERSONAL".

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## AN AXIOMATIC APPROACH TO ALGORITHMS

by Don Harper

Following Markov an algorithm will be represented by the ordered quadruple  $(F, I, \theta, S)$ , where

$S$  = The universe of discourse;

$F$  = A mapping from the set  $S$  to itself;

$I$  = The input set ( $I \subset S$ ) – the least member(s) of the set  $S$ ;

$\theta$  = The output set ( $\theta \subset S$ ).

$x_n$  = An element of the set  $S$  which describes the state of the algorithm at step  $n$ . The notation  $F^n(x_0)$  is used to denote the state of the algorithm after  $n$  steps, starting from state  $x_0$ ,  $x_0 \in S$ .

**Definition.** An algorithm is said to terminate in a 'finite' number of steps if and only if for some finite  $n$ ,  $F^{n+1}(x_0) = F^n(x_0)$ ; ie  $F(x_n) = x_n$ . (For logistic systems  $I$  = the set of axioms,  $\theta$  = the set of theorems,  $F$  = rules of inference,  $S$  = the universe of discourse.)

The axioms for a solution generator algorithm (or 'complete' logistic system):

- (1)  $\forall x: x \in S \text{ and } x > x_0 \Rightarrow F(x), F^{-1}(x), F(x_0) \in S$ ;
- (2)  $\forall x \text{ in } S, x_a \leq x_b \Rightarrow F^{-1}(x_a) \leq F^{-1}(x_b)$ ;
- (3) a. If the set  $S$  has no greatest member  $x_G$  then  $\forall x \in S F(x) > x$ ,  
b. otherwise  $\forall x \in S, x < x_G; F(x) > x$  and  $\exists p : F^p(x_0) = x_G$ .

**THEOREM I** If an algorithm satisfies the above axioms then it can be used to find all positive integer solutions from the initial solution(s). (Find all 'theorems' from the 'axioms'.)

**Proof** Suppose the axioms are satisfied but the algorithm fails to generate at least one solution, say  $w$ , from an initial solution. By axioms 1 and 3,  $\exists n : F^n(x_0) \leq w \leq F^{(n+1)}(x_0)$ . Apply  $F^{-1}$   $n$  times and  $x_0 \leq F^{(-n)}(w) \leq F(x_0)$  by axiom 2. By axiom 1,  $F^{-n}(w) \in S$ . But  $F^{-n}(w) = x_0$  is a contradiction, and if  $F^{-n}(w) > x_0$  then by axiom 1  $F^{-1}$  can be used again to generate a further member of the set  $S$  which is smaller than the least member  $x_0$  – a further contradiction. Hence  $w$  cannot exist and the theorem follows directly.

Examples:

1. For the set of positive integers the function  $F: x \mapsto x+1$  with  $S = \mathbb{Z}^+$  and  $x_0 = 1$  satisfies all three axioms. Hence  $F$  can be used to generate all positive integers by iteration.

(NB.  $F: x \mapsto x+2$  does not satisfy axiom 1.)

2. For algorithms for generating integer solutions simply let  $S =$  the positive integer solution set;  $x_0 =$  the initial solution (axiom). The relations  $R = \{<, =, >\}$  are defined on the set  $S$  as follows:  $x_a R x_b$  if and only if  $x_a(m) R x_b(m)$ , where  $x(m)$  is the  $m^{\text{th}}$  variable in the  $m \times 1$  vector  $x$ . We then prove that  $F$  satisfies the axioms, thus establishing that the algorithm generates all positive integer solutions.

(For non-linear equations it must also be demonstrated that either there is only one positive integer solution with the value  $x(m)$ , or that the algorithm finds all solutions having the value of the variable  $x(m)$ .)

3. It is fairly simple to show that if  $K, L, Q, R \in \mathbb{Z}^+$  then the transformation

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} K & RL \\ L & K + QL \end{pmatrix}^n \begin{pmatrix} K \\ L \end{pmatrix}$$

satisfies the axioms and consequently can be used to generate all positive integer solutions from the initial solution  $(K, L)$ .

THEOREM II If  $F$  satisfies axiom 3a then the set  $S$  is not finite. The proof is trivial.

Ed - This is the end of the article but there are some problems associated with it which will appear next month.

"It does not trouble the mathematician that he has to deal with the unknown. At the outset in algebra he handles unknown quantities  $x$  and  $y$ . His quantities are unknown, but he subjects them to operations - addition, multiplication etc. Recalling Bertrand Russell's famous definition, the mathematician never knows what he is talking about nor whether what he is saying is true, but we are tempted to add, at least he knows what he is doing." - Sir Arthur Eddington, NEW PATHWAYS IN SCIENCE".

(from Ron Davidson)

## PATAGIUM LEPIDOPTEROUS, SCILLA'S STRECK

by S Pantos

— Plumb my dyad and sum the phone! a collocation!

— Be yours ouzel!

— The egret mine! — Slate!

Taken among the stricture of pasture and the limping  
pasturing of lines, had he locked, or in what  
sweet rings of orphine

and caking goods, the looping  
of daughters by the rills of posture and their  
ideate contusions that certitude the  
pastorals. Steve looked in by the gauss adamant  
gauze of perfume emanating the

curtained tempters. Parsonage. Skin between.

He opened his looping emblems the  
weltering skeins and pealed a tonsil from the streats of  
shaling rustics.

— Long live his grace!

— Makes your tulip sing don't it!

— Give him a chance!

— I've been tripping along since the day of Saint-Fives  
and all's not well! no! not at all well!

it had been as place-meat to his yellowed cancer!

a ganglia of pretence! I'm not the

side-dish! no! never was nor will nor shall be

and it is to be made clear unto the lies and lies and likes and his lies of him  
that these are, that there be exigencies and the shoring amber and their

placating shades that haunt cantata with the broom and patee, a drooping  
pathogenesis that rhymes my clap of tans. No. So fiord. Be blessed. Let

in pastrami. Give bird's wing and patch pattenmar. Create Jade. No blow.  
Ram patchery. Slice the tart. Kill! Kill! To cry 'asphalt'! The passenger

pigeon is at your pastern gate. The surpassingly young.

— Strawberry leaves.

Passifloraceae. Ovaries filamentous.

Safe-conduct home. Passeriform and songbird.

Disraeli of the ghibelline shears.

— I see of yours. Tou gradiate of cursories and tankard.  
 But wait- please- move the minuets but a music of faster crone that our  
 ambivalents in horsemanship might  
 sides the forepaw and fetlock with joiner and hod that our thoroughfares  
 run entrails-gee:  
 the drooling endive. — Had a gittern that could dance it could on the stalks  
 and fonts of dear Brodowsky's furnace. Shipped it to Mother McGilly's  
 corridor, an alley of balsam black commutants' and the shrieking antic-cloth,  
 greying pia-matter (sepulchred).

— Yes. Old Joe's Trace. Hand in his pockets while perching the  
 passerine passerer chiefly clamatores on backboard and sheeting. Passant.  
 Walk to the left. Hand me the streale. Passegarde scamandrian; crayon  
 shield. Parturitive litigant partridgewood rasorial paschal facetious dexter  
 paturifacient  
 confederate parturiency Siva of pasch flower cabinetwork parvenu pericarp  
 parthenium pentine monticulate

## SUMMER LIGHTNING

Yvonne Kedge

Please may I return to the world of MOUTHS?

I finally sank on M331 in 1975 with a 4.6 grade for the project and  
 am hoping this counts as a pass. Still it was a good course and I am glad to  
 have done it.

I returned to full time teaching last year, after part time 'odd jobs'  
 while our children were too young for school, and cannot understand the  
 OU's or MOUTHS' attitude to teachers. Perhaps only other teachers know  
 how hard we work, how many parents' evenings etc there are and perhaps  
 the strains of teaching in a large comprehensive (or any school?). Also I now  
 find the middle class world of parenthood, ie swimming, spelling tests,  
 music lessons, making 'puffs' (don't ask me to explain that one) must now  
 push the OU far into the background despite the support of my husband.

This is where I have really missed a Summer School and envied the  
 weekend which was, very sadly, impossible for me to attend. So I would like  
 to add my voice to that of Patrick Sharkey's and plead for 3rd level Summer  
 Schools where Mathematics may be gluttonously imbibed.

## THE AXIOM OF ARCHIMEDES

Datta Gumaste

In M500 26 I had asserted that the "Axiom of Infinity" is equivalent to the Axiom of Archimedes.

By "Axiom of Infinity" I mean the following statement:

If  $n \in \mathbb{N}$  then  $n+1 \in \mathbb{N}$ , where  $\mathbb{N}$  is the set of natural numbers. Less formally, this means that there are infinitely many natural numbers.

By "equivalent" I mean the logical equivalence between two statements. Thus  $p$  is equivalent to  $q$  means  $p \Rightarrow q$  &  $q \Rightarrow p$ .

I will attempt to demonstrate the equivalence of Axiom of Infinity and Axiom of Archimedes.

Let  $p$  = Axiom of infinity,

let  $q$  = Axiom of Archimedes =  $\forall x \in \mathbb{R}^+, \exists n \in \mathbb{N}$  such that  $n \geq x$ .

This is equivalent to saying that  $\mathbb{N} \subset \mathbb{R}^+$  is unbounded.

I:  $p \Rightarrow q$ .

Proof: Suppose not. Then  $\mathbb{N} \subset \mathbb{R}^+$  is bounded. Therefore  $\exists x \in \mathbb{R}^+$  such that  $n \leq x$  for  $\forall n \in \mathbb{N}$ . Since  $x \in \mathbb{R}^+$ ,  $x$  is an infinite decimal. Consider the integral part of  $x$ . It is an integer. Call it  $k$ . For this  $k$ ,  $k \leq x$  and  $\forall n \in \mathbb{N}$ ,  $n \leq k$ . In words,  $k$  is the largest natural number such that  $k \leq x$ . Hence, for this  $k$ ,  $k \in \mathbb{N} \Rightarrow k+1 \in \mathbb{N}$  is false. But this contradicts  $p$ , ie the Axiom of Infinity.

II:  $q \Rightarrow p$ .

Again, suppose not. Then  $\forall n \in \mathbb{N}$ ,  $n \in \mathbb{N} \Rightarrow n+1 \in \mathbb{N}$  is false. This implies that  $\exists k \in \mathbb{N}$  such that  $k \in \mathbb{N}$  but  $k+1 \notin \mathbb{N}$ . This  $k$  is the largest natural number. Consider the real number  $k+a$ ,  $a \in \mathbb{R}^+$ .  $k < k+a$ . But for this  $k+a$  we cannot find  $\forall n$  such that  $n \geq k+a$  since  $\forall n \in \mathbb{N}$ ,  $n \leq k$ . This contradicts  $q$ , the Axiom of Archimedes.

Given that I have not made any mistakes, using similar argument as contained in I above, we can show that  $\mathbb{H} \subset \mathbb{Q}^+$  is unbounded, where  $\mathbb{Q}^+$  is the set of positive rationals. Are there ordered fields other than  $\mathbb{R}$  and  $\mathbb{Q}$  which satisfy the Archimedean Axiom?

29/6

M500

From Roger Bridgman - Every contributor must be allowed his personal style of course, but I have to say I find a lot of M500 heavy going first thing in the morning. I think there's a temptation to pack too many ideas into too small a space. M500 25 was full of articles I found indigestible. Actually I don't think I'm a natural mathematician, but I have survived M201 complete with condensed extracts of Nering so I am a bit distressed to find this happening.

I think M500 should be a happy mathematical relaxation, not a difficult supplementary course unit cunningly made compulsive with anecdote and agony column. Maybe people could just say how they stumbled on the results, instead of making them look so terrifyingly professional! Personally I find that about one simple result per page is about all I can take. If you look at a well-written undergraduate textbook like Spivak, you'll find that this is about the rate at which the material unfolds (allowing for the fact that a page of M500 is only about half a page of Spivak).

After all, if the stuff puts me off, what on earth will it do to any adventurer from the arts who chances to take a peek? Or do we want to frighten them all away?

From Michael Masters - I would like to say how much I have enjoyed M500. At first I was a bit terrified of all the long mathematical expressions and new words and I thought that as a mere M100 student the magazine would be too high for me. Luckily though the first impression soon gave way to pleasure and now I can at least understand a few of the words and symbols!

From Barry Chinchen - Many thanks for sending me M500. It is quite the best mathematical periodical I have seen. We used to get some when I was at school and I was not impressed - too much showing off in those. M500 has avoided this. I was most interested to read the OPUS problems.

From John Hampton - I have not been greatly impressed by M500 over the past year. I think £3 annual subscription for a student publication of this kind is too much. Have fewer issues with editorially selected articles with a real mathematical content been considered? Issue 27 was almost barren of mathematics.

From Maureen Childs - Please send 1 Dice Star Trek. Thoroughly enjoyed 27 - expect it's because Marion's on every page!!

From Alun Davies - I am fascinated by the idea of my own space wargame, that I can play for months on end. Please therefore send me your Dice Star Trek.

May I also thank you for the effort you put into M500. There must be plenty of subscribers like me who fell by accident into maths courses of the OU. But beauty and interest of the subject are brought out in the articles from M500. For this I thank you.

29/7

e  
John Reade

### Error Analysis of $e - (1 + (1/n))^n$

The error is roughly equal to  $\frac{1}{2}e \times 1/n$  for large  $n$  which bears out Peter Weir's guess (M500 28 5) that the number of correct decimal places in  $(1 + (1/n))^n$  is the same as the number of digits in  $n$ .

To see this consider the expansion

$$\begin{aligned}(1+x)^{1/x} &= e^{1/x \log(1+x)} \\ &= e^{\frac{1}{x} \left( x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right)} = e^{\left( 1 - \frac{x}{2} + \frac{x^2}{3} - \dots \right)} \\ &= e \cdot e^{-\frac{x}{2}} \cdot e^{\frac{x^2}{3}} \cdot \dots \\ &= e \left( 1 - \frac{x}{2} + \frac{1}{2!} \left( \frac{x}{2} \right)^2 - \dots \right) \left( 1 + \frac{x^2}{3} + \frac{1}{(2!)} \left( \frac{x^2}{3} \right)^2 + \dots \right) \\ &= e \left( 1 - \frac{x}{2} + \frac{11}{24} x^2 - \dots \right)\end{aligned}$$

valid for  $|x| < 1$ .

Writing  $x = 1/n$  we have  $(1 + \frac{1}{n})^n = e \left( 1 - \frac{1}{2n} + \frac{11}{24n^2} - \dots \right)$

which shows that  $e - (1 + \frac{1}{n})^n = \frac{e}{2n} + O(1/n^2)$  for large  $n$ .

### NOTATION

Ray Geo. Tiver

I started to do MST281 last year, I had been practising mathematics as I knew it for some time; I thought I would have an even chance. But I was totally unprepared for the American notation and phraseology. I have been informed that there is nothing wrong with it - many people of this country have managed to master it, why could not I? I find it illogical and impossible to understand. I read Marjoram, almost like a book, no bother at all. The only thing is Marjoram seems to wander from the point, and like many other writers does not mention the codomain.

This year I am doing S100. I would be much obliged if I could contact someone interested in Geochemistry and Geophysics. If I have any bother getting through it will be because of my lack of any interest in Biology (bugs and grubs) - stones and bones always seem so much cleaner to me. I spend most of my spare time in the Geological department of the Adult Education College. They have a fairly nice collection there.

## MATHEMATICS WEEKEND WORK-IN 1976

Marion Stubbs

Support for the Weekend is still coming along nicely. At present the voluntary £5 cheques to help meet the required deposit amount to £168, from 49 students. 49 other students indicate "interested" and "want details" when available. Last year, when we had 65 students "interested" I risked £50 deposit from my own pocket, booked up at Aston - and then 30 of them never booked at all. It is really very difficult to assess whether the 49 "interested" students for MWWI '76 are likely to book or not. Perhaps the 49 "interested" for '76 could find even a solitary £1 as hard evidence of serious intent, please? Post-dated quids would be acceptable in the intended spirit, if necessary. Sorry if I sound cautious but someone reporting on Aston in M500 27 thought that the responsibility ought to be more evenly divided for '76, and this is the kind of shared responsibility which is really needed! The clerical and admin side is trivial by comparison. Aston is still strongly supported as venue, despite its lack of beauty. Peter Weir, with local experience, says that Lanchester Poly is very similar, but older in some parts. Like Aston, it still has a ringway careering past and high-rise buildings. Since Lanchester's Conditions ref final deadline would have made '75 completely impossible to run successfully I see no reason to desert Aston, personally. The date is still stuck at September 3-4-5, since Region 08 (North-West, centred on Manchester) seems to have some sort of tutorial(s) on every Saturday thereafter, whatever the rest of the UK has.

Only M334 is showing any significant interest in an Easter Weekend. Further comments on this from M334 would be welcomed. It might be feasible, with luck, and then you could all come to Aston as well for your other courses!

Thanks to everyone who has shown interest, especially to the 36 deposit-helpers. Now I hope that you will ALL go out into the highways and byways and persuade non-M500 students to come along too. They are not forced to join M500 to enjoy this facility, the Weekend being completely Open. In fact even your spouses would not be rejected. How about this for a golden opportunity to show them what really goes on at "Summer Schools"? Prices for spouses not using tuition would be for accomodation only, details on request. One wife came to Aston in 1975, namely Linda Forbes (Mrs Tony), who could, presumably, be contacted by any other spouse wanting info.

CORRIGENDUM - Last month I told a fair number of people that Peter Weir had coped with 111 enquirers from Sesame in one week. It turned out that he was using Roman numerals, merely clocking up each enquiry as it arrived, and forgot to convert the total to 3 before posting. Three is, of course, the usual sort of response to Sesame publicity of any description. - MS

29/9

## TUESDAY'S CHILD

Steve Murphy

### An Alternative Method for Calculating: the Day of the Week

corresponding to a given date. (see M500 24 10.)

The following formula is said to give the day of the week corresponding to a given date. Spot checks seem to confirm it works, but is anybody prepared to comment?

$$F \equiv ([2.6M - 0.2] + K + D + [\frac{D}{4}] + [\frac{C}{4}] - 2C) \pmod{7}$$

where F is the "day-number"; Sunday = 0, Monday = 1, etc

M is the "month-number" taking March = 1, April = 2, etc

K is the "day of the month" - 1-31 as appropriate

D = year in century, ie 52 for the year 1952

C = century, ie 19 for the year 1952

[x] = the largest integer  $\leq$  x.

Thus for 17 December 1975 M = 10, K = 17, D = 75 and C = 19, leading to  $F \equiv 3 \pmod{7}$  so that the day was a Wednesday, (it was): Gregorian Calendar only. Any offers for the orresponding Julian formula?

PI

### To Marion Stubbs from Margaret Corbett

I am also ignorant about  $\pi$  and probably much more so than you but am using a Hewlett Packard calculator which "performs all calculations by using a 10-digit number and a power of 10" - and this is always done "internally regardless of how many digits are displayed." So its output will have less error than that of your calculator. According to my calculator

$$16 \arctan \frac{1}{5} - 4 \arctan \frac{1}{239} = 179.9999999 \quad (1)$$

$$180.9589195 - 0.95891958$$

$$4 \arctan 1 = 180.0000000 \quad (2)$$

$$r = \frac{180}{\pi} = 57.29577951 \text{ where } \pi = 3.141592654$$

$$(1)/\pi = 3.141592652$$

$$(2)/\pi = 3.141592654.$$

By H.P.  $4 \arctan \frac{1}{5} - \arctan \frac{1}{239} = 44.99999998$

$$\arctan 1 = 45.00000000.$$

All of which is only a comment on the relative accuracy of two machines. Your main query is untouched. Marvellous, how did he?

Perhaps this is rather feeble comment. M500 27 arrived for breakfast on a day when I didn't have a morning clinic so

got straight on with comparative calculations – intriguing.

I see you spell better than Eddy, but perhaps it is not very important.

PI

Marion Stubbs

When I bought my 3001M calculator I wanted to check some series, eg  $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$ . And of course it was not so.  $\sin 1 = 0.017452$  whereas  $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} = 0.841471$  and further terms contribute just 0. However,  $\sin 180/\pi = 0.841471$  and we are back to the point where I wanted to know how John Machin did it (see M500 27).

Actually Eddie Kent seems almost as hilariously wrong since the arctan x formula given in M500 28 is known as "Gregory's series", dated apparently c 1668. Gregory discovered the Taylor series more than 40 years before Taylor published it, says Boyer, in HISTORY OF MATHEMATICS, p422, but "neither Bernoulli nor Taylor was aware that both had been anticipated by Gregory in the discovery of 'Taylor's series';" (ibid, p462).

Brook Taylor's series were published in 1715, some 9 years after John Machin's approximation to pi. Did Machin know of Gregory's work? He is not even mentioned in Boyer. Pi was approximated by  $\frac{355}{113}$  by c 1573 and "Ludolph van Ceulen published in 1596 a twenty-place value obtained by starting with a polygon of fifteen sides and doubling the number of sides 37 times," (ibid p352).

The mind boggles - he had no calculator.

Ed - There are contribution in hand on Machin's  $\pi$  from Peter Hartley, Bill Shannon, John Reade, Tom Patton and Peter Weir. These will all get in eventually, no doubt. No more though, please, or not till next year.

And I spell better than Shakspear.

Prom Anne Williams - I'm sending a donation to the equipment fund since I believe in putting my money where my "MOUTHS" is.

Prom Tony Brooks - ...M332... is the worst I've done in any maths exam. The M332 exam relied too much on simple memorizing of definitions and proofs for my liking, and my memory is not all that good.

29/11

SKEWES' NUMBER

Max Bramer

If  $li(x) = \int_0^x du / \log u$  (the logarithmic integral function) and  $\pi(x)$  = number of primes < x, Then Skewes' number, S, is an upper bound for the first x for which  $li(x) < \pi(x)$ .

$$S = e^{e^{34}} \approx 10^{79} \cdot 10^{10}$$

The number of digits in S is approximately

$$10^{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

which compares well with 1000! (2568 digits),

$2^{2^{1945}} + 1$  (approx  $10^{60}$  digits) and the sixth-millionth prime (only 9 digits!)

For further information see Skewes' paper in London Mathematical Society Journal vol 8 (1933) p277; or John Littlewood's book A MATHEMATICIAN'S MISCELLANY (Methuen, London, 1960) p113. These notes are taken from EXCURSIONS IN NUMBER THEORY, C Ogilvy and J Anderson, OUP, 1966.

From Keith Charsley - Having read the Special Issue of M500 I find myself in agreement with most people's comments on the courses I have done, especially Tony Brooks's opinion on the requirements to do mathematics. However I feel that Hugh McIntyre is wrong in advising people not to take M201 and M202 simultaneously. If one feels able to take the two together there is no reason why one should not. In fact speaking from experience I found a welcome counterbalance between the applications of M201 and the "purity" of M202.

Congratulations on the success of both MOUTHS and M500, and I think that M500 is well worth the new subscription.

From Marilyn Howels - I think the idea of M500 and MOUTHS is great - but as usual it's taken me the best part of a year to do anything about it.

From Sue White - Please find enclosed cheque for another year's contact with sanity within the OU, with grateful thanks for this year.

From Major Peter Ost - Service requirements, exercises etc, plus lack of a tutorial service caused me to drop back past the point of no return in my course of M332. As I want a reasonable pass, and am prepared to wait for it I opted out of the exam this year and will repeat the course next year.

Anyone who is passing through Minden in West Germany is welcome to pop into my workshop.

29/12

## WILLOWBURN HOTEL

Riky Rickard

Sorry there's no donation this year but as you can see from the address I'm struggling to make ends meet opening this new hotel. In between growing vegetables, cooking enormous meals and uncorking wine bottles I've had little time for maths (particularly 3rd level!) except to observe that on certain days Telford's "Atlantic Bridge" (see below) with its reflection appears to form a perfect circle - I would like to see a proved solution, so all M500s BOOK NOW! How about a maths weekend up here? (Bring skis if winter, boats if summer.)

I can see it all now:



Scallops Spivak

-

Vectors of Venison

-

Gateau Galois

all washed down with Pouilly Fourier, chateau bottled of course!

If all goes well I promise a donation next year.

## ESSAYS

Tom Dale

I won't now take T321 in 1976 as I intended - I've been scared off first year technology courses as a result of T341, although, to be fair, I understand a lot of the trouble arose from a printing dispute. But if you don't want to write essays, don't take T341. The first TMA asked for 'about a thousand words' and the others have been just about as bad. (But TMA4 was drastically reduced - the course team seem to have realised that they have badly underestimated the amount of work.) I have found it a most frustrating course - partly perhaps because it is potentially so interesting, but the delays have made it so difficult to keep up with. The Summer School was frustrating too - again potentially interesting but never quite maintaining the interest. The lecture I enjoyed most had nothing to do with the course - it was about computer aided design. It was fairly technical and more suited to TM221.

I have tried too in my time to be a philosopher; but, I didn't know how, cheerfulness was always breaking in.

Oliver Edwards (in Boswell's Johnson)

## DICE STAR TREK AND CHEZ ANGELIQUE

Marion Stubbs

DST continued to sell steadily, often 2-copies-ordered-at-a-time, throughout December. A few copies still remain, and a Second Impression is under consideration, so you and your friends can still send along orders. If no reply by return post (second class mail), guess that I am running off a further supply of all 42 pages of it, which is no mean task! If your friends (or son's friends) are ordering, it would be interesting to know how they heard about it, so just ask them to mention it when ordering, please. I'm always very curious.

Chez Angelique is not even printed yet, but patience will be rewarded, never fear. CA is not one of the Stubbs Enterprises, but you can order via me if you prefer. I shall pass on the orders to the Appropriate Authorities, and may yet end up as the postal distributor of this anticipated masterpiece from our lords and masters. Please try to keep cheques for DST and CA separate from M500 subs and donations, as this makes bookkeeping so much easier. M500 subs and forms are supposed to go to Peter Weir, please, not to me. DST is definitely MINE, ALL MINE, and nothing to do with M500. CA is neither; but can be accommodated. If it is all too complex for you to work out send payment wherever you like! We maintain completely accurate ledgers and all "accounts" (Subs M500; Donations M500; Maths Weekend '76; DST; CA) are included, and the whole lot balances nicely, thank you, with the bank account with Nationwide.

We still want a Chartered Accountant to volunteer to produce an annual balance sheet, even if unwilling to do the whole job as M500 Treasurer, please. Surely, statistically, there MUST be at least one Chartered Accountant in the house? If not, or if some incognito C. A. member is unwilling to offer his/her expertise I can go it alone (bravely) in my annual arguments with the Inland Revenue. At least this ensures that your subs are as minimal as possible, since we have to make a neat little loss annually to avoid tax - and a constitution would not help either, Jim Marchant! Still, I would prefer a proper Treasurer, or at least a professional-looking balance sheet and a Constitution, not for financial gain but for democratic reasons. No Treasurer and no Constitution have yet shown themselves. I am flattered that the membership seems so satisfied with the present Capitalist Dictatorship, but it is definitely Not British, folks, and is perhaps a scandal in other OU eyes, if they care at all.

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From Peter Needham - I think it is a very good idea setting up a M500 Society. I think it would be a good idea if you state what you feel the aims of the Society would be; so we could have some debate in the matter.

I feel that it might be an idea to have short reviews on interesting articles in other maths mags. Something like Monitor in New Scientist. I also think that an article (every now and again) about maths outside the OU would be of interest.

From Margaret Kyle - I went to the exam sort of in a dream - I felt so unprepared but! if I get through (miracles do happen) I will certainly be at your next maths weekend in the midlands so £10 following... . Keep up the good work and, incidentally, I admired your thanks to 'Sacred Heart' in Sesame - good for you! - How would we manage indeed without Him.

From Mervyn Savage - Regarding the format of M500, I have no desire that it should be produced in a grander style than now. However I am quite prepared to pay for a printed version if this makes production less tedious.

From Jeremy Humphries - You ask how the exams went. Don't !!  
(Ed - Jeremy also came up with the solution to problem 28.1 but that page is finished.)

From Michael James - I have at last got round to doing something about M500 and MOUTHS, and would be grateful if you would include me as a subscriber and as a member of the MOUTHS list. What has really galvanised me is your excellent issue 24 which I found very helpful and drew me inexorably to enrolling for M202 (rather than my first choice of M201). I had found it very difficult to appreciate the merits of the various courses at the time the registration forms came round as my counsellor could only describe the courses in terms of future units of M100 and terminology which at the time was meaningless. Also I gained the impression that M202 had an ogre-ish quality and should be attempted only by the stout of heart and experienced. However, your correspondents do in the main help to dispel this impression and I feel it would be an interesting challenge, especially if it does for one all that Peter Weir suggests it will.

I am also tempted to drop at least one of my two exemptions (if that is possible) especially having read your advice and due to an unwillingness, now that I am hooked, to have to give up taking OU courses until absolutely necessary. If I were to drop both exemptions presumably I would have to take another Foundation Course, but that is perhaps to the good in that a Technology course would be useful in my occupation whereas Mathematics is just a fun activity.

I now feel that being involved with MOUTHS would certainly be of assistance at times as, being a member of a very small tutorial group I get a rather isolationist feeling at times. When I first heard about it I obtained the impression that the membership was basically confined to the South coast and that in the beginning there would not be much point in anyone telephoning me. However, I now have the confidence not only to seek assistance but to provide for others where I can.

POCKET CALCULATORS Michael Masters:

I comment I wanted to make about a suggestion for a future issue. I am bewildered by all the pocket calculators on the market and I was thinking that a number of readers must have one themselves and could provide a Which? type report for the assistance of those wanting to obtain one.

## PRONUNCIATION

Eddie Kent

There have been suggestions that we run a little series on the pronunciation of names and terms which turn up in mathematics and which place the Open University student at some disadvantage when chatting in pubs.

For instance ln is pronounced "log", not "IN"; Euler = oiler (not "youler"); Lebesgue goes something like Le-BAY-g. Icosahedron - I cossa HEAD (HEED?) ron; dodecahedron starts DOUGH... .

This is another area where Summer Schools would make a lot of difference to post foundation maths students, ensuring that even an OU degree implies some experience of the oral tradition which is one aspect of British University life. AM289 is likely to be rich in unpronounceables; how do you say Machin? (or Bernouilli?)

But why does everyone always try to pronounce  $\leq$  as "less than or equal to"? (I say try because it is one of our more difficult and less lovely English phrases) when it would be so much simpler to use "not more than". But if you use that to a mathematician he immediately gets suspicious and starts looking for hidden meanings.

## YOUR OWN COMPUTER

Marion Stubbs

Creative Computing intends to report "in future issues" on building your own computer from kits or plans; in particular building an Altair 8800 computer kit. The Altair 8800 claims, in its sales ads, to be the Number One hobby computer in the whole world. The total price is quoted (Christmas 1975) as \$544, being Altair \$439, Memory \$97 (1K), both unassembled; or \$621 for Altair assembled plus \$139 for Memory (1K) assembled. Extra Memory up to 4K and various interfaces are available at additional cost. I just wish I knew something about electronics.

## C H E S S

Rager Claxton

I wish to make contact with anyone who would like to play postal or telephone chess. If there is sufficient interest it might be worth setting up an M500 league or something, but I am quite happy just to play friendly games.

My experience of playing chess is severely limited, but I am keen to learn and happy to organise as necessary. Please write me or telephone - details in the MOUTHS list.

CONTRIBUTIONS - could contributors very kindly not write on the back of sheets (unless it is a continuation of what was on the front) as I have a proliferation of files with different headings and like to tear sheets up. Thank you. EK

## SOLUTIONS

27.1 PERRY

See diagram on cover of M500 27

Since  $P_1, P_2$  are points on the curve  $P_1B_1 - a = b(c + P_1A_1)^n$ ,

ie  $TA_1 = b(SB_1)^n$ , similarly  $TA_2 = b(SB_2)^n$ ;

therefore  $TA_2/TA_1 = (SB_2/SB_1)^n$ , ie  $1 + A_1A_2/TA_1 = (1 + B_1B_2/SB_1)^n$ ;

therefore  $1 + T_1A_1/TA_1 = (1 + B_1S_1)^n$  since  $\angle A_1A_2T_1 = \angle B_1B_2S_1 = 45^\circ$ ;

and  $1 + \tan \theta = (1 + \tan \phi)^n$  where  $\angle YTT_1 = \theta$ ,  $\angle XSS_1 = \phi$ .

(b)

Hence  $1 + T_2A_2/TA_2 = (1 + B_2S_2/SB_2)^n$ ;

therefore  $1 + A_2A_3/TA_2 = (1 + B_2B_3/SB_2)^n$  since  $\angle A_2A_3T_2 = \angle B_2B_3S_2 = 45^\circ$ ;

therefore  $TA_3/TA_2 = (SB_3/SB_2)^n$ .

But  $TA_2 = b(SB_2)^n$ ; so that  $TA_3 = b(SB_3)^n$ ,

$y_3 - a = b(c + x_3)^n$  and  $P_3$  is on the curve!

(a)

28.1 HINTS TO SURVEYORS

Divide a  $\triangle$  site in 2 equal areas

by fence at  $\angle$ rt to frontage.

Solutions: Henry Jones (1), Steve Murphy (2) and Michael Gregory (3).

(1) has  $CF = \sqrt{\frac{1}{2}BC \cdot CE}$

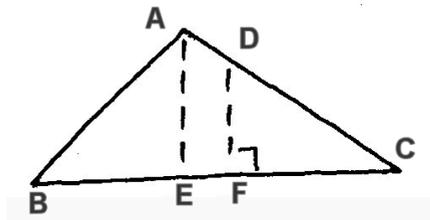
(3) derives the same solution thus:

equal areas  $\Rightarrow \frac{1}{4} AE \cdot BC = \frac{1}{2} DF \cdot FC$ ;

similar  $\angle s \Rightarrow DF/FC = AE/EC$ ; etc.

(2) has  $CF = \frac{1}{2} \sqrt{BC^2 + |AC^2 - AB^2|}$

with  $|AC^2 - AB^2| \leq 2 \cdot BC^2$ .

28.3 ROLL ME OVER - I have been told that this can be done in 38 moves.

Solution promised in time for next month.

28.4 INTO THE UNKNOWN

Bill Shannon says "Ship A (the pursuer) proceeds two-thirds of the distance to the point where ship B entered the fog-bank, then takes a course corresponding to a logarithmic spirral." Similar looking solutions have come from L S Johnson, Michael Gregory and Steve Murphy.

SOLUTIONS continued

28.5 UNDER-PRIMES

The number of odd primes  $< x$  is  $< (x + 2)/3$

If  $p_n$  is the  $n$ th odd prime then  $\forall x$ , the number of odd primes  $< x$  is  $P_n(x) \leq x(1 - 1/2)(1 - 1/3)\dots(1 - 1/p_n) - (1 + 1/2)(1 + 1/3)\dots(1 + 1/p_n) + n + 2^r - q_n(x)$  where  $r = n$  if odd otherwise  $n + 1$  and  $q_n(x)$  is the number of composite numbers  $< x$  which have a prime factor  $> p_n$ .

Taking  $n = 1$ :  $F_1(x) \leq 1/3 x + 1 - q_1(x)$ . If  $x \geq 16 \exists$  at least one composite number - 15 -  $\rightarrow p_1 = 3$ . Hence  $x \geq 16 \Rightarrow P_1(x) < 1/3 x$ . The gap up to  $x = 15$  is filled in by direct trial and the result follows. - Steve Murphy.

Another: There are  $x$  integers ( $\geq 0$ )  $< x$ . If  $x$  odd  $1/2x + 1/2$  even integers  $< x$ ; if  $x$  even,  $1/2x$  such. If  $x = 3k - r$  ( $0 \leq r < 3$ ) with  $k$  even,  $1/2k$  odd multiples of  $3 < x$ ; with  $k$  odd,  $1/2(k-1)$  such. Four cases result: i-  $x$  odd  $k$  even; ii-  $x, k$  odd; iii-  $x, k$  even; iii-  $x$  even  $k$  odd. Numbers of odd nonmultiples of  $3 < x$  are therefore by inspection i-  $x/3 - 1/2 - r/6$ ; ii-  $x/3 - r/6$ ; iii-  $x/3 - r/6$ ; iii-  $x/3 - i/2 - r/6$ . Setting  $x = 0, 1, 2$  we get in no case a number  $\geq (x+2)/3$ . In this number are all the odd primes  $< x$ . - Hugh McIntyre.

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P R O B L E M S

29.1 SEVEN SPIROS MAKE AN AGNEW - Sue Davies

But in how many ways?

S P I R O

That is, how many solutions are there to the problem on the right? Is there a pattern?

7  
A G N E W

29.2 NEWTON'S COWS - Jeremy Humphries

- a cows graze b fields bare in c days,
- a' cows graze b' fields bare in c' days,
- a'' cows graze b'' fields bare in c'' days.

What relation exists between the nine magnitudes a to c''? Make the obvious assumptions: cows are equi-voracious, fields are similar and grass grows at a constant rate &c.

29.3 DONUT SLICER - Marion Stubbs

What is the maximum number of pieces you can get with three simultaneous cuts through one doughnut? Generalise!

## EDITORIAL

This month there are no MOUTHS numbers shown. At our last conversation Marion had not finally decided on the form the list will take. If we had a section called, "notes on contributors" it might mention that Don Harper signed himself "ex M100", S Pantos sent his poem from 1200 Mass Ave, Apt 33W, Cambridge, Mass 02138; which makes us M500 International. John Reade is staff, as is Max Bramer. And the rest are just like us, more or less.

For people like John Hampton who want M500 to be exclusively mathematical there are two points to be made. In the first place we are a democratic society consisting fundamentally of MOUTHS members with M500 as packaging: members decide the basic form of the magazine. Secondly we can only print what you send. If you want more maths, write more. If you want something else write and say so and we'll put that in. MOUTHS exists because of academic isolation. M100 students don't realise yet what is going to happen to them: less students per course, more distance to travel, less tutorials, no counsellor, NO SUMMER SCHOOLS.

One letter that will not appear in M500 criticised the OU in strong terms for a certain failing and ended with a note asking me not to publish the author's name. I have never liked anonymity and have always regretted the OU policy on this point. I would like to make it clear that although occasional pseudonymous contributions have appeared in M500; and notwithstanding that I will print scurril till the paper cringes, if it is signed; I will not put in anonymous abuse.

Just recently people have started telephoning me to ask if I have received a contribution from them because it had not appeared in M500. The society's expense sheet will not run to acknowledgment cards like newspapers send but my postman is very friendly and I don't think he loses many things. The point is that at the moment there are a few items on hand so that I don't begin panicking as the month draws to an end and fill up with garbage. However, you and I both know that this situation will not persist throughout the year. As pressure builds up non-essential output falls and so I am trying to hold onto some of the non-ephemera to provide a backbone through the dog months. So don't necessarily expect anything you write to be in the next issue. And in any case each issue probably leaves me a fortnight before you get it. Not that I mind being telephoned - in fact I am just about to pay my bill.

*Eddie Kont.*

**No-one has yet sent anything for the special issue 1976. Deadline Jan. 31st.**

**I know just what it feels like: it was such a short time ago 24-Special appeared. But there are lots of people who had nothing in that - surely they have something to say. And what about comments from the staff. Time is getting short till the end of January.**

**PS: 3 bits have just arrived. Cheers - Marion Stubbs**