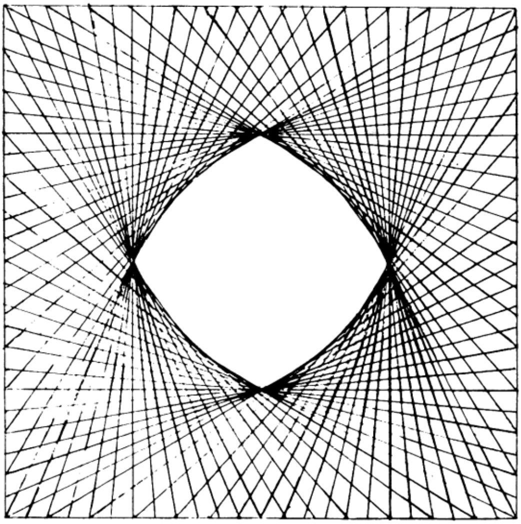


M500 45

M500 *is a student operated and owned magazine for Open University students and staff, and friends. It is designed to alleviate student academic isolation by providing a forum for public discussion of the mathematical interests of students.*

Articles and solutions are not necessarily correct but invite criticism and comment. Anything submitted for publication of more than about six hundred words will probably be split into installments.

MOUTHS *is a list of names and addresses, with telephone numbers and past and present courses of voluntary members, by means of which private contacts can be made, to share OU and general mathematical interests - or to form self help groups by telephone or correspondence.*

There is also a special list of those MOUTHS *members who have explictly volunteered for their* MOUTHS *details to be distributed to members in closed institutions such as prisons.*

The views and mathematical abilities expressed in M500 *are those of the authors and may not represent those of either the Open University or the Editor.*

PUBLISHER	Marion Stubbs	(In charge of cover designs and missing pages.)
EDITDR	Eddie Kent	(Articles and letters for M500.)
PROBLEMS EDITOR	Jeremy Humphries	(Problems and solutions for M500.)
MEMBERSHIP SECRETARY	Peter Weir	(Applications for membership, change of address, MOUTHS data.)
TREASURER	Austen F Jones	
PRINTER	Waterside Printing Company 11 Rumbridge Street Totton.	

M500 45 published August 1977 (but late); subscription £3 for 10 issues (but see page 8). Cheques and postal orders (which should go to Peter Weir) should be made payable to THE M500 SOCIETY and crossed a/c payee not negotiable for safety in the post.

ANYTHING SENT TO ANY OFFICER OF THE SOCIETY WILL BE CONSIDERED FOR POSSIBLE INCLUSION IN M500 UNLESS MARKED PERSONAL.

The cover is Tony Brooks' solution to Problem 43.3. He calculates the area of the forbidden territory in the middle of the diagram equal to

$$\tfrac{1}{2}\{ 1 - \ln\tfrac{1}{2} + \ln\tfrac{1}{4} \} = 0.153\,426\,409\,7$$

taking the square as unit area.

INTERSECTING DIAGONALS OF POLYGONS JOHN READE

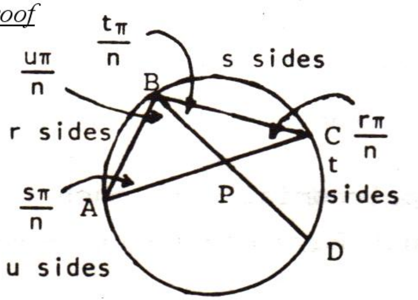
Lemma 1 A regular polygon with n sides has three diagonals intersecting whenever the Diophantine equation

$$\sin\frac{a\pi}{n}\sin\frac{b\pi}{n}\sin\frac{c\pi}{n} = \sin\frac{x\pi}{n}\sin\frac{y\pi}{n}\sin\frac{z\pi}{n}$$

$$a + b + c = x + y + z$$

has a solution for a, b, c, x, y, z all integers.

Proof



By the sine rule

$$AB = \frac{\sin\frac{r\pi}{n}}{\sin\frac{\pi}{n}} \text{ and}$$

$$AP = \frac{\sin\frac{r\pi}{n}}{\sin\frac{\pi}{n}}\,\frac{\sin\frac{u\pi}{n}}{\sin\,(s+u)\frac{\pi}{n}}\,.$$

If a third diagonal is to pass through P we must have

$$\frac{\sin\frac{r\pi}{n}}{\sin\frac{\pi}{n}}\,\frac{\sin\frac{u\pi}{n}}{\sin\,(s+u)\frac{\pi}{n}} = \frac{\sin\frac{r'\pi}{n}}{\sin\frac{\pi}{n}}\,\frac{\sin\frac{u'\pi}{n}}{\sin\,(s'+u')\frac{\pi}{n}}\,,$$

therefore $\sin\frac{r\pi}{n}\sin\frac{u\pi}{n}\sin(s'+u')\frac{\pi}{n} = \sin\frac{r'\pi}{n}\sin\frac{u'\pi}{n}\sin(s+u)\,\frac{\pi}{n}$. Note that $r+s = r'+s'$ so take $a = n-r$, $b = u$, $c = s'+u'$, $x = n-r'$, $y = u'$, $z = s+u$.

EXAMPLES

1. Pairs of equal diagonals intersecting on a diameter (n even) correspond to the solution

$$\sin\frac{a\pi}{n}\sin\left(a+\frac{n}{2}\right)\frac{\pi}{n}\sin\frac{2b\pi}{n} = \sin\frac{2a\pi}{n}\sin\frac{b\pi}{n}\sin\left(b+\frac{n}{2}\right)\frac{\pi}{n}$$

where a and b are any integers.

2. The off-diameter intersection when $n = 12$ is equivalent to

$$\sin\frac{\pi}{12}\sin^2\frac{5\pi}{12} = \sin^2\frac{\pi}{6}\sin\frac{7\pi}{12}\,.$$

If $\zeta = e^{2\pi i/n}$ the Diophantine equation becomes ➔

$$(\zeta^a-1)\ \zeta^b-1)\ \zeta^c-1)\ =\ (\zeta^x-1)\ \zeta^y-1)\ (\zeta^z-1)$$

where again $a + b + c = x + y + z$, which on expanding gives

$$\zeta^a+\zeta^b+\zeta^c-\zeta^{b+c}-\zeta^{c+a}-\zeta^{a+b} = \dots .$$

Lemma 2 If p is prime and $\zeta = e^{2\pi i/p}$ then $1,\ \zeta,\ \zeta^2,\ \dots,\ \zeta^{p-2}$ are linearly independent over the rationals.

Proof The polynomial $f(x) = 1 + x + x^2 + \dots + x^{p-1}$ is irreducible over the rationals since it equals

$$\frac{x^p-1}{x-1} = \frac{(1+(x-1))^p-1}{x-1}$$
$$= p+\frac{p(p-1)}{2}(x-1) + \dots$$

which is irreducible by Eisenstein's criterion.

Since $f(\zeta) = 0$ it follows that any polynomial $g(x)$ such that $g(\zeta) = 0$ must be a multiple of $f(x)$ so must have degree at least $p-1$.

Corollary For n prime the Diophantine equation of lemma 1 has only trivial solutions, hence the correspondinding polygon has no non-trivial triple diagonal intersections.

Proof We must have $a, b, c, y+z, z+x, x+y = x, y, z, b+c, c+a, a+b$ in some order, which can only happen trivially.

This clears up Richard Ahrens's problem for polygons with prime numbers of sides. I cannot decide yet whether his conjecture is true for all polygons with an odd number of sides. Can anyone else finish it off?

Anyone who can solve $x^2 - 92y^2 = 1$ within a year is a true mathematician.

Brahmagupta. (C7 Indian mathematician)

(*From JH, who writes*: $x^2 - Ny^2 = 1$ *is called Pell's equation because Euler mistakenly called it so, and the name stuck. Pell never solved it.*)

DIFFERENTIAL EQUATIONS ANNE ANDREWS

I hope I haven't depressed any readers by that title, but I thought what follows might provide a useful (and not quite straightforward) means of revising some aspects of M100 calculus. It also strikes me as a mildly interesting "application" of elementary differential equations - an application in the sense that it's the sort of thing biology students come across in undergraduate courses. It was provided by a class of (non-OU) biology students to whom I teach mathematics. They wanted me to solve it for them.

The first formula method for solving differential equations that is discussed in M100 is separation of variables. There are various simple applications of this type of equation: for example the discussion of population growth problems. The equation $Q' = kQ$ there discussed can also be used to study reactions involving an enzyme; then $Q(t)$ is the concentration of the enzyme at time t. The problem I intend to present also deals with an enzyme-catalysed reaction but there are two enzymes present. Let the concentrations of the enzymes at time t be $Q_1(t)$, $Q_2(t)$ respectively. The rates of change of these concentrations are governed by the equations

$$Q_1' = Q_2' = kQ_1Q_2. \qquad (1)$$

It is required to find the ratio of the two concentrations explicitly in terms of t - ie we want $Q_2(t)/Q_1(t)$ and we can assume we know the initial concentrations $Q_2(0)$ and $Q_1(0)$.

As a start it would be nice to disentangle (1) so that we have an equation in Q_1 and another in Q_2. To achieve this you can use the following:

Hint 1 Brood on the implications of $Q_1' = Q_2'$ for a bit. For example $Q_1' = Q_2'$ is the same as $Q_1' - Q_2' = 0$. What does this suggest?

If you use the hint - or another method - successfully you obtain equations in Q_1, Q_2, each of which can be solved by separation of variables - unfortunately the solutions are a bit messy. A more eleqant solution can be obtained by thinking again and, in particular, thinking of the result we want, namely Q_2/Q_1. Try giving this function a name, say P, and extract from (1) a single differential equation in $P = Q_2/Q_1$.

Hint 2 Differentiate P and use (1) and Hint 1.

(Note: Of course this is harder than most M100 problems but I hope you will find it solvable, with or without the hints, and be cheered up to have solved a non-standard problem by simple methods.)

(Ed - note: The solution appears later in this issue for those, like me, who would rather not think if it can be reasonably avoided.)

(Note 3: Anne Andrews is on the Staff.)

LETTER FROM AMERICA DAN FOX

It was very impressive to see that I have now been 'published'. M500 must be a saviour to budding authors.

In answer to a question from Marion, yes public education is free. That is to say we pay for it via taxes. The rule here is that property taxes (on land and buildings) levied by individual states/counties/cities are passed on directly to the schools: no more and no less. Thus in areas with lots of valuable property a relatively low tax rate can generate enough dollars to finance very good schools. On the other hand, poor counties (and most cities as of the last few years) have not enough property value and too many students to allow a good school system. Thus the rich have good schools for their kids while the poor uneducated masses living in the urban centres like New York, Chicago, Los Angeles, Washington DC, etc end up with depressingly underfunded schools. This shows up in lower test scores, high dropout rates, poor morale, violence and so on. Federalization of school funding seems to be the answer with a uniform tax rate across the country and dollars passed out on the basis of number of students. Unfortunately there is a significant block of people who panic whenever the government seems to be expanding its influence. The idea of the federal government 'controlling the minds of our children' just flips them out. By convention and law, education is deemed to be a local matter.

Thus the Fed has no direct hand in education here. At the college level the same prevails except that there are a lot of students who get loans and grants from the government. (Determined by financial need and not academics.) Also there are megadollars in research grants to colleges.

With regard to post offices, zip codes and so on: recently our dear postal dis-Service spent untold dollars on huge semi-automated sorting equipment with the result that an indecent percentage of boxes and all bulky letters are being crushed and torn to shreds - thereby decimating addressing etc. The letter sorting machines are little better. This has prompted some of us to wonder if the designers weren't originally (a la Monty Python) employed by a manufacturer of shredding machines. One hesitates these days about mailing packages and even important letters. Perhaps its all a great plot by the phone company to drum up business. They sure do need it too. Their profits from the first three months of this year were a meagre $\$1.0\times10^9$. Discounting oil companies, Ma Bell (as she is affectionately called) makes more money than anyone else. In fact, according to the People's Almanac (a great book):

"ATT is the biggest monopoly in the world. It holds assets of $\$60 \times 10^9$ which is more than the combined gross national product of every country in Africa. Only seven countries have GNPs greater."

Outside of the government, ATT also has the largest number of employees: one million.

The people's Almanac is a really great book and I highly ➔

recommend it to your library. It has almost 1500 pages of facts you don't normally find in a regular almanac. For example a woman pope, hollow earth theories, psychoanalysis of Nixon, drugs, human behavior experiments, famous wills, and 'the most odd, unusual, strange, unique, incredible, amazing, uncommon, unheard of, fantastic facts in the world.' As an example from that last chapter:

"the longest period of time for which a modern painting was hung upside down in a public gallery unnoticed is 47 days. This occured to Le Bateau (Matisse). In this time 116000 people had passed thru the gallery."

Or how about this one:

"There was once a Society for the Prevention of People Being Buried Alive, and a number of devices were patented to help further the members aims. The simplest of these was an ordinary electric bell, by means of which anyone who woke up down below could raise the alarm. A more cynical device was a coffin fitted with nails which, when driven home punctuated capsules of gas."

I see M500 has a piece about palindromes and the magick 196. (M500 35 1, 41 11, 42 5.) *The Best of Creative Computing* vol 1 has a neat little article which was my intro to the subject. 196 and its friends (295 394 493 592 691 689 788 887 986) all form 1675 after a few reversals. This number has so far failed to become palindromic after even 79K reversals. Interestingly enough, 1675 translated into base 2 is 11010001011: a number which is already a palindrome.

Problem for a rainy evening: what is

$$\int_0^\infty e^{i(ax+\sqrt{1-bx^2})}\,dx$$

for real a,b (both > 0). The sign of the $\sqrt{}$ is taken so the integrand goes to 0 as $x\to\infty$.

SQUARE ROOT COMPETITION POSTSCRIPT PETER WEIR

Mick Bromilow of the Maths Faculty sent in an entry for this competition (see M500 44) more than a month late - he must have thought it to be a course unit.

He combines Minimax (M201/M351) and Newton–Raphson combined:

(1) Scale the number, y, to (1, 100].

(2) Let $t_0 = (2y+20)/(11+2\sqrt{10})$. (Best error approximation for $y\in$ [1, 100] .)

(3) $t_{i+1} = ½(t_{i+y}/t_i)$ i=0, 1, 2; Newton–Raphson.

Mick reckons this gives excellent results quickly and I can't prove him wronq. His prize - a chance to pay for his own copy of M500 instead of pinching Alex Wilkie's.

DIFFERENTIAL EQUATION - SOLUTION ANNE ANDREWS

We seek to solve $Q_1 = Q_2 = kQ_1Q_2$ (1)

for Q_1/Q_2. Set $Q_1/Q_2 = P$.

Then $P' = (Q_1Q_2' - Q_1' Q_2)/Q_1^2$

$= (Q_1 \cdot kQ_1Q_2 - kQ_1Q_2 \cdot Q_2)/ Q_1^2$ (substituting trom equation (1))

$= (kQ_1Q_2 - kQ_2^2)/Q_1$.

We aim for an equation in P so note

$$P' = k\frac{Q_2}{Q_1}(Q_1 - Q_2) = kP(Q_1 - Q_2). \qquad (2)$$

To remove $(Q_1 - Q_2)$ use Hint 1. Note that $Q_1' - Q_2' = 0$ implies $(Q_1 - Q_2)' = 0$, ie $(Q_1 - Q_2)$ is a constant function, α say. Thus (2) becomes

$$P' = k\alpha P. \qquad (3)$$

We can now use separation of variables after all. Following the handbook (but of course we have P instead of Q and P' for DP) we see that g is $t \longmapsto 1/t$ and $h = k\alpha$. (I'm just forgetting about domains, but of course domain $P = \mathbb{R}_0^+$.) The solution set is given by $\ln P = (t \longmapsto k\alpha t) + (t \longmapsto c)$. Write as

$$\ln P = (t \longmapsto k\alpha t) + (t \longmapsto \ln c_1)$$

ie $$\ln\frac{P}{c_1} = (t \longmapsto k\alpha t)$$

ie $$P = \frac{Q_2}{Q_1} = c_1 \exp (t \longmapsto k\alpha t)$$

- or just write this down by inspection of the familiar equation (3).

Putting $t = 0$ we obtain $\frac{Q_2}{Q_1}\frac{(0)}{(0)} = c_1$, so

$$P(t) = \frac{Q_2}{Q_1}\frac{(t)}{(t)} = \frac{Q_2}{Q_1}\frac{(0)}{(0)} \exp k\alpha t.$$

Finally, recall that $Q_1(t) - Q_2(t) = \alpha$ for all t; in particular $Q_2(0) - Q_1(0) = \alpha$. Thus our final answer is

$$\frac{Q_2}{Q_1}\frac{(t)}{(t)} = \frac{Q_2}{Q_1}\frac{(0)}{(0)} \exp^{k(Q1(0) - (Q2(0)t},$$

which is of the form required.

THE M500 SOCIETY

THE CONSTITUTION - RON AITKEN and MICHAEL GREGORY

Understandably, there has been some conflict as a result of the appeal by two students In Broadmoor Hospital to join their fellow undergraduates of the Open University in M500/MOUTHS, and no one can be indifferent to the anxieties which their request has raised.

The main points we have thought of are:

(a) Each member of M500 should decide for himself whether his name and address should be sent to prisoners;

(b) We wish to avoid a significant drop in M500/MOUTHS membership;

(c) We don't want a "thin end of the wedge" after which more and more members are put on special lists - or excluded altogether;

(d) We fear that the Authorities might make checks on those who contact their involuntary guests;

(e) The existing M500 Constitution needs amending to allow for the existence of a special list, and for not circulating the full MOUTHS list to all members.

The Open University has provided restricted students with an interest which is creative and may also help to combat despair. Without the goodwill and active cooperation of their fellow students, OU courses alone will not suffice. As always, there is no easy and complete solution to all the problems, but we hope you will find the following proposals for amending the Constitution both constructive and acceptable.

AMEND RULE 2 to allow for three major publications.

add new sub-paragraph

"3. MATES

MATES shall be a directory of voluntary members who are also subscribers to M500. Members of MATES shall undertake to correspond with fellow members in closed institutions, to assist them-mathematicallv, or to share mathematical interests and problems. The MATES directory shall be circulated only to MATES."

AMEND RULE 3

delete: "directory"

substitute: "and/or MATES directories"

This contribution flows from exchange between a small group of students. We offer it now for discussion and further revision, if required, in the hope of optimising results for all members.

(*With acknowledgements to anonymous referees.*)

TREASURER'S REPORT AUSTEN F JONES

The results for 1976, which you should have received by now, were rather disappointing but 1977 appears to be an improvement. However, the subscription rate for 1978 will need to be raised to £4, following a period of two years when it was fixed at £3.

However, there are at least two alternatives to raising the sub, viz:

(1) Duplicating rather than printing M500 - this would save printing charges but would necessitate additional labour costs for compiling, stapling, etc.

(2) Affiliation with OUSA, where we received a capitation fee or other income from OUSA. However, this may involve obligations on our part. I intend making initial enquiries along these lines and will inform you of the discussions at a later date.

These are preliminary views and your thoughts or ideas will be appreciated by either Marion or myself .

The moon's D+tre gilds the trees.
And, blown from sections in the north,
The su%ed evening breeze
With 10der coaxing lures ¼

A love 6swain, I wander here,
An$ound the mighty pines
Their wide embracing branches rear
Deep rooted as the Apen9s.

On thi7tful night I've sent
My .ic billet-doux;
With lots and lots of sentiment
I've vowed 2 1 th@ I love true.

I've put the ? so profound;
I 1der i5 said it ½;
Ah, would I could my love X£
In one short, —ing ¶.

Her father's handy †s were
Of small a/c my love Bside.
I'll all dis* 4 her
Until I × the gr8 ÷.

Plus; percent; ten; one-fourth; six; dollar; nine; seven; period; two; one; at; question; one; five.; half; pound; dash; paragraph; dagger; account; asterisk; four; cross; eight; divide.

M202 - JOHN HAMPTON

I have just returned from Summer School at Reading. Without exageration I had a superb week with excellent tuition and enthusiasm from the staff (T C Lister (director), R W Ahrens, A Best, L M Smith and R J Webster) which has given me, and I think many other students, an entirely new confidence about M202. I started this course immediately following a rather nasty attack of depression and I had great difficulty in gettinq going with it at all. For weeks I could find no proper working schedule and despite good TMA/CMA results felt I should give it up. However, by June things were looking up and then Reading came along. Now I am converted and find it difficult to explain how I felt only a few weeks ago! I think that there can be little doubt that M202 is a difficult course, but I am convinced that this arises essentially from the abstract nature of much of the material it considers and some familiarity/experience of this is absolutely necessary before appreciation finally dawns. If students feel uncertain of their ability to cope with pure mathematics then I think that they should give M202 a miss and take M203 instead. However, those that feel more confident and like a challenge might like to take M202 in 1978, its last year of presentation. At Reading Professor Rourke told us that a third level Galois Theory course is planned for presentation from about 1980, with another possible course on Algebraic Topology sometime later. Both of these have a significant overlap with the later parts of M202. If you want two easier third level half credits later on do M202 <u>now</u> before it disappears.

A LOOK BACK AT MST281 - DERYK JENKINS

It is to me a mystery as to how I survived it at all. Volume 1 of the set book is the most annotated and most absolutely worn out book I have ever had in my possession. I am a great one for trying to link things so that all my maths books contain cobwebby lines which sometimes go across and around two open pages. It is remarkable that I never had to continue a line <u>over</u> a page but I would have managed it had it been necessary.

It never ceases to amaze me how for some people the penny drops immediately; in fact for some gravity is not enough, their pence are accelerated by mental kicks which sometimes startle the Tutors at Summer School into the error of thinking that the rest of us are as bright. There was a good looking ginger girl who, regrettably, had a consort apparently as bright as her. Between them they would chatter incomprehensively while the tutor was trying to make his point to the rest of us and come up with the answer a full ten seconds before it was necessary. By the end of the week the protection afforded her by her good looks even to a sex maniac had worn extremely thin; the rest of us were prepared to bop her on her pretty clever head.

For me the penny never dropped until, thank God, just before the exam. I was aided by our local M100 tutor who really knew what it was all about and who, unpaid, gave us a lot of encouragement as well as tuition. I was also aided by two very informative and able letters written to me by David Crowe of the OU Maths Faculty. If ever one is in complete despair over a problem I do recommend a ➔

letter to the Faculty.

About text books, I thought the OU-authored *An Introduction to Algebra and Calculus* vols I II & IIIwere terrible, and for this reason: it was written by people who never had to struggle and because of this they entirely missed the places where step by step explanations were necessary and the places where little intuitive jumps could be taken safely. I have noticed this before: the briqht ones never know where the dullards can make the jumps. Since the bright ones are the folk who write the books the books are full of tedium in the wrong places.

The crowning point of bitterness with MST281 is only to be awarded ½ credit. This, to me, throws the whole of the credit system out of the window. I did both S and T 100 together in the previous year. At no time was I subjected to the same agonies of understanding as happened with the maths course but I was awarded 2 credits for almost less effort. For what then are credits awarded?

PM951 - COMPUTING AND COMPUTERS. MAX RRAMER

Very little publicity seems to have been given to this course which is being offered to Undergraduates (as well as Post-Experience/ Associate students) for the first time next year, as a half credit at second level.

The course has been running since 1973 (as a PE course only) and is completely different in emphasis from M251. with which it will be an excluded combination, because of a common element of BASIC programming. Students who already have a course certificate in PM951 will be able to transfer it for a half credit (as long as they have not also gained a ½-credit in M251 of course). There are no prerequisites, certainly not Ml00 Units 8 and 20 from which it is also radically different in approach. A very wide field is covered and illustrated with a number of case studies of real-life systems. In fact, it is probably best summarised as a serious introduction to computers and their uses for those who are not and do not necessarily intend to become computer specialists themselves. Full details are in the Associate Student publications. If any readers have taken this course it would be interesting to have their opinions - alternatively I could be prevailed upon to supply further information if anyone is interested.

He has 150 cattle.
He thought there were 147
Until he rounded them up.

(Sent by Nick Fraser who took it from *A Pun My Soul* by Alan F G Lewis, obtainable from 27 Odds Farm Estate, Wooburn Common, High Wycombe, Bucks, HP10 01A. Nick wants M500 readers to submit puns. He says it is easy: You jest, Put pun. To paper!)

OBITUARY - NAUM GABO

Naum Neentia Pevsner, the Constructivist sculptor who took the name Gabo late in life was the hero, for those that can remember that far back, of Michael Gregory's articles on *Constructions* (M500 6 3 and 28 1).

He was born at Bryansk, Russia, 87 years ago; the son of an engineer. His elder brother trained as a painter but Naum was sent to Munich to study medicine. While there he transferred to the natural sciences and, excited by the experimental work going on in Munich he made his first attempts at sculpture. He went to Norway in 1914 where he made his first heads of sheet metal and celluloid in what was to become his characteristic, open stereometric form.

Back in Russia after the war, he produced *Realist Manifesto* (1920) where he argued (it is still readable) the necessity for pure art.

In 1922 he went to Berlin for the exhibition of soviet art and stayed there, producing works which "have the appearence of models for monuments in some machine-city of the future." In 1931 he designed a project for the Palace of the Soviet - but Constructivism was now discredited in Russia.

He went to Paris in 1932, and England in 1936, settling in Hampstead and joining the circle that included Henry Moore, Ben Nicholson, Herbert Read, Imre Moholy-Nagy, Piet Mondrlan and Walter Gropius. He became joint editor of *Circle*. In 1936 he moved to Cornwall and in 1946 to the US.

He had a retrospective exhibition at the Tate in 1966 when Michael Gregory saw and was impressed by his *Linear Construction Number* 2. I believe this work is still at the Tate, or there is another similar one (perhaps *Number* 1 as it is not numbered). For comparison there is a work by his brother, Antoine Pevsner called *Macquette of a Moment Symbolizing the Liberation of the Spirit* 1952, in bronze but having the same linear form. It was said of Gabo and Pevsner that their linear and sheet forms "may exist in three dimensional space but are not themselves in any but the most tenuous way three dimensional." They frame but do not fill space.

In 1952 Gabo took American citizenship and recleved many honours, but was still never given a chance to realise his plans for really architectural sculpture. (The only large thing he ever made was the 50ft monument for the Bijenkorf Building in Rotterdam, 1957 although most of his works aspired toward the monumental.)

His aim-was to express the scientist's conception of space. He made the first example of kinetic sculpture ever - it is now in the Tate. A single vertical steel wire is vibrated electrically to produce a wave movement expressing perfect, clear yet insubstantial form.

His materials: glass, clear plastic, thread strung over an open form, were expressions of the ideal expounded in *Realist Manifesto*, "we renounce ... mass as a sculptural element." About 1936 he began to search for a spherical expression of space; but he never used mathematical formulae as the practice later became. He always insisted on his intuitive approach.

He was made an Hon KBE in 1971.

(See Read and Martin: *Gabo*.)

EK

PROBLEMS SECTION EDITED BY JEREMY HUMPHRIES

I've been getting some solutions to dead problems lately. That's fine with me - all letters are welcome - but unless they break new ground late arrivals probably won't get into the magazine.

Also, some work which I get is good but long; I just haven't got room for it. Sometimes I must cut, condense, summarise, even merely print results. If you are particularly interested in anything and want more details don't be afraid to ask - I can always send you a copy; and if you're not as poor as the Problems Editor, a stamped addressed envelope would be nice.

My friend ROGER STANGAR hopes to run a course of ten lectures next year on Lebesgue Integration. This will be at Hatfield Polytechnic, in the evening, and free. Roger says that "the course is necessarily introductory and limited in application, but someone who has attended will be able to start on the theory of partial differential equations." Prospective 321s and 331s send for details.

I shall be going away for a few days about the middle of August and I hope to send an early Problem Corner to Eddie for 46 before I go. Please send your solutions for the 44 problems as early as possible or they might not get in. It's probably already too late to tell you this, but if your bit doesn't appear you'll know why.

* * * * * * * *

MIKE PURTON has sent some work on Problem 41.5 - Disc Covering, but I haven't managed to look at it properly yet, so I'll put it into 46. (There are great demands on my time lately.)

SOLUTI ON 43.1 OLYMPIAD III *If one places as many cubes as possible, of volume* 2, *in a rectangular box of integral dimensions, the box is* 40% *full. Find all such boxes*.

STEVE AINLEY, STEVE MURPHY, CHRIS PILE and MIKE PURTON sent two boxes: $2\times 5\times 3$ containing 6 cubes, and $2\times 5\times 6$ containing 12 cubes. If we must place the cubes orthogonally, which is what they all assumed, then they are right. But the problem doesn't say that. On the 5×6 floor of the $2\times 5\times 6$ box we can put five cubes along a diagonal, then three down each side of this line, then one in each of the empty corners. Total 13.

Note that the cubes don't lie exactly parallel to the diagonal. If they did we could squeeze in only 4.998999... down the middle. However there is plenty of room for all the others, so a bit of judicious twisting gets them in.

I haven't done any more investigating. There may be another box of which a fiendish maximal arrangement of '2' cubes fills 40%. Otherwise $2\times 5\times 3$ alone is correct. In which case L S JOHNSON inadvertently got it right using a method which should ➔

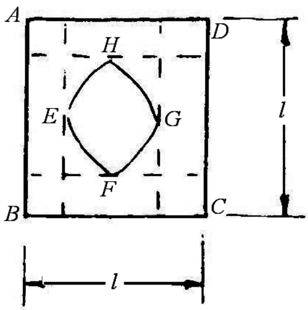
also have found 2×5×6 but didn't. So no marks all round I'm afraid.

When Mike had his first go at this he was working from memory and thought the problem said "40% empty". He assumed orthogonal packing again and says: 'This was a very much more difficult problem, involving heavy use of the calculator.' He sent in six pages of work which produced 40 boxes. I don't fancy going through them to see which ones are eliminated by removing the orthogonal restriction, but I shall be pleased to hear from anyone who does; and from anyone who can find any more solutions to the original problem.

(*Ed - The problem said "Find all such boxes". It might be an idea to try and construct a proof that there is only one. And what about generalising the problem: How many sets of* (*orthogonal*?) *blocks of dimension a×b×c are there which with judicious packing will occupy no more than d% of an integral rectangular box*? *You might even get a PhD for it.*)

SOLUTION 45.3 FORBIDDEN TERRITORY *On a square of side l a straight line is drawn which divides the area of the square in the ratio* 3:1. *What is the area of that part of the square over which the line cannot pass*?

Six people sent four answers to this. The three who got it right were STEVE AINLEY, MIKE PURTON and my friend TOM KITTLE (not Kettle).



The required area is $EFGH = \frac{l^2}{2}(1 - \ln 2)$

$= 0.1534...\, l^2$.

It's fairly easy to find the area by calculus by using eg AB, AD as x and y axes; thougn one of the wrong answers got this far and merely made a mistake in the calculus.

Note that the four boundaries of the forbidden territory are rectangular nyperbolas, and are asymptotic to the sides of the square, since the product of a tangent's intercepts on the appropriate two sides is constant. if we take AC as the x-axis, A as origin, then the equation for EH is

$$y = \pm\sqrt{x^2 - a^2}.$$

We can find the area under EH by integration; and multiplying by four and adding to the area of the square $EFGH$ ($=l^2/8$) gives the same answer as before.

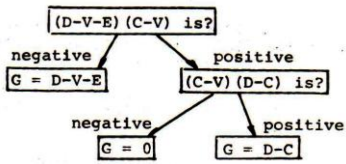
SOLUTION 43.2 TAXATION

Only STEVE AINLEY tried this. He writes

C = cost; D = amount received on disposal; V = value on 6/4/65; E = expenses; G = gain/loss.

* shows that Jane has a chargeable gain of 0. I think the expenses point is ambiguous, but Jane's case is clear. I usually get flow charts wrong, so I confidently expect this is:

(D-V-E)	(C-V)	(D-C)	G
+	+	+	D-C
+	+	-	0
+	-	+	D-V-E
+	-	-	D-V-E
-	+	+	D-V-E
-	+	-	D-V-E
-	-	+	0*
-	-	-	D-C



SOLUTION 43.4 CRIBBAGE *What is the probability of getting a* '29' *hand at two-handed cribbage*?

Several people sent attempts at this. Only two got the same answer, which I think is right. STEVE AINLEY and STEVE MURPHY gave 1/216,580.

My statistical friend STUART SIMPSON did it by three different methods and got that each time too.

Any 1 of 4 fives for turn up = $\binom{4}{1}$ ways.

The other 3 fives and the jack out of 51 = 1 way.

Any other 2 out of 47 = $\binom{47}{2}$ ways.

$\therefore$ probability = $\frac{\binom{4}{1}}{52} \cdot \frac{\binom{47}{2}.1}{\binom{51}{6}} = \frac{1}{216580}$.

SOLUTION 45.5 PRIME SQUARES *Arrange the numbers* 1 *to* 16 *in a* 4 × 4 *array so the sum of any two which are horizontally or vertically adjacent is prime. How many can you find*? *Why can't the sum of two diagonally adjacent numbers be prime also*?

When MICHAEL MCAREE sent me this he gave five examples and stated without demonstrating that these led to another 183 solutions giving a total of 188. STEVE AINLEY sent one solution: "and it took some hours to find that!" I was consequently a little taken aback to receive from MIKE PURTON ten closely written pages showing, by means of various ingenious dodges, how to find 2640 solutions. It's too long to be published but anyone ➔

interested, ...

Mike says at the end: "I believe this is known as the method by exhaustion. I find that the main effect of my joining M500 is that I have spent more time than I can really afford in trying to solve your problems."

Ah, yes. The M500 officers know the feeling well. Still, keep at it. We do.

The answer to the last part, also from Mike, is:

Since primes (except 2) are odd, adjacent vertical and horizontal squares must be even and odd. Therefore diagonally adjacent squares must be either both even or both odd.

Summary of results: With 15 and 14 in the central squares there are no combinations. With 11 in the central block we have, reading from left to right along each row in turn, 14, a, b, 13, 5, 8, a, b, c, 11, 12, a, 7, c, 1, b (128 combinations) or 14, a, b, a, a, 2, 1,b, 8, 11, 6, 13 ,5, 12, 7, b (112 combinations), where a is 3,9 or15; b is 4,10 or 16 and c is 6 or 12. If 12 is central we have 13, b, a, b ,b, 1, 2,a, 7, 12, 5, 14, 6, 11, a, a (112 combinations) or 13, b, a, 14, 6, 7, b, a, d, 12, 1 ,b, 8, d, 2, a (128) with d as 5 or 11. 5, 6 central gives 12, 7, b,13, 11, 6, 1,b, 8, 5, 2, a a,14, a, b (112) or 14, a, b, a, a, 2, 1, b, 8, 5, 6, 13, 11, 12,7, b (112). 7 central: b, a, f, 5, 13,b, a, f, 6, 7, b, a,11,12,1,2 (128), f is 8 or 14. 8 central: a, b, a, h, 6, 14, a, b, h, 5, 8, a, b, 12, 11, 2, 1 (128); h is 7 or 13. 1,2 central: f, a, f, a, d, 2, a, b, k, 1, b, h, d, k, h, b, (1008) or k, d, 8, a, d, 2, a, 14, k, 1, b, a, h, b, h ,b, (672); k is 6 or 12. Total 2640. Each case of course excludes those before it. All symmetrical variations are included. (Any arrangement produces 8 others by rotation and reflection in a diagonal.)

PROBLEM 45.1 OLYMPIAD V (18th International Mathematical Olympiad 1976, Lienz Austria. 2nd question July 13.)

Consider the system of p equations in q unknowns, where $q = 2p$.

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1q}x_q = 0$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2q}x_q = 0$$
$$\ldots$$
$$a_{p1}x_1 + a_{p2}x_2 + \ldots + a_{pq}x_q = 0$$

with every coefficient a_{ij} a member of the set {1,0,-1}.

Prove there exists a solution ($x_1, x_2, \ldots, x_q$) of the system such that:

(a) all x_j ($j = 1,2, \ldots ,q$) are integers
(b) there is at least one value of j for which $x_j \neq 0$
(c) $|x_j| \leq q$, ($j = 1 ,2, \ldots ,q$) .

PROBLEM 45.2 DISTANT POINTS MAX BRAMER

In an infinite two-dimensional lattice of points with integer coordinates, the distance between two horizontally, vertically or diagonally adjacent points is defined as one unit.

The distance between two points is defined as the length of the shortest path between them, traversed by way of adjacent points.

The distance of a point (x, y) from a fixed point (a, b) can be easily shown to be

$$\max(|x-a|, |y-b|).$$

Suppose that the point (p, q) and all eight adjacent points are removed from the lattice. How is the distance formula affected?

PROBLEM 45.3 BLIND CHESS TONY FORBES

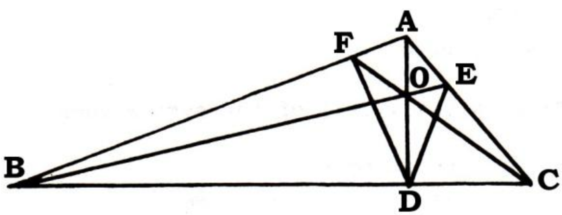
You (white) have a King and two Rooks. Black has only a King, which is invisible. Either devise a checkmating strategy or show that black can escape when you play on:

(a) a normal board,
(b) an infinite quarter plane,
(c) an infinite half plane.

Black will tell you if he is mated. Your strategy must ensure that you never make an illegal move.

PROBLEM 45.4 BISECTOR C FWRIGHT

This is your chance to earn some money for M500. Fred offers his thanks and £1 to the equipment fund if anyone solves this.



AD is perpendicular to BC

0 is any point on AD.

Show that AD bisects angle FDE.

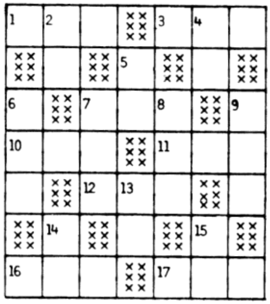
PROBLEM 45,5, HOW MANY WERE THERE AT ST IVES?

(From *Fun with Figures* by L H Clarke; by kind permission of Heinemann Educational Books Ltd.)

The 'bus, making its daily journey between Cadford and St Ives, has three intermediate stops - at Wallop, Odham and Salchester. At each of these stops some people got out and others got in. How many were in the 'bus when it arrived at St Ives?

ACROSS

1 The cube of the number who got in at Salchester.
3 The cube of the number who got out at Odham.
7 One third of 1 ACROSS (to nearest whole number).
10 The cube of the number who got out at Wallop.
11 One half of 1 ACROSS.
12 Double 8 DOWN.
16 The same as 7 ACROSS.
17 Twice 16 ACROSS.



DOWN

2 The total number of passengers who got out before the final stop.
4 The total number of passengers who travelled by the 'bus that day.
5 The total number of passengers who got in at intermediate stops.
6 The cube of the number who got in at Odham.
7 This contains the same figures as 1 ACROSS but in a different order.
8 Five times the number of passengers the 'bus started with.
9 nine times the total number of passengers carried.
13 One fifth of 12 ACROSS.
14 The cube of the number who got in at Wallop.
15 The cube of the number who got out at Salchester.

I've been dipping into *Laws of Form* by G Spencer Brown, which I found in the library. (See M500 39 18.) I can't say what value it has but the bits I've tried make very nice reading. He writes with wit (... writing on the surface of the Earth. Ignoring rabbit holes etc, we may take it to be a surface of genus 0.) and his 'primary algebra' will solve the typical Lewis Carroll sorites in one line. I shall carry on with it - perhaps get some ideas for Problem Corner.

There is a rave review in the front by Bertrand Russell; though that may carry questionable weight. I believe that by the late sixties (the book is 1967 - 1969) poor old BR had long since fallen amongst weirdos and, some say, gone dotty.

JH

EDITORIAL

Jerremy Humphries has come up with six pages again, which is considerably more than was ever allowed to problems before he took over the section. I am of course very pleased: less work for me and, from the response he is qetting, popular with the readership. I for one have stopped getting letters to M500 complaininq of the difficulty of reading the magazine.

On the other hand I've almost stopped getting letters at all. Fine, except that we may yet end up with several blank pages in some issues.

Last month some people got messed up copies it seems. Those that wrote to me about it suffered considerable delay in getting their complaints dealt with, largely through my lethargy and inefficiency. It would be a good idea in the future to remember that Marion Stubbs, as publisher, is the best person to get in touch with about anything to do with the magazine as a physical entity. For those that don't know her I can assure them that she is almost friqhteningly efficient.

I have been asked to give Dan Fox's address (see page 4). It is: 6802 Lake Kenilworth, #216, New Orleans. La 70126, USA.

And Marion has asked me to say that Professor Pengelly has written a nice letter thanking everyone for their efforts over the Special Issue.

I am still getting letters about subjects from the past which could go in if there was room and if I had time to sort out the wheat from the chaff, as it were. John Wills for instance has written to point out that 29830 Judeans returned from captivitv to take up residence in Jerusalem. About 20000 nonexiles had never left the place and nerhaps a few came from Egypt. The congregation of *Ezra* 2:64 is the <u>whole</u> church in Jerusalem after Sheshbazzar took over. Well, I am glad to get that cleared up. It may teach those cynics among us not to interfere with other people's scholarship. But I doubt it.

Letters on some other subjects are unlikely to get published however, at least in the forseeable future. And heading the list of these is the subject Prisoners and MOUTHS; although I imagine we will find room for comment on the constitution.

I will end with the next stanza of our continuina saga, which unaccountahly seems to have been missed out of the main bodv of the magazine this month:

...
Ter, quater. atque iterum cito vorpalissimus ensis.
    Snicsnaccans penitus viscera dissecuit.
Exanimum corpus linquens caput abstulit heros
    Quocum galumphat multa, domumque redlt.
...

Eddie Kent