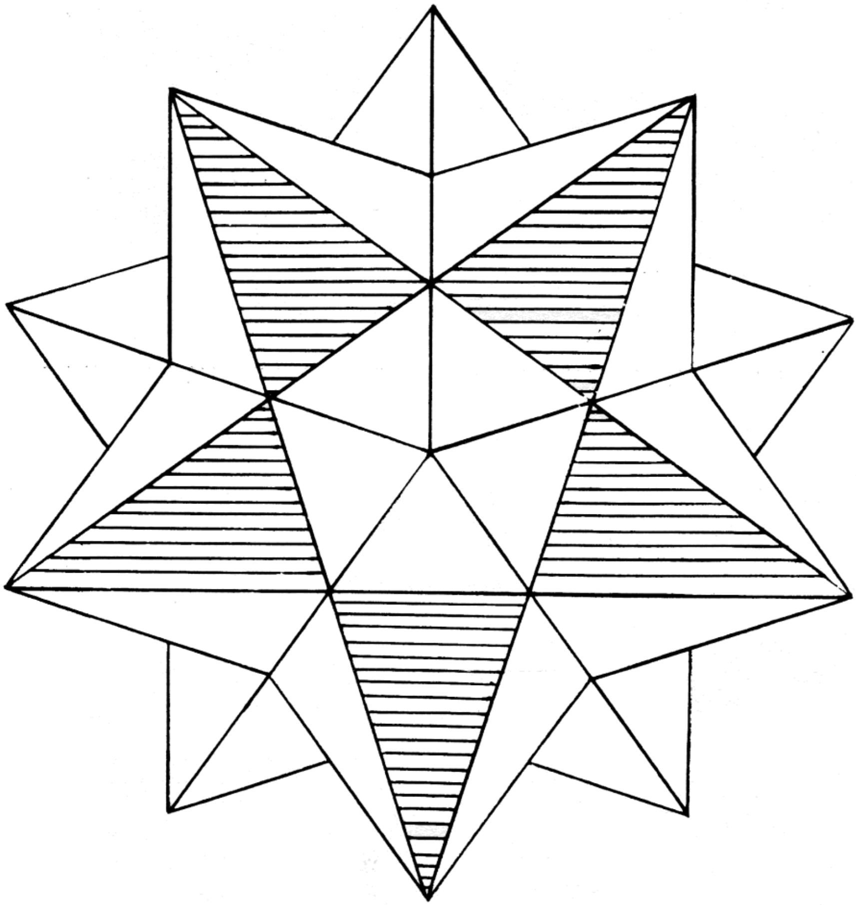

M500 46

M500 *it a student owned and operated magazine for Open University students and staff, and friends. It it designed to alleviate student academic isolation by providing a forum for public discussion of the mathematical interets of members.*

Articles and solutions aree not necassaruly correct but invite criticism and comment. Anything submitted for publication of mone than about six hundred words will probably be split into installments.

MOUTHS *it a list of names and addresses with telephone numbers and past and pnesent courses of voluntany members by meant of which private contacts can be made - to share OU and general mathematical interests, or to form self-help gnoups by telphone or correspondence.*

There is also a sppecial list of those MOUTHS *members who have explicitly volunteered for their* MOUTHS *details to be distributed to members in closed institutions such as prisons).*

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The cover design is a diagram of the small stellated dodecahedron by Chris Pile.

ARITHMETIC OPERATIONS TONY BROOKS

In M202 Unit 3 sections 3.3.0 and 3.3.1 there is a definition of the familiar operations of addition, multiplication and exponentiation given in set-theoretic terms. This is achieved by using the successor function

$$+ : n \longmapsto n+ \qquad (n \in \mathbb{N})$$

and the recursion theorem as starting points. Anyone interested in a formal development of these ideas can consult sections 3.2 and 3.3 of Unit 3; this note has been written at a much more informal level.

The successor function simply maps a natural number to its successor so that for example in the usual notation

0+ = 1,
1+ = 2,
2+ = 3, ...

Addition can then be regarded as repeated applicatipn of the successor function. (This is where the recursion comes in.) For example $m+n$ means apply the successor funcion n times to m, which will yield the familiar result for addition. Applying the process to $n+m$, $m+(n+p)$ and $(m+n)+p$ will show that addition thus defined has the familiar properties of commutativity and associativity.

Similarly multiplication can be defined by repeated application of the addition operator, thus $m \times n$ means

$$\underbrace{m + m + \ldots + m}_{n \text{ times}}$$

($m,n \in \mathbb{N}$). Applying the same principle yet again yields the power operator:

$$\underbrace{m \times m \times \ldots \times m}_{n \text{ times}}$$

gives m^n. The obvious thing to do now is to carry the process a stage further and it is the operator thus derived that is the real point of this note. Let $m * n = \underbrace{m^{m^{\cdots m}}}_{n \text{ times}}$. Thus for example $2 * 1 = 2$; $2 * 2 = 2^2 = 4$; $2 * 3 = 2^{2^2} = 16$; $2 * 4 = 65536$.

Care is needed in interpreting these expressions. For example let $m\rho n$ stand for m^n then 2*4 needs to be calculated as $(((2\rho 2)\rho)\rho 2) = 65536$ and not as $(2\rho 2)\rho(2\rho 2) = 4\rho 4 = 256$.

This function is discussed in M202 3 33 in the solution to SAQ14. The solution points out that '*' is not commutative: easily shown by $2*3 = 16 \neq 27 = 3*2$. Nor is the function associative: $(2*1)*2) = 2*2 = 4 \neq 2 = 2*(1*2) = 2*1$.

➔

I have been investigating further properties of *. Perhaps readers may know of some reference to the function, or be able to fill in some gaps. So far the following points have occurred to me:

1. By analogy with $a^{(m+n)} = a^m \times a^n$ it seems hopeful to define $m*(n+p) = (m*n)^{(m*p)}$, but this does not work as a simple example shows: $3*(1+2) = 3^{27} \neq 27^3 = 3*(2+1)$. However it is satisfactory to use

$$m*n = m^{(m*(n-1))}.$$

2. It seems natural to extend the use of * beyond the natural numbers. There is no problem in allowing m to be positive and rational or real:

$3.5*3 = 4.372 \times 10^{43}$;
$\pi*3 = 1.34 \times 10^{18}$.

It would also be nice to extend n beyond the natural numbers. By analogy with multiplication and exponentiation extension I can define $m*r$ where $r = 1/n$:

Since $a \times (½)$ means b such that $b + b = a$;
and $a^{(½)}$ means b such that $b \times b = a$:
then $a*(½)$ means b such that $b^b = a$.

Hence $10*½ = 2.507$; $10*⅓ = 1.897$; $10*¼ = 1.658$ and $\pi*½ = 1.854$. Could r be extended to all rational and real numbers?

3. Perhaps something similar to logarithms can be devised for *. Let them be called 'clogs' (from an idea in M100). Then $\text{clog}_m(m*n) = n$, and in general, if $a*b = c$, $\text{clog}_a c = b$. Thus $\text{anticlog}_{10} 1 = 10$ and $\text{anticlog}_{10} 2 = 10^{10}$. It is fairly easy to see that using conventional logarithms

$$\log_a(a*m) = a*(m-1).$$

4. The use of * gives a very compact notation for large numbers. The number of 'elementary' particles in the universe is less than $4*3 = 10^{154}$. Also I have read that the largest decimal number that can be printed in an average size 400 page book is $10^{1000000}$ which is less than $10*3$.

Can any reader develop this operator further or provide references to information on it?

In English no-one can count higher than nine hundred and ninety-nine thousand nine hundred and ninety-nine decillion nine hundred and ninety-nine thousand nine hundred and ninety-nine nonillion ... nine hundred and ninety-nine thousand nine hundred and ninety-nine. ... The trouble is that the rule for naming the successor of a number after you have named the number is not everywhere defined; there is no successor function.

Carl E Llnderholm

WEEKEND JOYCE MOORE

Since the Lanchester Maths Weekend was to be the last of the three organised by Marion Stubbs, it was suggested there that we might express our thanks for all her work in some tangible form, and a collection was therefore made on the Sunday afternoon.

I was asked to choose a suitable present mainly because I was thought to have a aood idea of the kind of thing she would like (we work in the same library), and so I had the rare pleasure of visiting all Southampton's jewellery shops, settling in the end for a beautiful silver locket. This was engraved with the words O.U. Grateful thanks from the 1977 weekenders - 75, 76, 77, and presented to Marion at Walton Hall by Professor Pengelly. She will no doubt describe this to you herself.

I'm sure you will all agree that Marion has earned the thanks of everyone who attended any of the three Maths Weekends that she organised. I think most people don't realise the sheer nervous stress involved. All through the summer vacation in the library Marion muttered darkly about 'bankruptcy' and produced utterly incomprehensible letters from Lanchester for us to decipher. With 120 being the magical 'break-even' point we played a numbers game that was positively unnerving. Next Summer, when Sidney Silverstone will be coping instead, we'll probablv settle for a little quiet Russian roulette to calm our nerves!

So thanks Marion, for everything; and good luck Sidney.

* * * * * * * * * *

MARION STUBBS

I must express my delight and thanks to all Weekenders 1977 who contributed to the very beautiful silver pendant locket. This was presented to me at Walton Hall following the Maths Faculty Board meeting on October 4th 1977, at which I was supposed to spend an hour persuading the entire assembled maths academics that THE M500 SOCIETY was worth supporting, and indicating ways and means bv which this support might be given.

Now while I am deeply grateful, beyond expression, to the Weekenders who took such an enormous amount of trouble, effort and sheer cash to give me this exquisite memento (which incidentally is "just what I wanted", as I "collect" pendants) and will always deeply cherish and frequently wear this gift, yet the occasion itself caused me immense suffering, I kid you not. I'm a type which FEARS talking to large audiences! It took 3 × 2mg valium tranquillisers to get me to that Faculty Board at all!

Once there the valium took charge and I managed to sound sensible, according to academic critics afterwards. The Maths Faculty itself, apparently, was largely totally apathetic - duly chided by tranquillised me. (Heaven would have perhaps needed to help them if I had not been tranquillised!!) However, offers of quite massive support, from the academics who tolerated my "speech", have rolled in since the event. Particularly, Robin Wilson is now volunteering publicity for M500 in all five of M231 Stop Presses in 1978, since he is i/c M231/78 maintenance - and also ➔

wants to come to Weekends. Jean Nicolson volunteered SP publicity for M500 in AM289/78 SPs. Anne Andrews (assistant staff-tutor, Yorks, not to be confused with M334 C-T down south of same or similar name, which is doubly confusing because both ladies are or were members of M500) is busy distributing copies of the Special Issue to Yorkshire counsellors and tutors (some of whom have never even heard of M500); and sundry Walton Hall staff are apparently takinq an enlivened interest in our publications. Also, the publication of Special Issue 1978 was unanimously approved. It will be edited by Richard Shreeve (and Lytton Jarman, Steve Murphy and Tony Forbes, with the usual three staff editors). It will be distributed probably at ConReg meetings and/or Summer Schools. Get writing, folks. They want it printed and published by early April. Send contributions to Richard Shreeve .

Note however that Howard Thomas, staff-tutor, Wales, opines strongly that our Weekend should be in Cardiff so that the Welsh can get there - and meant it seriously and at length, folks! I guess the Welsh students need to read Joyce Moore's *Brotherhood of Man* in M500 38 and should learn to be a bit less insular?

The implication of this criticism in general was that if M500 ever expects to get any Welsh members then it will need to go to Wales, not *vice versa*. Suggestions - to Eddie for M500 publication - on how to attract the Celtic fringe in the current political climate would or could be useful. Frankly, I feel that If anyone puts nationalism and politics before mathematics in M500 then I don't want to know them. Over to you. You may have some positive, creative suggestions.

The only other criticism came from Allan Solomon, thought that M500 should be by and for students, not (much) including staff. This one tended to fall on deaf ears all round, as did his argument that yours truly *ought* to be militant; and that maths students ought to be militant instead of the decent, solid, fairly conservative types that most of us are. Since A.S. is (apparently) eagerly awaiting the "next" issue of M500 - and is thus totally "converted" as a new reader (though he hasn't yet paid a sub) - I leave it to members to tell him why you all are, or are not, "militant".

The whole event was quite a traumatic experience for me, faced with about sixty academics some of whom were known (by me, but not by you) to be hostile towards M500 and its aims. I went solely because M500 was my baby - although truly it now mostly the baby of Eddie Kent, Peter Weir, Austen Jones, Nick Fraser and Sidney Silverstone; with me just madly addressing envelopes and dealing with printers in the back seat. Actually, I wouldn't mind going again, now that I have done it once, and it was a great delight to receive that pendant locket as a "reward" afterwards: privately presented at a luncheon party consisting of Professor Pengelly, Chris Rowley and Richard Ahrens (Academic Directors of Weekends '75, '76 and '77) and Peter Thomas. (The WH chef deserves some similar award for the menu! It was served in the private dining room at WH, obviously reserved,for VIPs, and I felt duly ➔

honoured - or would have done if not totally doped by Valium!)

Anyway, thank you, everyone for the gift. I hope you will feel like doing it again for any future retiring Weekend organisers, but for their sakes would beg you to enquire first just where and how they would like to receive their gifts! I would rather have had mine at Weekend 78, actually, not at WH 4-10-77. Still the occasion was useful to M500, so I hope I coped with it adequately for the sake of THE SOCIETY and not for my own pleasure.

I N F I N I T Y OR NOT SID FINCH

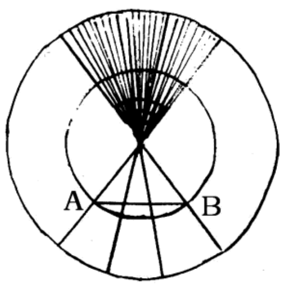
The area of a circle is the circumference multiplied by the radius and divided by two. This consists of an infinite number of triangles the height of which is the radius, and the sum of infinitly small bases which equals the circumference of the circle.

Now this suagests a contradiction as we can progressively halve the arc AB (in the diagram) an infinite number of times until the division is infinitely small, so we could likewise infinitely double the radius, so that the arc is infinitely large. This therefore suggests that: either infinity is relative (to the mode of conception), or, that there can be more than one infinity, or, finally that infinity does not exist at all.

It is suggested that if we only keep our equations within the bounds of our smallest and largest known measurements, then mathematics can be regarded as logical.

The 'fanning out' shown in this diagram might open the way for new equations in the sphere of astro-mathematics: are orbiting masses carried from point to point (however small the distance between the points) by continuous forces? Also would the fanning out explain the infra-red shift, rather than the concept of an expanding universe?

Please refer to the diagram.



INTERSECTING DIAGONALS OF POLYGONS II

POLYDIAGONALS JOHN READE

Problem 36.3c (*Polygons), by Richard Ahrens, asked "Can you prove that in a regular polygon with an odd number of sides, at most two diagonals meet at a point other than a vertex*?" *In* M500 45 1 *John Reade proved the assertion for the case where the number of sides is prime..*

We had reduced the problem to showing that the equation

$$(1-\zeta^a)\,(1-\zeta^b)\,(1-\zeta^c) = (1-\zeta^x)\,(1-\zeta^y)\,(1-\zeta^z)$$

has no non-trivial integer solutions a, b, c, x, y, z satisfying $a + b + c = x + y + z$, where $\zeta = e^{2\pi i/n}$.

This is in fact true for all odd n and can be proved as follows.

Since n is odd we must also have

$$(1-\zeta^{2a})\,(1-\zeta^{2b})\,(1-\zeta^{2c}) = (1-\zeta^{2x})\,(1-\zeta^{2y})\,(1-\zeta^{2z}).$$

This depends on the fact that if p is any polynomial with rational coefficients for which $p(\zeta) = 0$ then also $p(\zeta) = 0$ for all k coprime to n. (See M202.) - Factorising and cancelling out we get

$$(1+\zeta^a)\,(1+\zeta^b)\,(1+\zeta^c) = (1+\zeta^x)\,(1+\zeta^y)\,(1+\zeta^z).$$

Expanding this expression and the original one we have, on adding and subtracting

$$\zeta^a + \zeta^b + \zeta^c = \zeta^x + \zeta^y + \zeta^z$$

$$\zeta^{b+c} + \zeta^{c+a} + \zeta^{a+b} = \zeta^{y+z} + \zeta^{z+x} + \zeta^{x+y}$$

and therefore

$$(t-\zeta^a)\,(t-\zeta^b)\,(t-\zeta^c) = (t-\zeta^x)\,(t-\zeta^y)\,(t-\zeta^z)$$

for all t.

Hence $\zeta^a, \zeta^b, \zeta^c = \zeta^x, \zeta^y/\zeta^z$ in some order.

QED.

CARL FRIEDRICH GAUSS PART I

JEREMY GRAY

Prefatory note. Since I attended a two-day conference on Gauss (who's bicentennial year it is) in June and heard and read several papers about him recently, I wrote to Eddie and said: how about a series of articles by way of a scientific biography of "him (Gauss, that is). Eddie fell for the idea, and so here is episode one of the not-to-be-so-called Gauss centre-fold. Pull out and keep in a dry place, watch then accumulate issue by issue into your very own historical encyclopedia. Better still, argue with me and contribute your own knowledge to the difficult task of understanding the 'Prince of mathematicians'!

Here are a few good things to read:

K O May, "Gauss" *Dictionary of Scientific Biography* " 1972 pp 298 - 315;

I N Stewart, "Gauss" *Scientific American* July 1977;

and for those of you who read German:

F Klein, M. Brendel and L Schlesinger: *Materielen für eine wissenschaftliche Biographie von Gauss*, much of which is reprinted in the *Werke* (12 volumes). Above all, read the *Werke* themselves.

There is hardly a topic in the history of mathematics which has been worked on with any thoroughness; nor can there be more excitement in reading mathematics than is to be gained from reading a great mathematician. I urge you to try it.

Carl Friedrich Gauss was born in Braunschweig (Brunswick) (now in West Germany) on 30 April 1777; his parents were poor and scarcely literate. When he died a little short of his 78th birthday he was rich, and famous as a mathematician, astronomer ➔

and physicist. So his story is in many ways a success story, which by its diversity and brilliance makes it difficult to tell - frankly one is overcome by each part of it in turn, and the totality is awsome. I shall concentrate on his mathematics, partly because that is my subject, and partly because, I shall suggest, it is the subject in which he made his greatest achievements. As it happens his career divides, somewhat schematically, into three parts:

mathematician, 1796 - 1809,

astronomer, 1805 - 1855, and

physicist, 1818 - 1842;

so I may at least begin where Gauss did, even if I must fail to follow him all the way.

By starting with Gauss the mathematician we encounter at once a matter of great importance in the history of mathematics, for it is not merely that Gauss is a giant among mathematicians. He became their "Prince" (the appellation was agreed by all) at a time when the subject was widely regarded by its practitioners as all but worked out. And he earned this title by redefining the subject; by introducing into it wholly new objects of study and new approaches to that study. Gauss is really the father of modern mathematics, and it is that which makes him great.

Let us attend to the two halves of this paradox. First, his isolation. At no time since the rediscovery of mathematics in the West (which I provocatively set late, at 1600, with the French mathematician Viète) has there been anything like it. Merely to list some of the names is to sense the swell of continuous development: Viète, Descartes, Fermat, Pascal, Wallis, Barrow, Newton, Leibniz, the Bernoulli family, Euler, D'Alembert, Lagrange, Laplace, Legendre, and so on. One might consider Descartes, Newton, Leibniz and Euler as peaks, but undoubtedly the two centuries are one continuous progress in the scope and power of mathematics. So it is very surprising to find a *fin de siècle* weariness setting around 1800.

➔

One aspect of this can be discussed straight away. The appalling decline in mathematics and the sciences in Britain during the eighteenth century is well known. The descent from Newton to nothing is broken only by a few minor figures who are usually remembered for publishing results already known to the master (Taylor and McLaurin series, for example). The true home of mathematics was in France and Germany, although it was well paid in St Petersburg and well stocked with Swiss, one suspects that national chauvism amongst the commercially and militarily successful British put them off learning such a 'foreign' subject. In its homeland mathematics was successfully applied to various subjects: Calculus of Variations, differential equations (ordinary and partial), Celestial mechanics, differential geometry, probability theory. There was no division into pure and applied mathematics, rather into mathematical subjects. Rather crudely, the century saw the working out of the calculus in one and several variables: it was given definitive text books, and employed in signal victories in the struggle to understand nature. I might mention Euler's *Institutiones calculi* ... and *Introductio in analysis infinitorum* amongst the first kind and the works of Lagrange (*Mécanique analytique*) and Laplace (*Mécanique céleste*) in the second. They are all great unifying works, every one repaying study. But perhaps they do look like 'last words', the new fields to conquer did seem at last to lie elsewhere. Problems began to seem bitty: Euler had eliminated the gross observational disparity between Newton's theory and the actual motion of the moon (*Theoria motus lunæ* 1753), and the immense theory of pertubations now only had the task of mopping up errors whose source (the effect of Jupiter, for example) was assumed known. Lagrange considered abandonning mathematics for a while, and other still alive in 1800 were past their prime: Laplace 51, Legendre 48, Monge 54, Lagrange himself 64; and there were no successors in sight. This break in the development of mathematics is unique, it has never been repeated since. Its very existence provides a puzzle I cannot explain, but the solution of which must reside in the political and social circumstances of the time. The French Revolution gave a boost to mathematics, largely in the hands of the radical Monge, but Napoleonic times marked a gap (Laplace's work notwithstanding). When mathematics really began again it was to be in three directions: mathematical physics of ➔

the kind Fourier inspired, complex analysis, and what I shall call abstract mathematics. In each area Gauss was a giant, and in two supreme, so to him I shall now return in his profound loneliness.

Gauss displayed his brilliance at an early age. It seems, if we are to believe the stories he told in later life, that he virtually taught himself to read and to do arithmetic. At the age of three and before anyone had thought to try and teach him to count, he corrected an error in his father's wage calculations. At eight he solved instantly the problem his schoolteacher had set to keep the class quiet: to find the sum ot the first hundred integers. Presumably he had already spotted the trick (which I leave to the reader as an exercise!) This brilliance fortunately brought him encouragement. His teachers set him to reading the classics of both literature and mathematics - incidentally Gauss found learning languages an easy task, he taught himself Russian in his sixties - and there is no better diet for a first-rate mathematician than to read his peers. He entered the local Gymnasium in 1788 and the Collegium Carolinum in 1792, by then supported with a stipend from Duke Ferdinand of Brunswick. Before entering Gottingen University in 1795 he had read Newton's *Principia* and Bernoulli's Ars conjectandi, although it was at Göttingen, the leading research university of the day, that Gauss was first able to find out easily what had already been done. He found out, of course, that many of his discoveries had already been made by others, but that is to put the point the wrong way round. The remarkable thing is that he had discovered so many things on his own, and he was only eighteen years old. Furthermore he had by then hit upon the pattern of work he was to use throughout his life: enormous quantities of detailed calculation leading to a discovery, followed by repeated attempts to polish and simplify the proofs. This prodigious effort had also equipped him with an immense battery of minor results, techniques, and an amazing familiarity with numbers. Numbers, rather than people, were to be his lifelong friends. He knew their behaviour intimately, perhaps indeed there are few people whose acquaintance could survive such prolonged scrutiny.

Even so he hesitated before committing himself to mathematics, which we have seen languished internationally in despond. He dallied for a time with the scholarly German tradition in philology, but in 1796 he made the discovery that decided him to be a mathematician. He found the answer to a problem which had defied the Greeks and everyone after them: how to decide when a regular polygon is constructable using ruler and compass alone.

PROBLEMS - JEREMY HUMPHRIES

Again I can't think of much to say in the preamble. I've been away on holiday; been doing a bit of house improvement and the like; and haven't been thinking much about maths or related subjects. (Though I did have to calculate the Airey points for some shelves.)

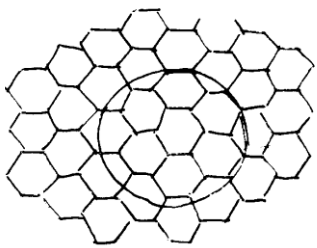
Contributions are few lately, and I hope things will pick up a little when the exams are out of the way.

Contributions with new usable problems are very few indeed - I really hope that will pick up.

And don't forget about writing on one side of the paper, etc.

SOLUTION 41.5 DISC COVERING *How many discs of radius* 1 *cover the disc of radius n*?

MIKE PURTON has sent some work on this in which he considers the unit discs in a regular hexagonal array. ie each hexagon vertex is the intersection of three discs. This is the same as having the disc centres form a net of equilateral triangles of side √3 which I mentioned in 44 (though Mike wrote before 44 was published.) He says

> The most efficient way for unit discs to cover a given area is for them to be in a hexagonal array.
>
> We may depict this configuration as an array of hexagons:
>
> 
>
> The problem can be reformulated as: Place a circle of radius *n* so that it encloses the minimum number of vertices. I assume that such a circle will either be centred on the centre of a hexagon (model I) or on a vertex (model II) - this looks right but I haven't proved it.
>
> We can then consider hexagons in a 60° segment from the origin. If the nearest vertex of a given hevagon is less than *n* include it in total. Multiply result by six, allowing for hexagons shared between segments.
>
> A hand calculation gives: ➔

n		1	2	3	4	5	6	7	8	9	10	11	12	13	14
number of	model I	1	7	19	31	37	55	73	91	121	151	163	199	229	259
discs in	model II	3	12	18	27	42	54	75	102	120	141	174	198	225	270

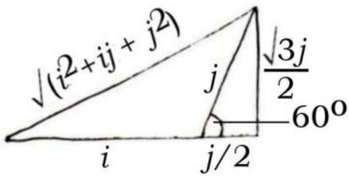
A computational formula is, take the minimum Q where

$$Q = 1 + 6\sum_{i=0}^{n-1} K, \text{ where } K = \begin{cases} 1, & \text{if } \sqrt{(i^2+ij+j^2)} < n \ \& \ j=i-1 \text{ or } i<n \text{ and } j = 0 \ \& \ i \bmod 3 = 2 \\ 2, & \sqrt{i^2+ij+j^2} < n \ \& \ j = 0 \ \& \ i \bmod 3 = 2 \\ 0, & \text{otherwise} \end{cases}$$

(model I); or

$$Q = 6\sum_{i=0}^{n-1} \sum_{j=0}^{i} L, \text{ where } L = \begin{cases} ½, & \text{if } i < n, j \neq 0, i \bmod 3 \neq 2 \\ 1, & \sqrt{(i^2+ij+j^2)} < n, \ j \neq 0, \ (i-j) \bmod 3 \neq 2 \\ 0, & \text{otherwise} \end{cases}$$

(model II).



This hexagonal packing is the best way to cover arbitrarily large areas - that is, it is the most economical way to 'tile' the plane with overlapping discs. (Note that model II beats CHRIS PILE'S method at n = 4.)

Machinery's Handbook (Machinery Publishing Company, Brighton; most reference libraries) has a section entitled "Diameter of Circle Enclosing a Given Number of Smaller Circles." This is not really what we are doing here, but it contains many things of practical interest. eg: For N enclosed circles of diameter d. when N is large (> 10 000) the diameter D of the enclosing circle is given to within 2% by

$$D = d\left(1 + \sqrt{\frac{N}{0.907}}\right).$$

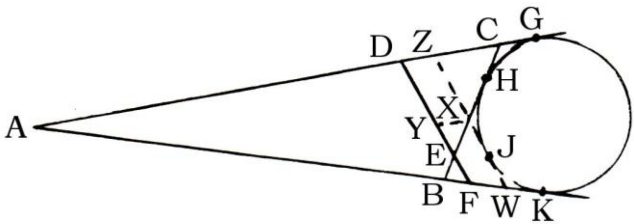
(Mike also sent some 60° segment drawings to illustrate his ideas; but unfortunately they would take up at least a page of the magazine. If anyone wants a sight of them as an aid to understanding, please get in touch.)

SOLUTION 44.1 ST SWITHIN'S SCHOOL

Across; 1:11; 3:51; 5:151; 7:51; 8:84; 10:1946; 13:25; 15:22; 16: 151.

Down; 1:112; 2:15; 3:55; 4:110; 6:1895; 9:44; 11:625; 12:42; 14: 51.

Correct solutions from NICK FRASER, MIKE PURTON, ARTHUR THOMSON, CHRIS LYONS and DAN FOX. There were no misprints this time. I shall spare the blushes of the member with four maths credits who wrote indignantly to me, saying that 13 across (= 25) is inconsistent but must be 75, since 11 down = 5^4 = 675.



SOLUTION M.7 GEOMETRY (*See diagram above*): *Given* AB + BE = AV +DE
Show AC + CE = AF + FE.

STEVE MURPHY and MIKE PURTON sent demonstrations of this and a couple of others mentioned in passing that they couldn't do it. Steve said "44.2 must have a simple proof but all I can discover is a rather elaborate one - unprintable because of its length". Mike's was even more complicated, and he says "There must be an easy way."

As Steve and Mike suspect, there is an easier way. RICHARD AHRENS showed me this at the recent Week-End:

Construct the circle which has as three tangents the lines AC produced, AF produced, and BC. Construct a fourth tangent WZ parallel to DF, and XY parallel to AC as shown. By using the fact that tangents from a point are equal it is easy to show that

AZ + ZX = AB + BX.

But AD + DE = AZ + ZX – XY + YE

and AB + BE = AB + BX – EX.

Since, also, AB+BE = AD+DE (given), we can subtract to give 0 = –XY + YE + EX which can be true only if X, Y and E are coincident. Therefore DF is also a tangent to the circle. It is now easy to show that AC + CE = AF + FE.

.

SOLUTION 44.3 JOBS *These men have two jobs each. The chauffer offended the musician by laughing at his long hair. The musician and the gardener used to fish with John. The painter bought a quart of gin from the consultant. The chauffer courted the painter's sister. Jack owed the gardener* £5. *Joe beat Jack and the gardener at quoits. One of them is a hairdresser and no two have the same job. Who does what*?

I think Andy intends that we assume:

1. No one is schitzophrenic so; The chauffer is not the musician; the painter is not the consultant; John is not the musician; John is not the gardener; Jack is not the gardener; Joe is not the painter.
2. No one is incestuous so: The chauffer is not the painter.
3. And further that: The musician is not the gardener; Jack is not the painter.

Assuming all these gives the solution

Jack is the musician and the consultant;
Joe is the chauffer and the gardener;
John is the painter and the hairdresser.

This was sent by MAX BRAMER, NICK FRASER, BRIAN GROVES , CHRIS LYONS, MIKE PURTON, HUGH TASSELL and DAN FOX.

Dropping assumption 3 can give

Jack is the chauffer and the consultant; Joe is the musician and the gardener; John is the painter and the hairdresser.

If we drop 2 also we can get

Jack is the hairdresser and the musician;
Joe is the consultant and the gardener;
John is the chauffer and the painter.

Both these solutions come from JOHN HALE.

Nobody dropped any of the assumptions in 1. If you did you could probably get pretty well what you liked as a solution.

SOLUTION 44.4 OLYMPIAD IV: *Determine with proof the largest number which is the product of positive integers whose sum is* 1977.

The answer is 3^{659}, which was sent by NICK FRASER, STEVE MURPHY, MIKE PURTON and DAN FOX. This is neat because 3 divide 1977. For numbers x which 3 does not divide there is a slight modification. If the remainder is 2, use it.

$$\max = 2 \times 3^{(x-2)/3}.$$

If the remainder is 1, replace the 1 and one of the 3s by a 4 (or two 2s).

$$\max = 4 \times 3^{(x-4)/3}.$$

If we drop the integer restriction the maximum number obtainable by this method is $e^{x/e}$ as is easily shown and intuitively obvious. This was Mike's proof, for number $x = 1977$.

S = sum of terms; P = product of terms. Let S_1 = series giving maximum product, n = arbitrary term in S_1. Obviously $n \neq 1 \ \forall x \neq 1$.

➔

Suppose $n > 3$. We can write

$$S_1 = n + \sum a_i \qquad S_2 = m+2 + \sum a_i \qquad \text{where } n = m+2.$$

Corresponding products are

$$P_1 = n(\sum a_i) \qquad P_2 = 2m(\sum a_i).$$

But $P_1 \geq P_2 \Rightarrow n \geq 2m \Rightarrow n \geq 2(n-2) \Rightarrow n \leq 4$.

Now suppose $n = 4$. Then $m = 2$ and $P_1 = P_2$. It follows that the maximum product can be obtained from a series of 2s and 3s such that

$$S_1 = 2\alpha_2 + 3\alpha_3 = 1977 \qquad P_1 = 2^{\alpha_2}3^{\alpha_3} = 2^{\alpha_2}.3^{(1977-2\alpha_2/3)} = 3^{659}(2^3 3^{-2})^{\alpha_2/3}.$$

P_1 is maximised when $\alpha_2 = 0$. Therefore max product $= 3^{659}$.

SOLUTION 44.5 QUICKIES AND TRICKIES

I. *Find the first and last numbers of* -, 121, 144, 202, 244, 400, 1210, 10201, -.

II. *Next two terms* of 1, 2, 4, 11, 24, 112, 1000, -, -,

This is a coincidence. TONY BROOKS sent I. to Eddie, without a solution and Eddie sent it to me when I became PE. I sent II. to Eddie ages ago, and that duly came back also. Having used them side by side I now find that the ideas are the same. ARTHUR THOMSON has sent both solutions:

I. 100, ... , 1100100 (100 denary, expressed in (10-k)ary notation, k = 0, 1, 2, ... , 8.)

II. 11202, 100 000 000 (2^k in (10-k)ary notation, k = 0, 1, ... , 8).

III. *Which salary scheme would you prefer*: a): £200 *increse per year or* b): £50 *increase per half year*?

Arthur and a couple of my HSD colleagues said that tney couldn't understand "£50 increase per half year", but nobody had any trouble with "£200 increase per year" and I fail to see how you can understand one and not the other.

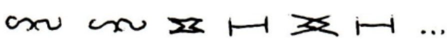
If you like money scheme b is better for you. It's true for any initial salary of course so for demonstartion let's assume £100 per month. Thus the accumulated pay under each scheme is

Time (years)	0	½	1	1½	2	2½	3	3½
Scheme a	0	600	1200	1900	2600	3400	4200	5100
Scheme b	0	600	1250	1950	2700	3500	4350	5200

etc.

IV. *What did Mark Twain say when asked "Why do you wear a white suit?"*

Nobody sent anything for this. Even MT himself was a little vague in his reply. He said "Naked people have little or no influence in society". He meant naked men.

V. *What's missing*? 

➔

Arthur and Eddie had  . RON AITKEN sent two solutions because, he says,

sausages seem much tastier when totally cooked, and Saturday Sunday Monday Tuesday Wednesday Thursday payday. 

PROBLEM 46.1 OLYMPIAD VI From 18 IMO 1976, Lienz Austria: by permission of the Mathematical Association.

A sequence u_n is defined by $u_0 = 2$, $u_1 = 5/2$ and $u_{n+1} = u_n(u_{n-1}^2 - 2) - u_1$, for $n = 1,2,\ldots$.
Prove that for positive integral n, $[u_n] = 2^{(2^n - (-1)^n)/3}$.
[x] denotes the greatest integer ≤ x.

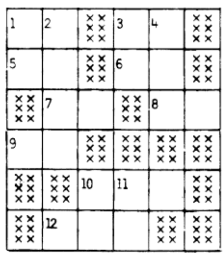
PROBLEM 46.2 MINIMUM POINT STEVE MURPHY

Given three points A,B,C; determine the point O such that the sum OA + OB + OC is a minimum.

This is taken from an SMP textbook. Steve says that solutions intelligible to a teenage son would be appreciated.)

PROBLEM 46.5 THE MEDWAY LEAGUE (*Another one from Fun with figures by L H Clarke, used by kind permission of Heinemann Educational Books Ltd*)

Six teams, the Eagles, the Hawks, the Lions, the Redwings, the Tigers and the Etceteras, compete in the Medway Football League. Each team plays every other team, home and away; two pointa are awarded for a win and one for a draw. The winning team did not lose a match but drew two. No two teams scored the same number of points and the Tigers came immediately above the Lions in the final order. The last game was luckily between the two leaders and decided the championship so that a good muster of spectators turned out. Put the six teams in their final order and find how many spectators watched the last match.



ACROSS
1 The number of points scored by the Eagles.
3 The number of points scored by the Hawks.
5 Twice the number of points scored by the Hawks.
6 The same figure twice repeated.
7 Four times the number of points scored by the Eagles.
8 The total number of points scored by all the teams.
9 The square of the number of points scored by the Redwings.
10 The square of the number of points scored by the Eagles.

DOWN
1 The sum of the points scored by the Lions and the Tigers.
2 The number of spectators at the final match.
3 The number of points the second team would have scored had they won the final match.

➔

4 A perfect square.
10 Twice the number of points scored by the Eagles.
11 The square of the number of points scored by the Etceteras.

PROBLEM 46,4 MEASURES

You have three measures: 15 pint, 10 pint and 6 pint and an unlimited water supply. An operation is filling or emptying a measure, or transferring water from one to another. Obtain ONE PINT in each of two measures, using the smallest possible number of operations.

PROBLEM 46,5 CLOCK PATIENCE JOHN READE

John would like to know the probability of getting clock patience out.

The 52 cards are placed face down in 13 piles of 4. The piles are labelled A,2 ,3,... ,Q,K. The top card of the K pile is turned up, and is placed next to the pile of its denomination. The top card of that pile is then taken and placed next to its corresponding pile, and so on. Success is when all cards are turned over.

OBITUARY - PROFESSOR J E LITTLFWOOD

Anyone who has read the 1967 edition of Hardy's *A Mathematician's Apology* will remember the forword by C P Snow. I hope his Lordship won't mind if I lift one or two bits from it.

> The Hardy–Littlewood researches dominated English pure mathematics, and much of world pure mathematics, for a generation. ... Of its enduring value there is no question. ...
>
> ... But no one knows how they did it: unless Littlewood tells us no one will ever know. I have already given Hardy's judgment that Littlewood was the more powerful mathematician of the two. ... Littlewood was and is a more normal man than Hardy ... He never had Hardy's taste for a kind of intellectual flamboyance, and so was less in the centre of the academic scene. This led to jokes from European mathematicians, such as that Hardy had invented him so as to take the blame in case there turned out anything wrong with one of their theorems....
>
> ... Through their most productive period they were not at the same university. Harald Bohr ... is reported as saying that one of their principles was this: if one wrote a letter to the other, the recipient was under no obligation to reply to it, or even to read it.

I must not go on any longer wearing another's plumes but just mention that every time I come back to that book it is better than I remembered it. But then I assume that none of my readers is without a copy.

Professor J E Littlewood FRS FRAS Rouse Ball Professor of Mathematics ➔

in the University of Cambridge from 1928 to 1950 died on September 6 at the age of 92.

He was born John Edensor Littlewood to a father who had been ninth wrangler in the mathematical tripos in 1882. After some time in South Africa where his father was a schoolmaster he came back to St Paul's School in 1900, where the scholarship class was taught by F S Macaulay (later FRS) . Macaulay taught him to distinguish between mathematics and examination questions (see *A Mathematician's Apology* again) to the effect that in 1905 he was senior wranqler, bracketed with J Mercer of Christ's; he was at Trinity. In 1906 he was placed in Class I, Division I of Part II of the Tripos. His tutor, E W Barnes, later Bishop of Birmingham, put him on to research some problems in Integral Functions. He recalled that he "rather luckily struck oil at once."

In 1908 he took a Smith's Prize and was elected a Fellow of Trinity the same year. After lecturing at Manchester he returned in 1910 as a College Lecturer at Trinitv, then as Cayley Lecturer in the Universitv of Cambridge from 1920 to 1928 when he became the first Rouse Ball Professor, holding the chair till he retired in 1950. He was a life fellow of Trinity.

His 35 year collaboration with Hardy began after he proved the Abel–Tauber theorem (in the Theory of Series). Together they worked on Series, particularly Fourier; Distribution of Primes; the Riemann Zeta Function; Diophantine Approximation; Inequalities; the Theory of Functions; and of course the famous papers on "Partitio Numerorum", 1920, applying the Hardy–Ramanujan–Littlewood analytical method in the "additive" theory of numbers.

Since I have used too much space I will ignore his honours, his later collaborations, his music and his mountaineering, and I hope someone else will describe some of his discoveries. The only books he appears to have written are *Elements of the Theory of Real Functions* (1926) and *Lectures on the Theory of Functions* (1944), but after retirement he published *A Mathematician's Miscellany* (1953) which presumably (since it antedates him) does not answer Snow's question about how they did it. He was still publishing classical analytical papers well into his 80s and solving problems which even others considered important. EK

*   *   *   *   *   *   *   *   *

ERRATUM M500 45 6, solution to DIFFERENTIAL EQUATION by Anne Andrews. Line 2 should read

for Q_2/Q_1. Set $Q_2/Q_1 = P$.

(Not Q_1/Q_2.) It's the right way up everywhere else.

Anne didn't mention that in line –2 on the same page I put $\exp^{k...}$ instead of either $\exp(k...)$ or $e^{k...}$.

Of course nothing in M500 is guaranteed. It is up to the membership to point out any mistakes, whether they are trivial or conceptual. That way I will at least get letters.

And that's my EDITORIAL for this issue. Number 47 is on its way!

Eddie Kent.