M 500 19

M500 is a student-operated and student-owned newsletter for Open University mathematics students and staff. It is designed to alleviate student academic isolation by providing a forum for public discussion of individual's mathematical interests. Articles and solutions are not necessarily correct but invite criticism and argument.

MOUTHS is a list of names, addresses, telephones and courses of voluntary members, by means of which private contacts may be made by any who wish to share OU and general mathematical experiences or who wish to form telephone or correspondence self-help groups.

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The cover design is, with apologies, only an outline trace of a composition by Theodore Bally, from THEODORE BALLY II: MECANIQUES. Neuchatel, Switzerland: Editions du Griffon, 1968. (The original is composed of solid black shapes, but experiments proved that there was too much ink area for reproduction by stencil).

Bally's experiments consist of arrangements of lines, curves and "basic forms" which incorporate "gear" teeth, spokes of unequal length, eccentric hubs and pieces of irregular shape. He rarely uses any parallel lines or right angles or perfect conic sections. This construction is one of a set designated as "personnages", the 'programmed interaction' of shapes presenting the illusion of a figure. The viewer observes a set of irregular polygons and curves which he may associate with phenomena from the range of his own experience - in this case perhaps we may see a personnage holding a telephone and calling for a unit! By other arrangements of basic forms, Bally creates constructions which give illusions of skeletal structures of machines, while others are simple or complex abstract geometrical patterns called 'fabriques', all of which explore the aesthetics of combinations of two-dimensional shapes.

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SOME SPECIAL POINTS

Under this heading let's include points in a curve which have no unique tangent or at which two or more branches meet. They provide features in the curve whose shape may well justify investigation, while names like limaçon and rhamphoid cusp add spice for the lexicologist. Assuming that x is real, and $y = x^2$ means $y = \pm\sqrt{x}$ we must accept that a mapping may not be a function although still a valid subject for study (1a). (References which I hope will satisfy the most fastidious are listed at the conclusion.) Three large collections of mathematical curves (2, 3, 4) will not be referenced separately for each curve.

The isolated point has no shape at all; an extreme example is $y = \sqrt{-x^2}$ which consists of just the originl parabola (5):

$$z = (x - a)^2(x - b) = x^3 - (2a + b)x^2 + (a^2 + 2ab)x - a^2b$$

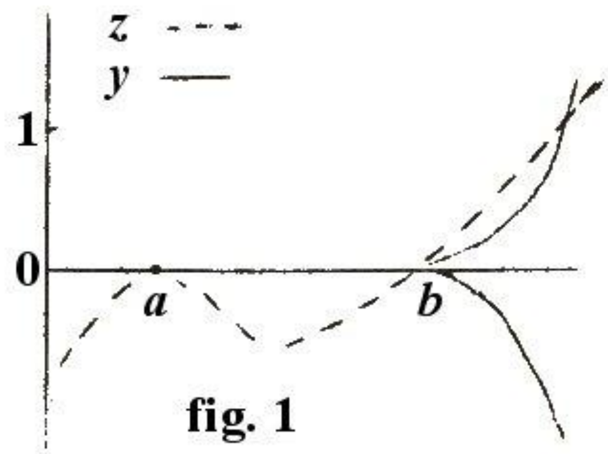

fig. 1

has a double root at $(a, 0)$ which ensures that $z' = 0$ at a. If $z'' = 6x - 4a - 2b < 0$ at a, that is $a < b$, then z is negative on both sides of a (fig. 1). Now consider $y^2 = (x - a)^2(x - b)$. y is not defined for $a < x < b$ nor for $x < a$ so $(a, 0)$ is an isolated point in

$$\{(x, y): y^2 = (x - a)^2(x - b),\ a < b\}.$$

Infinite sets of isolated points are found in

$$y = \begin{cases} 0, & x \text{ irrational,} \\ |x|, & x \text{ rational} \end{cases} \qquad (1b)$$

and

$$y = \begin{cases} x, & x \text{ integral,} \\ 1, & x \text{ nonintegral.} \end{cases} \qquad \text{(fig. 2)}$$

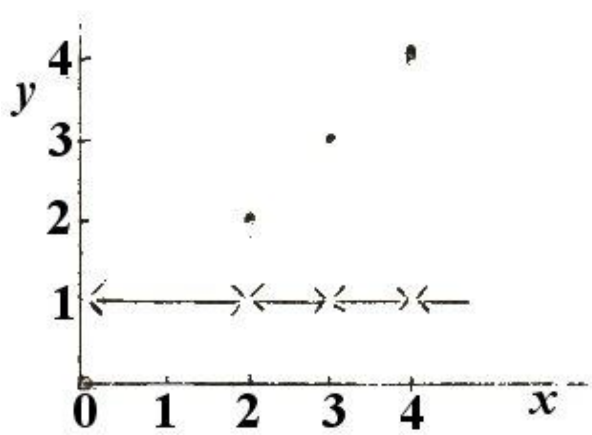

fig. 2

An example of historical interest is the conchoid of Nicomedes (5):

$$(x - a)^2(x^2 + y^2) - l^2x^2 = 0$$

with an isolated point at (0, 0) when $l < a$. You might like to investigate

$$x^2 - xy^2 + y^2 = 0$$

which has a maximum, a minimum and an isolated point.

The limiting case of $y^2 = (x - a)^2(x - b)$ with $a = b = 0$ is Neil's parabola (5) in which the isolated point merges with the main curve giving a ceratoid cusp at (0,0) as in fig. 3. At a cusp the tangents to the two branches coincide, for Neil's parabola $y' = \frac{3\sqrt{x}}{2}$ and

$$\lim_{x\to 0^+} y' = \lim_{x\to 0^+} \frac{3\sqrt{x}}{2} = \lim_{x\to 0^+} \frac{-3\sqrt{x}}{2} = 0.$$

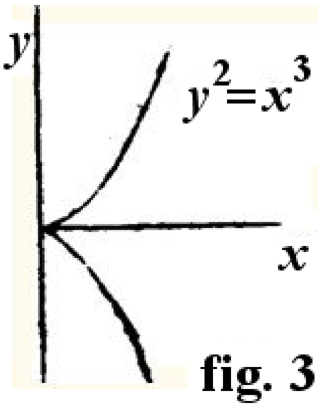

fig. 3

This contrasts with a branch point such as (0,0) for $y = |x|$ for which $\lim_{x\to 0^+} y' \neq \lim_{x\to 0^-} y'$. With a ceratoid cusp the branches leave the cusp on opposite sides of the tangent, whereas the branches leave the rhamphoid cusp on the same side of the tangent.

One of the simplest examples of a rhamphoid cusp occurs at the origin of

$$(y - x^2)^2 = x^5 \quad \text{or} \quad y = x^2 \pm \sqrt{x^5}$$

$$\text{so } y' = 2x \pm \frac{5\sqrt{x^3}}{2},$$

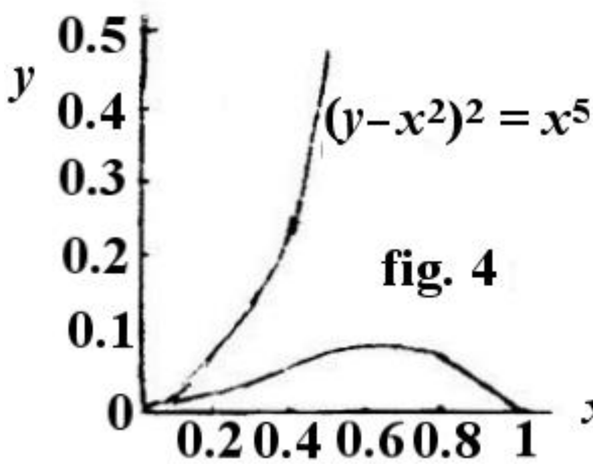

fig. 4

at $x = 0$ $y' = 0$ for both branches and both lie above the tangent. In the negative branch there is a maximum at (0.64, 0.082) and this branch also passes through (1,0). The graph is shown in fig. 4; y is not defined for $x < 0$ because of the square root. Examples of both types of cusp will be found in a fascinating analysis of about300 curves, without using calculus (6).

Points of stopping are points at which a curve ends; we met examples in the derivative of $f(x) = |x|$ (1c). Examples also occur at the boundary of the domain of a function, for instance $y = x \log x$ with domain $(0, \infty)$, using L'Hopital's rule $\lim_{x\to 0^+} y = 0$ (1d) so the origin is the stopping point. Functions of the type $y = 3^{1/x}$ (see fig. 5) provide good examples. An extra stopping point will be obtained by reciprocating again, e.g with $y = \frac{1}{3^{1/x} + 2}$.

Corrections, constructive corrments and interesting examples will be gratefully received.

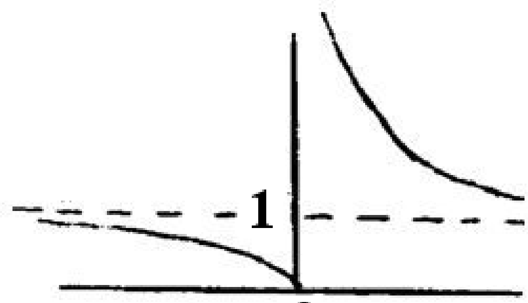

fig. 5

References:

1. The Open University, Course Units
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(b) M231 2.4
(c) M100 12.2.1
(d) M231 10.6.

2. *Encyclopaedia Britannica*, 14th edn., article ‘Curves, special’.
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5. Bronshtein, I. N., and Semendyayer, K. A., *A guide-hook to mathematics for technologists and engineers*. Pergamon, 1964.
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Michael Gregory

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COMPUTING – IS NOT MATHEMATICS

Commercial computering is not mathematical and systems analysts and commercial programmers do not claim it is. (Personnel officers like one to have A-level maths but what do they know about it?)

The confusion started in the early sixties when no-one knew what to recruit as trainee programmers and some twit from Manchester University claimed that only people with a degree in maths could write programs.

Ex-students who had failed the first year’s exam were favourite up to that time. The confusion has since been compounded by degrees in computer science. The name of these degrees causes the naive to think that they include the aim of teaching one how to be a commercial programmer. In fact they allow mathematicians to use the computer as a tool, a very necessary ability for some of them.

This brings us to M251. I do not see how a university can offer a degree in maths unless some basic computing is an option. True M251 tries to teach one about computers rather than how to use them but I suppose learning about rather than learning how is the usual academic approach.

Mike Stanton - Another Glassblower

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OBITUARY

I had a note from George Dingley's people saying that George died on December 1st after 3 weeks in hospital. I had not realised that George was not the writer of his letters. He must have suffered a lot, and yet none of that came through. A certain wry humour is what I saw, for the most part. His interests ranged from mathematics to man-powered flight to the social history of the Potteries. Correspondence between mathematical amateurs is an essential but rarely achieved concomitant to mathematical maturity. George was a voluminous correspondent and I learned a great deal during my brief association with him. I will be sending a donation to Cancer Research in his name.

Hugh Mdntyre

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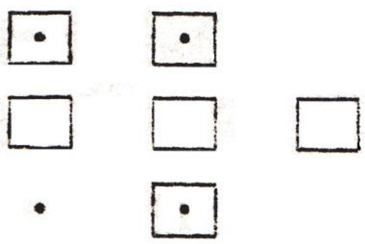
PROBLEMS

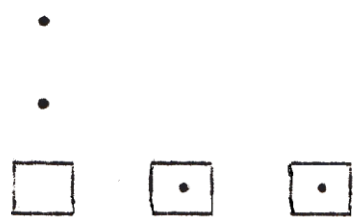
1. INSCRIPTIONS - Roger Claxton

Prof. Charo A. O’Gleist discovers the following stone inscriptions in an ancient Indian site in South America:

1st inscription

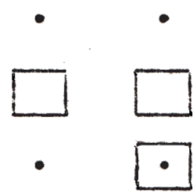


2nd inscription



3rd inscription

4th inscription



By external evidence Prof. O’Gleist knows that the first three sets of symbols refer to the numbers of prisoners sacrificed after a series of famous battles and represent 836, 1597, and 133 respectively (in decimal). Skeletal remains indicate that this was a tribe with 12 fingers and toes instead of the usual number. How many prisoners were sacrificed after the 4th battle?

2. TORELLI MURDER - Eddie Kent

Benno Torelli, a nightclub owner, was shot and killed by a racketeer gang because he fell behind in his protection payments. After considerable effort on the part of the police 5 men were invited to assist in their enquiries. When asked what they had to say for themselves each of the men made 3 statements, 2 true and 1 false:

LEFTY: I did not kill Torelli. I never owned a revolver. Spike did it.
RED: I did not kill Torelli. I never owned a revolver. The other guys are all passing the buck.
DOPEY: I am innocent. I never saw Butch before. Spike is guilty.
SPIKE: I am innocent. Butch is the guilty man. Lefty lied when he said I did it.
BUTCH: I did not kill Torelli. Red is the guilty man. Dopey and I are old pals.

Which one did it? In your head, please.

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HOOPS – SUMMARY OF PROBLEM AND OF MAJOR RESULTS PUBLISHED IN M500

If a hoop is a set with a binary operation (H, o) such that for all $x,y,z \in H$:

$$x \circ x = x \qquad \text{(idempotent law),}$$

$$\left.\begin{aligned} x \circ (y \circ z) &= (x \circ y) \circ (x \circ z) \\ (x \circ y) \circ z &= (x \circ z) \circ (y \circ z) \end{aligned}\right\} \qquad \text{(distributive laws),}$$

$$\left.\begin{aligned} x \circ y = x \circ z &\text{ implies } y = z \\ x \circ y = z \circ y &\text{ implies } x = z \end{aligned}\right\} \qquad \text{(cancellation laws),}$$

does there exist a finite hoop of order 10? (order of H = o(H) = no. of elements in H.) This question was asked by John Bennett (in January 1974) who quoted the arithmetic mean as an operation on the set of real numbers as the archetypal hoop, other known hoops being the geometric and harmonic means and taking the midpoint in n-dimensional space. John asked whether hoop theory was part of field theory, hence required a hoop of order 10.

RESULTS

FROM M500/10 – Bob Margolis: SOME THOUGHTS ON HOOPS

1. First result: There can be no hoop of order 2. There exists a hoop of order 1.

1. Definition: A subhoop of a hoop H is a non-empty subset of H which is closed under the hoop operation.

2. Definition: A coset of a subhoop S of a hoop H is defined as the set x o S = {x o s : $s \in$ s}.

2. Theorem: Every coset of S is also a subhoop.

1. Lemma: All cosets of a given subhoop have the same number of elements.

2. Lemma: Let S be a subhoop of a hoop H, and x an element of H.

(i) $x \notin S$ implies $x \notin (x \circ S)$ and $S \cap (x \circ S) = \varnothing$.
(ii) If T is another subhoop of H and $S \cap T = \varnothing$ then $(x \circ S) \cap (x \circ T) = \varnothing$.
(iii) If T is as in (ii) and $t \in T$ then S, T and (t o S) are pairwise disjoint.

3. Theorem (Margolis): If S is a proper subhoop of H then o(s) ≤ 1/3 o(H).

4. Theorem (1) (Wilkie): If a finite hoop has a subhoop of order 3 then the order of the hoop is divisible by 3.

5. Theorem (Ahrens 1): There is no hoop of order 10. (Statement only).

FROM M500/11 – Richard Ahrens: THERE IS NO HOOP OF ORDER 10, Part 1.

SAQ 1: Hoops are never associative unless trivial (1 element).

SAQ 2: The intersection of 2 subhoops is a subhoop.

SAQ 3: Show that a hoop of order 6 has no proper subhoop.
(Proper = more than 1 element but not the whole hoop).

FROM M500/12 – Bob Margolis: ALL HOOPS ARE FINITE

3. Definition: A hoop morphism is a function f: (H, o) (K, ✳) where (H, o) and (K, ✳) are hoops and $f(h_1$ o $h_2) = f(h_1)$ ✳ $f(h_2)$ for all $h_1, h_2 \in H$.

4. Definition: If a relation $\mathcal{R}$ on H is defined by $x \mathcal{R} y \Leftrightarrow f(x) = f(y)$ then $\mathcal{R}$ is an equivalence relation (see M100 Unit 19).

3. Lemma: If f: $H \to K$ is a hoop morphism and $\mathcal{R}$ is the equivalence relation on H induced by f then:

   (a) The equivalence classes are subhoops.
   (b) Every equivalence class is of the form (x o s) for a fixed equivalence class S.
   (c) $\mathcal{R}$ is compatible with o.
   (d) O(S) divides o(H).

5. Definition: A *minimal* subhoop M of a hoop H is a proper subhoop of H such that M has no non-trivial proper subhoops.

6. Theorem (Sue Ahrens): If M is a minimal subhoop of H then o(M) divides o(H).

7. Theorem (Richard Ahrens 2): If H is a special hoop (see below), S a subhoop, then o(s) divides o(H).

Corollary: The function m: $H \to \{x$ o S: $x \in H\}$, where m is defined by m: $h \to (h$ o $S)$, for $h \in H$, is a hoop morphism.

FROM M500/16 – Richard Ahrens: YET MORE OF THE HOOP SAGA

SAQ 6: For all $x, y, z, w \in K$ (x o y) o (z o w) = (x o z) o (y o w)

Every hoop so far found satisfies this rule. A hoop with this property is called a special hoop.

8. Theorem (2) R. Ahrens: If (K, ✳) is a special hoop take any element a of K and define a new binary operation o on K by:

$$\text{For all } x, y \in K, x \text{ o } y = R_a^{-1}(x) \ast L_a^{-1}(y).$$

Then o is commutative associative with cancellation and a is an identity element for o.

(This means that if K is finite then (K, o) is an Abelian group.) Also, R_a, L_a are 2 commuting automorplsms of (K, o) where $R_a(x)$ o $L_a(x) = x$ for all $x \in K$.

If we can show that a hoop of order 10 must be special and then show that an Abelian group of order 10 is no good for constructing hoops then we have finished the proof that there can be no hoop of order 10.

The Hoops research continued in M500 19, January 1975.

ALL DEFINITIONS, LEMMAS AND THEOREMS HAVE NOW BEEN ALLOCATED A NEW REFERENCE NUMBER (IN LEFT-HAND MARGIN) FOR GREATER CLARITY IN M500 20 onwards.

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There is no hoop of order 10. R. Ahrens

Third and last part.

9. Theorem 3 (Alec Wilkie): A hoop *H* which is not special, has a proper subhoop.

Proof: In such a hoop there must be a set of 4 elements a, b, c, d such that

$$(a\ b)\ (c\ d) \neq (a\ c)\ (b\ d).$$

Now consider the set $S = \{x : (a\ b)\ (x\ d) = (a\ x)\ (b\ d)\}$.

We know that $S \neq H$ because $c \notin S$ and $o(S) \geq 2$ because $a \in S$ and $d \in S$. (Proof SAQ 9)

S is closed because if p and q are in S we have

$$\begin{cases}(a\ b)(p\ d) = (a\ p)(b\ d) \\ (a\ b)(q\ d) = (a\ q)(b\ d)\end{cases}.$$

So multiplying these together

$$((a\ b)\ (p\ d))\ ((a\ b)\ (q\ d)) = ((a\ p)\ (b\ d))\ ((a\ q\ )\ (b\ d)),$$

$$(a\ b)\ [(p\ d)\ (q\ d)] = ((a\ p)\ (a\ q))\ (b\ d),$$

$$(a\ b)\ ((p\ q)\ d) = (a\ (p\ q))\ (b\ d),$$

but this says $p\ q \in S$.

So S is a proper subhoop of H.

Cor. A hoop of order 10 must be special because we have shown that it has no proper subhoop.

We are getting close.

Theorem 2 tells us that if H is a hoop of order 10 then we must be able to construct from it an abelian goup $(H, \ast)$ of order 10 together with 2 automorphisms σ and τ such that $\sigma(x) \ast \tau(x) = x \;\; \forall x \in H$. Sylow's theorem tells us that this group must have a subgroup of order 2 and clearly there can only be one such subgroup (if we had 2 subgroups they would generate a subgroup of order 4 which is impossible).

Let b be the *unique* element in $(H, \ast)$ whose order is 2. An automorphism of a group preserves the order of an element so $\sigma(b) = \tau(b) = b$

$$\therefore \; \sigma(\mathrm{b}) \ast \tau(b) = b \ast b = a \text{ (the identity in } (H, \ast)\text{)}$$

but $\sigma(x) \ast \tau(x) = x \;\; \forall x$ CONTRADICTION!!!

There is no hoop of order 10.

Cor. The above argument does not depend very much on 10. There is no hoop H with the following properties:

1. $o(H)$ = twice an odd number,

2. H is special.

Prove that if $o(H) = 14$ then H has no subhoop and so does not exist (Hard) (SAQ 10).

Sequences

May I quote Carl Linderholm on ‘Guess the Next Number’'? For example, what is the next number in the following sequence: 1, 2 ,4, 8, 16, ...

He analyses various methods of solving this sort of problem, concluding that “the numbers 1,2,4,8,16, if exhanged for the letters of the alphabet that correspond to them, are just the letters of the words: ‘Alien birds do have peculiar...’; and that these are the words of an old Kentish proverb, the last word 'feathers' being omitted. Arranging the alphabet in a circle with z next to a, and counting on past z to a, which then becomes 27, we see that the letter f, with which the extra word 'feathers' begins, gets the value 32 ... .” He doesn't think, for various reasons, that this is the best way to proceed in all cases and proposes the polynomial function:

$$\sum_{k=0}^{n} y_k \frac{\prod_{i\neq k}(x - x_1)}{\prod_{i\neq \mathrm{k}}(x_k - x_i)}$$

(under the hypothesis about (x_1)) has degree $\leq n$ and sends $x_1 \longmapsto y_1$ for $0 \leq i \leq n$; it must be the unique polynomial that does so. This method provides a systematic method for solving our problem, which gives the formula

$$1 + \frac{7x}{12} + \frac{11x^2}{24} \quad \frac{1x^3}{12} + \frac{1x^4}{24} + \ldots .$$

Thus, if $x = 0$ we get 1 + 0 + 0 – 0 + 0 = 1; for $x = 1$ we get 1 + 7/12 + 11/24 – 1/12 + 1/24 = 2. For $x = 2$ the sum of the series is 4 and if we apply this formula to the case $x = 5$ we get the answer = 31.

Lindeholm admits that if we think of taking logarithms to the base 2 and plot the graph, thus: $\log_2 1 = 0$, $\log_2 2 = 1$, $\log_2 4 = 2$, $\log_2 8 = 3$, $\log_2 16 = 4$ we get a straight line. The extrapolation gives 5 and taking 2^5 we get 32. But as he says: “Who could guess that we would get a straight line by taking the logarithms? Why not take the exponential function of the numbers, or the inverse tangent?” As he points out, the polynomial method is to be preferred since it is “simpler, easier to use, and is obtained by a more general method.”

Eddie Kent

I do not intend to offer the ‘correct’ solution to Problems 18 (a,b) but I am more concerned with asking what justifies any one answer as the right one? To illustrate my point more clearly, consider the simple sequence 1,2,4,8,... How should one continue this sequence? I expect that most people would answer with 16, 32,... on the assumption that the first four terms are of the sequence $\{a_n\}$ where $a_n = 2n - 1$. But why should this assumption be made in preference to any other? If I calculate the polynomial of degree 3 that passes through the points (1,1), (2,2), (3,4), (4,8), I get: $a_n = n^3/6 - n^2/2 + 4n/3$.

The first 6 terms obtained from this equation are:

n	1	2	3	4	5	6
a_n	1	2	4	8	15	26

Now is anything wrong with this solution? I can give a good reason for continuing the sequence 1, 2, 4, 8, ... with 15, 26, ... instead of 16, 32, ... Once launched on this line of thought then any number of other solutions for continuing the sequence can be given.

How about 1, 2, 4, 8, 1, 2, 4, 8, 1, 2, 4, 8, ... where $a_n = 2^{(n \bmod 5)}$ or 1, 2, 4, 8, 8, 8, 8, ... where $a_n = \begin{cases} 2^n, & 2^{n-1} \leq 8 \\ 8, & 2^{n-1} > 8 \end{cases}$.

If you now think that I am mad for suggesting any solution other than the ‘obvious’ one then this is the point I am coming to. I am concerned by the fact that we tend to judge a person's intelligent by such tests, or asking them to continue a sequence of numbers. But are we really judging their intelligence? Surely it is wrong to look for some occult justification behind any rule that makes that one any more correct than other rules. I feel that in exams and I.Q. tests we are too often measuring the ability of an individual to conform to what others believe to be right and not the ability to be original. So if I continue the sequence 1, 2, 4, 8, 18, 52, 206, 1080, 6994 with 4996, 801, 602, 25, 81, 8, 4, 2, 1 what, if anything, is wrong with this or anything else I care to wite down?

Tony Brooks

(Ed: M100 new subscribers may care to reply, or at least to bear in mind the above, after or during M100 Unit 4. Meanwhile, sorry I spoke!)

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MATH-COMMERCIAL

Military affairs are the most important of all in the nation, for upon them its security depends... Therefore, a general of wisdom and bravery must first of all have acknowledge of geometry, otherwise his knowledge and bravery will be without practical value.

Matteo Ricci, S.J. (1607) Preface to the first Chinese translation of Euclid's ‘Elements’

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hilbert matrices

I was interested by Percy Sillitto's work on Hilbert matrices (M500/17) and in my limited experience I have not met his expression (1):

$$(-1)^{i+j}\binom{2j+k-2}{j-1}\binom{i+j+k-2}{i-j}$$

for the (i, j)th element of L_k^{-1}. I would have understood the article more quickly if A_k had been labelled A_{k_n}, although I eventually understood that the size of the $n \times n$ submatrix does not affect the algebra at all. (I am a bit slow!)

As for the orthogonal polynomials my experience of M201 proved more valuable than Percy's; but the change of basis bit at the beginning of the course always is a problem! The connection between the kth Hilbert matrix A_k and orthogonal polynomials is as follows:

If $\sum_{i=1}^{n} a_i\, x^{i-1}$ is the least-squares polynomial approximation of degree $n - 1$ for $f(x)$ using the inner product $(p, q) = \int_0^1 p(x)q(x)x^k\, dx$ then the normal equations for the coefficients $\mathbf{a} = (a_1, ..., a_n)$ are $A_k\,\mathbf{a} = \mathbf{b} = (b_1, ...,b_n)^{\mathrm{T}}$ where $b_i = \int_0^1 f(x)x^{k+i-1}dx$.

Now Percy's statement of the LDL^{T} decomposition theorem is equivalent to the fact that any symmetric matrix can be transformed into a diagonal matrix by identical row and column operations, i.e. if $A = A^{\mathrm{T}}$ then there exists a non-singular triangular matrix P such that PAP^{T} is diagonal, and if A represents a linear transformation with respect to given bases then PAP^{T} represents the same linear transformation with respect to new bases; the co-ordinates of the jth new domain basis vector (with respect to the old basis) are the elements of the jth column of the change of basis matrix (P^{T}) and the relationship between new ($\hat{\mathbf{a}}$) and old ($\mathbf{a}$) coordinates of a general vector is $\hat{\mathbf{a}} = (P^{\mathrm{T}})^{-1}\mathbf{a}$.

In the particular example being discussed $A_k = L_kD_kL_k^{\mathrm{T}}$, i. e. $L_k^{-1}A_k\,(L_k^{\mathrm{T}})^{-1} = D_k$, therefore $P = L_k^{-1}$. Hence, by factorising A_k Percy is effectively changing the normal equations to $L_k D_k L_k^{\mathrm{T}}\mathbf{a} = \mathbf{b}$, i.e. $D_k(L_k^{\mathrm{T}}\mathbf{a}) = L_k^{-1}\mathbf{b}$, i.e. $D_k\,\hat{\mathbf{a}} = \hat{\mathbf{b}}$, say. Such diagonal normal equations occur if and only if the polynomial approximation is expressed in terms of an orthogonal basis $\{p_1(x), ..., p_n(x)\}$, i.e. $\sum_{i=1}^{n} \hat{\mathbf{a}}_i p_i(x)$.

My preceding remarks then imply that the coordinates (coefficients) of p_j are the elements of the *j*th column of $(L_k^{\mathrm{T}})^{-1}$. At this point I have to differ with Percy for I believe this implies that one has to *pre*multiply $(L_k^{\mathrm{T}})^{-1}$ by $(1\ x\ ...\ x^{n-1})$ to obtain $[p_1(x)\ \ p_2(x)\ \ ...\ \ p_n(x)]$.

Peter Hartley (Course Tutor, M31)

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Don't waste time looking in the envelope for a cheque - there isn’t one and I shall be unable to renew my subscription until the end of the month.

In September last I started a 3-year course at our local teacher-training college. It could be four years if I qualify for the B.Ed. Honours degree course. As it is, I have been accepted on the strength of my OU work so far (M100, T100, M201, M251, TS282). I have only 3 O-level GCEs. I have made some sort of history as this is the first time that the OU credits have been considered as an academic qualification. That it was accepted without reservation was, I found out later, due to the fact that one tutor had received his B.A. (OU) that summer and the Head of the Maths Dept. had been an OU counsellor, in 1971–1973.

As I am now dependent on a Local Authority grant, my means are determined by the calendar. The grant has been split into thirds, each cheque to be collected during the first week(s) of term. While the grant is adequate, it has been virtually impossible to obtain sufficient to survive the holidays.

Away with the violins and tears - all the best for the New Year.

Harold Moulson

(Ed: This all seemed too good to keep from publication. Of course Harold, and any similar, ranks as a 'deserving case' and can have a free sub. if wanted.)

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Many thanks for the sample copy of M500/18. I particularly enjoyed Vera Keates' excursion to Wonderland and I'm wondering what I have let myself in for, since I do not possess any form of footwear and must travel barefoot!

However, I have got as far as page 31, Unit 1, Foundation Level, so perhaps there's hope yet...

Pretty mazy it looks, too.

Leon Dunmore

Here is a slightly amusing tale. Having completed my last M100 TMA on the day before cutoff date I decided that posting it would be too risky, so I asked a non-OU friend who was going near my tutor's house to deliver it for me. This friend later remarked that he was disappointed to find that the OU was based in an ordinary house, and not in a vast complex of shiny buildings as he had imagined!

Re Peter Arnold's 'Rotten Egg' (M500/17) I agree that the Foundation year should be used to sort out students' problems–these problems being in most cases the very reason why the student was not able to pursue his studies at school level etc. Without help in this first year the same situation arises again and many students will have dropped out of the OU system.

Next point–whether the OU has tried to help. I can only go by personal experience. From the sample of staff that I have met I would say that mostly they are not aware of, or not made aware of, these difficulties. But I have been fortunate to have found an exception, and as a consequence I have managed to overcome a certain personality problem–an inferiority complexwhich impeded any kind of progress, and I am grateful.

Maybe given a little more time and persuasion some 'non-cream laden' OU graduates may penetrate the system and then we may see a little more understanding of these problems.

I'm glad that Peter's shot missed slightly (he should take up Netball)–not all staff deserve it, but I'm glad he tried!

Helen Gevers

Herewith my sub for '75 for your excellent magazine, without which I would go mad.

Here's a comment to be going on with: anyone who prefers 'Open Set' to 'M500' lacks the humour to be a mathematician.

Steve Osborn

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solutions

16.1 TMA M500 05 - Bob Escolme (cf TMA M100 05, 1974) [Dining club]

Problem, in brief: Club P with elements 'diners'; room Q with elements called 'dinners'; set of meals N called the attendance relation. 4 given axioms:

(1) N is a subset of the Cartesian product $P \times Q$. If $(p, q) \in N$ we say diner p attended dinner q and ate one meal.

(2) $p_i, p_j \in P \Rightarrow \mathbf{a} \vee (\mathbf{b} \wedge \mathbf{c})$, where $\mathbf{a} \Leftrightarrow p_i = p_j$, $\mathbf{b} \Leftrightarrow \{x \in Q\colon (p_i, x) \in N\} \cap \{x \in Q\colon (p_j, x) \in N\} \neq \emptyset$, $\mathbf{c} \Leftrightarrow$ the soln. set of **b** contains ≤ 1 dinner.

(3) $q_i, q_j \in Q \Rightarrow \mathbf{e} \vee (\mathbf{f} \wedge \mathbf{g})$, where $\mathbf{e} \Leftrightarrow q_i = q_j$, $\mathbf{f} \Leftrightarrow \{y \in P\colon (y, q_i) \in N\} \cap \{y \in P\colon (y, q_j) \in N\} \neq \emptyset$, $\mathbf{g} \Leftrightarrow$ the soln. set of **f** contains ≤ 1 diner.

(4) There exist 4 distinct diners p_1, p_2, p_3, p_4 and $\{x \in Q\colon (p_i, x) \in N\} \cap \{x \in Q\colon (p_j, x) \in N\} \cap \{x \in Q\colon (p_k, x) \in N\} \neq \emptyset$ (or no 3 of the 4 diners attended a dinner at the same time)

Show that any dining club geometry contains more than 4 diners. Find (a) minimum no. of diners, (b) minimum no. of dinners, (c) minimum no. of meals eaten, (d) minimum no. of chairs needed, all to be consistent with the axioms. Draw a geometrical figure.

Solution: The game is given away, or a method of solution is indicated, by the last part of the problem - draw a geometrical figure to illustrate your answers. Little more is necessary than to substitute the word 'point' for 'diner' and the word 'line' for ‘dinner’. Axioms (2), (3), and (4) then become:

(2) Every pair of distinct points are joined by a unique line, or have a unique line in common.

(3) Every pair of distinct lines intersect in a unique point, or have a unique point in common.

(4) There are given 4 distinct points, no three of which are collinear.

In fig. 1 below the four given points are set down and joined by the lines to which they give rise under axiom (2).

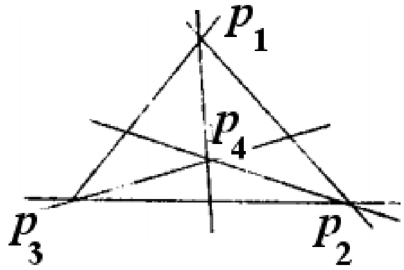


fig. 1

In fig. 2 the new intersections to which the lines give rise under axiom (3) are labelled.

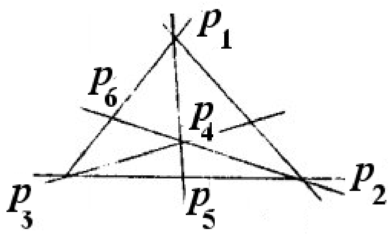
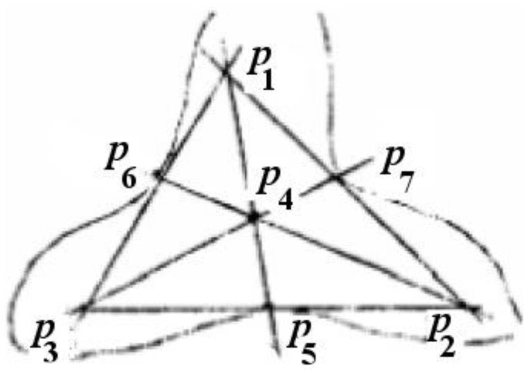


fig. 2

Now by axiom (2) the pairs of points (p_5, p_6), (p_6, p_7) and (p_7, p_5) give rise to new lines. However, remembering that we are after a minimum solution, and that there is nothing in the axioms which demands that the lines are infinite in length, or straight, we join p_5, p_6, p_7 by one line and produce no further new intersections or points, as indicated in

fig 3



Counting up the points and lines we get: 7 diners 7 dinners.

Every point lies on 3 lines, or every diner attended 3 dinners (i.e. every diner consumed 3 meals): 21 meals.

Every line contains 3 points or every dinner had 3 and only 3 diners attending: 3 chairs..

A proof of these results using the terminology of the original axioms (set theory and logic) would probably be rather tedious.

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17.3 <u>ARRAY</u> - 11 × 11 array. Find 22 points : no 3 are co-linear.

<u>Soln</u>. (1,1), (1,2). (2,7), (2,8). (3,2), (3,3), (4,8), (4,9), (5,3), (5,4), (6,9), (6,10) .(7,4), (7,5), (8,10), (8,11), (9,5), (9,6), (10,11), (10,1), (11,6), (11,7).

Marion Stubbs. (Further details and a rigorous specification of the second part of this problem are awaited from Dr. Earl.)

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17.4 Fishermen - Tom Dale's solution to follow in M500/20

18.1 (called ALPHAMETICS, according to Harold Moulson!)

SEND + MORE = MONEY: 9567 + 1085. = 10652 (Tom Dale, Harold Moulson)

FORTY + TEN + TEN = SIXTY: 29786 +850 + 850 = 31486 (H.McIntyre)

29468 + 650 + 650 = 31768 (H. Moulson)

18.4 FIND THE NEXT TERMS (and A rule)

(a) 53228, 462342 ((1) A. Winter, (2) M. Stubbs).

Rule: (1) $\triangle^2 u_r = u_{r+2} - 2u_{r+1} + u_r = r!$ ($\triangle = u_{r+1} - u_r$)

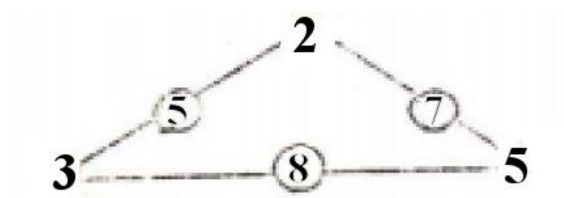
or (2) Second differences form the sequence $\{n!\}$.

(b) 8,8 (A. Winter, M. Stubbs)

Rule: u_r = no. of letters in r when written out in English.

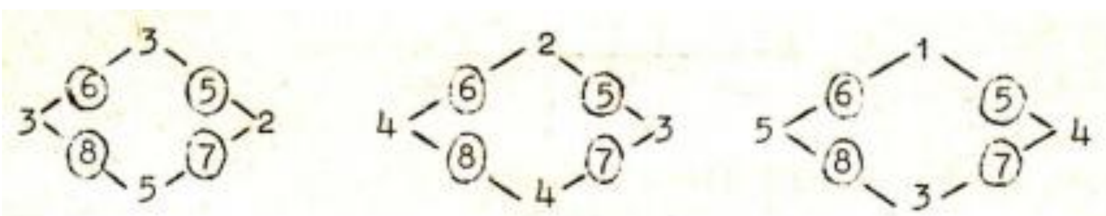
18.3 ARITOMOGRAMS - Hugh Mclntyre

Unique solution is



Uniqueness can be seen by using either matrix or simultaneous equations:

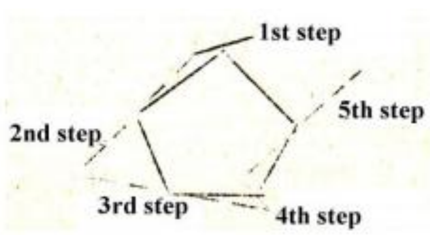
$x + y = 5$, $x + z = 7$, $y + z = 8$ so that $y = 3$, $x = 2$, $z = 5$, or some trivial re-naming. For the other problem there is no unique solution. Just consider:



etc.

I don't know if this is true in general, i.e. for all *a, b, c, d,* or for what kinds of values it holds. Presumably by ‘the kernel’is meant ‘for $a = b = c = d = 0$’. This is, starting at the top and reading either way round: $x, -x, x, -x$. Or, $-x, x, -x, x$. For any x.

The polygon problem is currently under observation. I need drawing instruments for such things, to keep out the blunders due to inaccuracy. A cursory appraisal to see what the problem was about indicates that the first step is crucial. For instance: - Take a pentagon for the midpoint points. It is easy to see that the first step absolutely determines whether or not a polygon results.



Presumably we just examine what kinds of first steps are OK and which are NBG. The mind boggles.

STOP PRESS

On the above pentagon: it appears that for a regular 5-agon there is only one circumscribed figure, i.e. another such. Obviously we can, given any 5-agon, inscribe another 5-agon, using the midpoints of the sides, and continue the process... It seems that the inscribed figure approaches a regular 5-agon. Is this true? And for other figures?

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POST-MORTEMS

MST282

Like Russell Brass I also found MST282 very time-consuming and also quite difficult. Too much reliance was placed on the textbook by the authors and there were insufficient worked examples. I suppose they were using them for TMAs and exam questions. There weren't even any recommended books for some units. Worst of all was the assumption of prior knowledge. Some of the TMAs were most intractable without knowledge of techniques which were not given to us. Still, it's over now - I hope!

The lesson to come out of MST282 both for myself and Russell Brass is to make enquiries about any intended course from students who have already taken it before signing up for it. The length and difficulty can be gauged by comparison with previous courses. Most of us have completed at least M100. The sort of survey published in M500/17 about M231 can also be most helpful. There are, I believe, many more of these surveys and I am sure we could persuade the I.E.T. to co-operate and allow us to see the results of these surveys in exchange for our co-operation in completing the various questionnaires.

Sidney Silverstone

M202/M231

M202 this year compared with 1973? For me it was much easier. Obviously some of this was due to my having done the course before. Not much, though, for I didn't cover more than 50% of M202 in 1973, which was why I didn't sit the exam. The assignments must have been easier, for the averages were much higher. I rate M231 as more difficult than M202. Less interesting, though, to me. Excellent material, however.

Hugh Mclntyre

MDT241

At first (as has happened before with OU courses) I thought MDT241 was a silly course: too numerical for non-mathematicians and too wordy for anyone who had done M100, and that they had left out a lot of bits that would have been enthrallingly interesting to anyone who had done M201. About halfway through I found that a lot of stuff which I had picked up by long experience was beginning to slot into place and make very good sense. The course became much more interesting to me and I must have been getting increasingly interesting to the people I have seen for 'work work' and to whom I have tried to explain some point of statistical practice (if not theory). Since mid-course I have felt strongly that I was giving much better value in my work which is chiefly arranging, testing and helping with the conclusions to be drawn from other people's data. By the end of the course it looks as if the course team have included quite a few ideas which run strongly on a basis of theory beneath nearly all the work we have done in the course, and a lot of techniques. I don't know whether the unifying effect of the underlying ideas is clear unless one understands the bulk of the course material fairly well, and it seems rather arrogant to say that and then claim that it is what I noticed, but I do think that the choice of material for the course and the level of thoroughness at which it is taught are cleverly chosen.

My experience has been that new mathematical ideas take a long time to sink in – sometimes months – and that statistical ideas if anything take even longer. Probability is intuitively obwious to a few people and I suppose if they have problems these are different ones, but the rest of us can hardly bear to believe the facts at first sight. The way that this course brought together facets I knew from experience and joined them with a skeleton of theory have been very striking and I am sure that I have gained a great deal from this course. Whether such an effect is possible far most people who do not have a background of practical experience is another matter, though I would expect that there is enough in the course to give anyone a good start who wanted to use statistical ideas in practice.

Well, the exam is now done. I don't know whether to be relieved or sad!

Margaret Corbett ('resting' 1975)

EDITORIAL

M500/20 is PRACTICALLY EMPTY. HELP! Gregory, Jones, Dale & Margolis are in; 12 pages to fill. M100 are busy sending for sample 18. Publicity in 2nd-level rather stunning, and not written by me! M500 still winning as title. Renewals arrive e.g. £15 from 5 people in ONE DAY. Many, many thanks. Post-dated cheques acceptable if you are short and want to renew. Ed. now a BA with 1948 sandals and very cheerful. It's perseverance wot counts, say I. This A4 paper leaves me no room to say anything, but Happy New Year anyway, and look forward to Grand Opening MOUTHS plus indexes to 1-8 and 9-18 with No. 20– IFF anyone writes No. 20!!

Cheers, M.

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