Answers to M500 Revision Questions

1. C 2. B 3. D 4. A 5. B 6. C

7. $y = x^3 - 1/x^3$ 8. $y = \frac{1}{2}(1 - e^{-x^2})$ 9. $y = \frac{1}{2}(x^2 + 3x^{-8})$

10. $y = \frac{2}{5}(\sqrt{x} - x^{-2})$ 11. $y = (3 - x^{-1/2})^2$

12. $y = \tan\left(\frac{1}{2}\tan^{-1}(2t) + c\right)$, $y = \tan\left(\frac{1}{2}\tan^{-1}(2t) + \frac{\pi}{4}\right)$ 13. $y = -\frac{3}{3x - x^3 + c}$, $y = \frac{3}{3 - 3x + x^3}$

14. $y = Ae^{-2x} + Be^{-4x} + \frac{1}{2}x + \frac{1}{4}$ 15. $x = -\frac{1}{18}(1 + 3t)e^{-3t} + \frac{1}{9} - \frac{1}{18}\cos(3t)$

16. $x = e^{-t}\big(2\cos(2t) + \sin(2t)\big) - \frac{1}{5}\cos(t) + \frac{2}{5}\sin(t)$ 17. $x = -e^{t}(\cos(2t) + \frac{3}{2}\sin(2t) + 1) + 2 + 5t$

18. $y = e^{2x}(A\cos(x) + B\sin(x) + 3)$ 19. 1.375

20. B 21. C 22. D 22(a) D 23. A 24. A

25. C 26. D 27. B 28. A 29. A 30. D

31. C 32. A

33. (i) $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda + 2 & 1 \\ 0 & 0 & (\lambda + 3)(-\lambda + 2) & -\lambda + 2 \end{array}\right]$ (ii)(a) $\lambda = -3$, (b) $\lambda \neq -3$, $\lambda \neq 2$, (c) $\lambda = 2$.
(iii)(a) $[1\ \ 1/4\ \ 1/4]^T$, $[5k\ \ 1 - 4k\ \ k]^T$, $k \in \mathbb{R}$

34. (i) $\left[\begin{array}{ccc|c} 1 & 4 & 3 & 3 \\ 0 & 13 & 5 & 10 \\ 0 & 0 & 0 & k + 4 \end{array}\right]$ (iii) $x = -(1 + 19\alpha)/13$, $y = 5(2 - k)/13$, $\alpha \in \mathbb{R}$ 35. $\begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

36. Eigenvalues are 7, –5 and 2, $[1\ \ 0\ \ 0]^T$ 37. 1, $[3\ \ 4]^T$; 2, $[2\ \ 3]^T$

38. (i) 3, $[1\ \ 2]^T$; –2, $[1\ \ -3]^T$; (ii) $x_1 = -1.25e^{-t} + 0.25e^{3t} + 3e^{-2t}$, $x_2 = 2.5e^{-t} + 0.5e^{3t} - 9e^{-2t}$.

39. (i) 1, $[-2\ \ 1]^T$; 2, $[5\ \ -3]^T$; (ii) $x_1 = -3e^{-t} + 3e^{t} + 5e^{2t}$, $x_2 = 1.5e^{-t} - 1.5e^{3t} - 3e^{-2t}$.

40. (i) 1, $[1\ \ -3]^T$; 5, $[1\ \ 1]^T$; (ii) $x_1 = -e^{t} + 2e^{5t} + e^{-t}$, $x_2 = 3e^{t} + 2e^{5t} - 5e^{-t}$.

41. $2 + 10(x - 1) + 3(y - 2) + 12(x - 1)^2 + 11(x - 1)(y - 2) + (y - 2)^2$.

42. (2, 1) saddle 43. (0, 0), saddle; (2, 0) local minimum;

44. (1, 1), (–2, –2) saddle points, (4, 2) local minimum

45. (i) (1, 1), spiral source; (–2, –2) spiral sink; (–1, 1), (2, –2) saddles.

(ii) (0, 0), star sink; (1, –1/2), (–1, 1/2), saddle points.

(iii) (1, 1), saddle : (-1, 1) linear system has a centre, for original non-linear system EP may be a centre, a spiral sink or spiral source.

(iv) (0, 0) source, (0, 2), (2, 0), (–2, –2) saddles

(v) (0, 0) star source, (0, 2), (2, 0) sinks, (2/3, 2/3) saddle point.

46. (a) (i) $x = x_0e^{qt}$, (ii) No, exponential growth,

(b)(i) (0, 0), (s/r, q/p) (ii) (0, 0) saddle, (s/r, q/p) centre

(c)

Point	Type of stationary point
(0, 0)	unstable source
(0, 64)	unstable saddle
(500, 0)	unstable saddle
(450, 100)	stable spiral sink

47. (i)

(0, 0) Unstable source

(r/s, p/q) Unstable saddle

(ii)

Point	Type of stationary point
(0, 0)	unstable source
(0, 1600)	stable sink
(2500, 0)	stable sink
(1500, 400)	unstable saddle

48.	B	49.	D	50.	D	51.	A	52.	B	53.	D
54.	D	55.	C	56.	C	57.	A	58.	C	59.	C

60. C 61. (i) $\sqrt{17}$, (ii) $\sqrt{14}$, (iii) $-\frac{5\sqrt{17}\sqrt{14}}{119}$ 62. (ii) $P = mg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)$

63. Mgcosec(θ)/2 64. $mg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\phi + \mu\sin\phi}\right)$

65

65(c) $\mathbf{N} = |\mathbf{N}|\,\mathbf{j}\,,$

$\mathbf{F} = -|\mathbf{F}|\,\mathbf{i}\,,$

$\mathbf{T}_1 = |\mathbf{T}_1|\,\mathbf{j}\,,$

$\mathbf{W}_1 = Mg\sin\theta\,\mathbf{i} - Mg\cos\theta\,\mathbf{j}\,.$

The forces acting on m are rather simp

$\mathbf{T}_2 = |\mathbf{T}_2|\,\mathbf{j}\,,$

and

$\mathbf{W}_2 = -mg\,\mathbf{j}\,.$

(d)

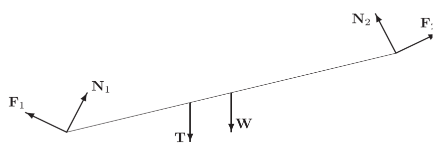
$$|\mathbf{N}| + |\mathbf{T}_1| = Mg\cos\theta.$$

$$|\mathbf{T}_2| = mg.$$

(e) $|\mathbf{T}_1| = |\mathbf{T}_2|.$ $\quad |\mathbf{F}| = \mu|\mathbf{N}|.$

(f) $$\mu = \frac{Mg\sin\theta}{Mg\cos\theta - mg} = \frac{M\sin\theta}{M\cos\theta - m}.$$

66 (a)



W — the weight of the rod

T — the tension in the rope at a third of the distance from the left end

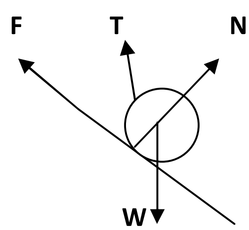
N_1 — the normal reaction from the plane on the left-hand-side of the rod

N_2 — the normal reaction from the plane on the right-hand-side of the rod

F_1 — the friction from the plane on the left-hand-side of the rod

F_2 — the friction from the plane on the right-hand-side of the rod

66(b)



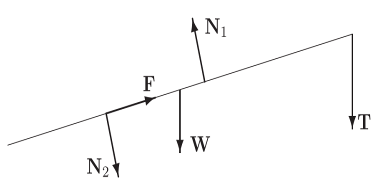
W – weight of the pipe

T – Tension in the rope

N – Normal reaction of the plane on the pipe

F – Friction force between pipe and plane

67.



There are five forces acting on the rod.

W — the weight of the rod

T — the tension in the rope at the right hand end

N_1 — the normal reaction from the freely-rotating cylinder on the rod

N_2 — the normal reaction from the fixed dowel on the rod

F — the friction force between the dowel and the rod.

68. C   69. C   70. A

71. For axis pointing upwards, upward motion: $a = -g - kv^2$, downward motion: $a = -g + kv^2$.

72. (i) 4**j**, (ii) (2t + 1)**i** – 12sin(3t)**j**, 1, (iii) $0.25\sqrt{4 + 36^2\cos^2(3t)}$, (iv) t =0, 2**i** – 36**j**, π/3, 2**i** + 36**j**

73. $v_T = 4$ 74. (ii) $v_T = 4v_0$, (iii) $x = \frac{8v_0^2}{g}\ell n\left(\frac{15v_0^2}{16v_0^2 - v^2}\right)$, (iv) $\frac{8v_0^2}{g}\ell n\left(\frac{15}{7}\right)$

75. - 76. (i) $5\mathbf{i}\ ms^{-1}$, (ii) $\mathbf{i}\ ms^{-2}$ (iii) 2 N 77. - 78. -

79. $x = \frac{v_0^2}{4g}\ell n\left(\frac{3}{2}e^{2gt/v_0} - e^{-gt/v_0}\right)$, 80. C 81. D 82. B

83. $\tan^{-1}(11/2)$, 60m 84. Speed = $5\sqrt{ag}/2$, angle = $\tan^{-1}(3/4)$

85. (i) $v_0 = d\sqrt{\frac{g}{2h}}$, (ii) $\theta = \tan^{-1}(d/h)$, (iii) $x = \frac{d^3}{2(h^2+d^2)}$, $y_{max} = \frac{d^4}{4h(d^2+h^2)}$ 86 $\tan^{-1}\left(\frac{u^2}{gd}\right)$

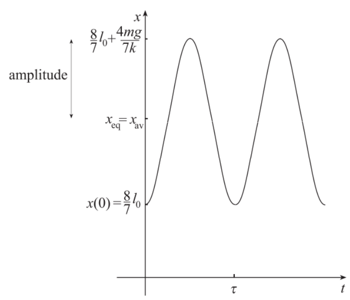
87. - 88. A 89. D 90. B 91. D 92. D

93. D 94. $\ell_0 + \frac{mg\sin(\theta)}{2k}$, $m\ddot{x} + 2kx = mg\sin(\theta) + 2k\ell_0$.

95. (c) $\frac{8\ell 0}{7} + \frac{2mg}{7k}$, (d) $\ddot{x} + \frac{7k}{2m}x = \frac{4k\ell_0}{m} + g$, (e) $x = C\cos\left(\sqrt{\frac{7k}{2m}}\,t\right) + D\sin\left(\sqrt{\frac{7k}{2m}}\,t\right) + \frac{8}{7}l_0 + \frac{2}{7}\frac{mg}{k}$.

(f) $x = -\frac{2}{7}\frac{mg}{k}\cos\left(\sqrt{\frac{7k}{2m}}\,t\right) + \frac{8}{7}l_0 + \frac{2}{7}\frac{mg}{k}$ $\tau = 2\pi\sqrt{\frac{2m}{7k}}$. $\frac{2mg}{7k}$

(h)



96. (b) $\frac{d+\ell_0}{3}$ (c) $m\ddot{x} + 3kx = k(d + \ell_0)$

97. $x = 0.5 + \frac{2g}{25}\sin\left(\frac{\pi}{8}\right)$

97(a). (a) $4\ddot{x} + 100x = 50 - 4g$

(b) $1/2 - g/25$

98. (a) $E = 2v^2 - 4gx + 50(x - 0.5)^2$ (b) $x_{max} = 0.5 + 0.04g + 0.04\sqrt{g(g+25)}$

99. (iii) $x(0) = 15/2$, $\dot{x}(0) = 0$, (iv) $x = \cos(2t)/3 - 4\cos(t)/3 + 17/2$ 100. C

101. D 101(a) A 102. $E = -mgx + m\dot{x}^2/2 + k(x - \ell_0)^2 + k(3\ell_0 - x)^2/2 + k(2\ell_0 - x)^2$.

103. - 104. 0.2 m 105. $E = v^2 + 20(x - 2)^2 + 20x$, min length = ¾ m, max = 9/4 m

106. $E = v^2 + 25x^2 - 7x + 0.25$, Yes 107. Maximum compression is $\ell_0/3$ (spring length is $2\ell_0/3$)

108. Min length = $2\ell_0/3$, max length is $7\ell_0/3$ 109. C

110. $E = \frac{1}{2}mv^2 - mgx + \frac{1}{2}k(x - \ell_0)^2 + k(x - d - \ell_0)^2$, $m\ddot{x} + 3kx = mg + 3k\ell_0 + kd$

111. (a) $2(x - 3)^2$, $5(2 - x)^2/2$, $3(2 - x)^2/2$, $E = mv^2/2 + 2(x - 3)^2 + 4(2 - x)^2$, (b) 8/3 m, 1/3 m

112. C 113. A 114. A 115. $5\ddot{x} + 50\dot{x} + 100x = 100$, x measure from ceiling, 1 m

116. (iii) $x = Ae^{-2t}\cos(t + \phi) + 1 + \cos(t)/8 + \sin(t)/8$, (iv) amplitude = $\sqrt{2}/8$, phase = $-\pi/4$

117. (ii) $m\ddot{x} + r\dot{x} + 2kx = r\dot{y} + kd$, strong damping as $r^2 = 16 > 4 \times 1 \times 2$,

P.I. $x = 1.5 + 4\cos(t - \phi)/\sqrt{17}$, $\phi = \cos^{-1}(1/\sqrt{17})$ (HB 56); or $x = 1.5 + 4\big(\cos(t) + 4\sin(t)\big)/17$

118. (iii) $A(0) = 0$, Dominating term in denominator is $\sqrt{\Omega^4} = \Omega^2$ so $A(\Omega) \to 0$ as $\Omega \to \infty$.

System exhibits resonance as limits = 0 and $A(\Omega) \geq 0$ for all positive Ω.

119. D 120. A | 121. (iii) $\sqrt{\frac{k}{m}}$, $[1\ \ 2]^T$; $\sqrt{\frac{11k}{2m}}$, $[4\ \ -1]^T$,

120(a) D **120(b) C**

121 $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A\begin{bmatrix} 1 \\ 2 \end{bmatrix}\cos\left(\sqrt{\frac{k}{m}}t + \phi_1\right) + B\begin{bmatrix} 4 \\ -1 \end{bmatrix}\cos\left(\sqrt{\frac{11k}{2m}}t + \phi_2\right)$ (iv) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}\cos\left(\sqrt{\frac{11k}{2m}}t\right)$

122. (c) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A\begin{bmatrix} -0.998 \\ -0.017 \\ 0.060 \end{bmatrix}\cos(2.249t + \phi_1) + B\begin{bmatrix} 0.162 \\ 0.627 \\ 0.762 \end{bmatrix}\cos(0.534t + \phi_2) + C\begin{bmatrix} 0.120 \\ -0.920 \\ 0.372 \end{bmatrix}\cos(1.380t + \phi_3)$

(d) $k\begin{bmatrix} 0.162 \\ 0.627 \\ 0.762 \end{bmatrix}$, where k is non-zero real, (e) A and B (same sign in the eigenvector).

123. (iii) $\sqrt{\frac{k}{2m}}$, $[3\ \ 2]^T$; $\sqrt{\frac{6k}{m}}$, $[1\ \ -3]^T$ (iv) $\mathbf{x}_0 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $\dot{\mathbf{x}}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A\begin{bmatrix} 1 \\ -3 \end{bmatrix}\cos\left(\sqrt{\frac{6k}{m}}t\right)$

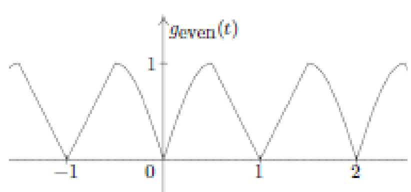
124. $2m\ddot{x}_1 = -4kx_1 + x_2$, $m\ddot{x}_1 = kx_1 - 3x_2$.

125. $\sqrt{\frac{2k}{m}}$, $[2\ \ 3]^T$; $\sqrt{\frac{15k}{m}}$, $[-3\ \ 2]^T$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{\ell_0}{10}\begin{bmatrix} 3 \\ -2 \end{bmatrix}\cos\left(\sqrt{\frac{15k}{m}}t\right)$ 126. C 127. D

128.

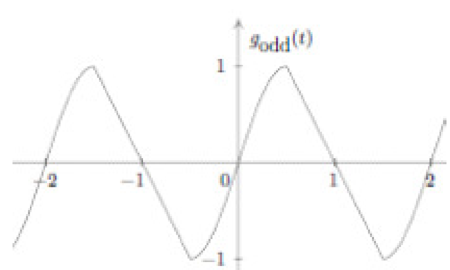
$$g_{even}(t) = \begin{cases} 2 - 2(-t) = 2 + 2t & (-1 \leq t < -\frac{1}{2}) \\ \sin(\pi(-t)) = -\sin(\pi t) & (-\frac{1}{2} \leq t < 0) \\ \sin(\pi t) & (0 < t \leq \frac{1}{2}) \\ 2 - 2t & (\frac{1}{2} < t \leq 1) \end{cases}$$

and $g_{even}(t + 2) = g_{even}(t)$,



$$g_{odd}(t) = \begin{cases} -(2 - 2(-t)) = -2 - 2t & (-1 \leq t < -\frac{1}{2}) \\ -\sin(\pi(-t)) = \sin(\pi t) & (-\frac{1}{2} \leq t < 0) \\ \sin(\pi t) & (0 < t \leq \frac{1}{2}) \\ 2 - 2t & (\frac{1}{2} < t \leq 1) \end{cases}$$

and $g_{odd}(t + 2) = g_{odd}(t)$,



(b) 0

129. (a) $1-\frac{2}{\pi}\sum_{r=1}^{\infty}\frac{(-1)^r}{r}\cos(r\pi t)$

(b) $g_{even}(t) = 1 - t^2, -1 \le t < 1$; $g_{odd}(t) = t^2 - 1, -1 \le t < 0, 1 - t^2, 0 \le t \le 1$.

130. (i) $f_{even}(t) = 1 + t, -1 \le t < 0, = 1 - t, 0 \le t < 1$; $f_{odd}(t) = -(1 + t), -1 \le t < 0, 1 - t, 0 \le t \le 1$.

(iv) Fourier cosine series as even extension is continuous at x = 0.

131. (i) $f_{even}(t) = |\sin(t)|, -\pi \le t \le \pi$, $f_{odd}(t) = \sin(t), -\pi \le t \le \pi$. (iii) $B_1 = 1$, = 0 otherwise.

132. (i) T = 8, $\omega = \pi/4$, even, (ii) M = 3/2, $A_n = 0$, $B_n = \frac{3(-1)^{k+1}}{\pi(2k-1)}$ 133. B 134. B

135. B 136. C

137. $$\frac{d^3X}{dx^3}+\lambda\frac{dX}{dx}-\alpha X=0 \quad \text{where } \alpha \text{ is a constant} \qquad \mu\frac{d^2T}{dt^2}-\alpha T=0$$

138. $X'' - \lambda X = 0$, $Y'' + \lambda Y = 0$, $X'(0) = 0$, $X'(2) = 0$, $Y(0) = 0$

139. (i) $T'(0) = 0$, (ii) $X(0) = X(L) = 0$, $\mu = -n^2\pi^2$, n = 1, 2, 3, …

(iii) $u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi ct}{L}\right)$, (iv) $f(x) = 2x/5, 0 \le x \le L/4, = -2(x - L)/15, L/4 < X \le L$

140. (ii) $\beta = \frac{(2n-1)\pi}{L}$, n = 1, 2, 3, … (iii) $\theta(x, 0) = 100$, $\theta(x, t) = \frac{400}{\pi}\sum_{n=1}^{\infty}\frac{\sin((2n-1)\pi x/2)e^{-\alpha((2n-1)\pi)^2 t/4}}{2n-1}$, $\to 0$

141. (a) $X'(0) = X'(L) = 0$, $\mu = -(r\pi/L)^2$, r = 1, 2, 3, .., $C_0 = \frac{1}{L}\int_0^L f(x)dx$, $C_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{r\pi x}{L}\right)dx$

142. (a) $X'(0) = X'(L) = 0$, $\mu = -(r\pi/L)^2$, r = 0, 1, 2, .., $A_0 = \int_0^1 f(x)dx$, $A_n = 2\int_0^1 f(x)\cos(r\pi x)dx$

143. B 144. D

145. $(\rho^2 + z^2)^{1/2} + 2\rho\sin(\phi)$, $\left((\rho(\rho^2 + z^2)^{-1/2} + 2\sin(\phi)\right)\mathbf{e}_\rho + 2\cos(\phi)\mathbf{e}_\phi + z(\rho^2 + z^2)^{-1/2}\mathbf{e}_z$, $r + 2r\sin(\theta)\sin(\phi)$,

$\left(1 + 2\sin(\theta)\sin(\phi)\right)\mathbf{e}_r + 2\cos(\theta)\sin(\phi)\mathbf{e}_\theta + 2\cos(\phi)\mathbf{e}_\phi$.

146. (a) $\sqrt{\rho^2 + 2z^2 + 1}$, (b) $(\rho\mathbf{e}_\rho + 2z\mathbf{e}_z)/\sqrt{\rho^2 + 2z^2 + 1}$, $\sqrt{\frac{\rho^2 + 4z^2}{\rho^2 + 2z^2 + 1}}$

(c) $\sqrt{r^2(1 + \cos^2\theta) + 1}$ (d) $\frac{(1+\cos^2\theta)r}{\sqrt{r^2(1+\cos^2\theta)+1}}\mathbf{e}_r - \frac{r\sin 2\theta}{2\sqrt{r^2(1+\cos^2\theta)+1}}\mathbf{e}_\theta$

147. (a) $r^2\sin^2\theta - r\cos\theta$, (b) $(2r\sin^2\theta - \cos\theta)\mathbf{e}_r + (r\sin 2\theta + \sin\theta)\mathbf{e}_\theta$ (c) No, depends on θ.

148. (a) $3\mathbf{i} + 4\mathbf{j}$, (b) $-1/\sqrt{2}$, (c) 5, $(3\mathbf{i} + 4\mathbf{j})/5$, (d) $\mathbf{j}$, (equation of tangent is x = 0).

139(a) $X''(x) = \mu X(x)$, $\dot{T}(t) = \mu DT(t)$ $X(0) = 0$ $X'(L) = 0$ $k = \frac{(2n-1)\pi}{2L}$, $n = 1, 2, 3, \ldots$

$$\theta(x,t) = 0.2\exp\left(-\frac{25D\pi^2 t}{4L^2}\right)\sin\left(\frac{5\pi x}{2L}\right).$$

149. C 150. (a)(ii) $-26/3$ (ii)(a) 119/33 (b) **curl F** = $z^2\mathbf{j} + 2y(1 - x)\mathbf{k}$

151. (i) 4 152. (i) 4π, (ii) $\phi = -(x^2 + y^2)z^2$. 153. 128 154. 1.9

155. B 156. A 157. - 158. 0 159. (i) **0**, (ii) $4z(1 - 2\rho\cos(\phi))$

160. B 161. 1/24 162. A 163. A 164. 1/8 165. D

166. (i) 1/3 (ii) $M = \pi\sigma_0 ha^2/6$ 167. $I = \int_{z=-1}^{4}\int_{\theta=0}^{2\pi}\int_{r=0}^{6} k\left(\frac{5r^3}{4+r^2\cos^2\theta}\right) dr\,d\theta\,dz$

168. $M = \int_{x=0}^{3}\int_{z=0}^{3-x}\int_{y=0}^{4} k(x^2 + y^2 + z^2)\,dy\,dz\,dx$ 169. $M = 14\pi\sigma/3$, $I = 15\pi\sigma/4$.

170. $M = 2\pi\sigma_0 a^3/3$ 171. (a) (1, 1), (5, 9) (c) $M = 544\sigma/15$, (d) $x_G = 1888\sigma/15$, $y_G = 1184\sigma/7$

172. (b) z = 4, ρ = 1, (c) $M = 247\pi\sigma/30$ (d) $I_z = \int_{\rho=0}^{\rho=1}\int_{\theta=-\pi}^{\theta=\pi}\int_{z=1}^{z=5-\rho^2} \sigma(3-\rho)\rho^2\,(\rho\,dz\,d\theta\,d\rho)$

$I = \frac{774}{1729}M$ (e) $I_x = \int_{\rho=0}^{\rho=1}\int_{\theta=-\pi}^{\theta=\pi}\int_{z=1}^{z=5-\rho^2} \sigma(3-\rho)\left(z^2 + \rho^2\sin^2\theta\right)\,\rho\,dz\,d\theta\,d\rho$

173. $A = \int_{\sqrt{3/8}}^{1}\int_{-\pi}^{\pi} r\sqrt{1+8r^2}\,d\theta\,dr.$, $19\pi/12$ 174 (c) 40σ (d) (2.5, 2) 175. D

175(a). A 176. D 177. (a) $2mu\mathbf{i}$, $2mv\mathbf{i}$, (b) $v = (2 - e)u/3$, $w = 2(1 + e)u/3$

178. (i) $(3\mathbf{i} + \mathbf{j})/3$ (ii) 16/3 J 179. $(-\mathbf{i} + 2\mathbf{j})$, 12

180. Before: $mu\mathbf{i}$, after: $mw(\cos\beta\mathbf{i} + \sin\beta\mathbf{j}) + mw(\cos\alpha\mathbf{i} - \sin\alpha\mathbf{j})$ 181. $\frac{1}{2}m(u_1^2 + u_2^2 + v_1^2 + v_2^2)$

182. B 183. A 184. B 185. (b) $\ddot{\mathbf{r}} = -R\dot{\theta}^2\mathbf{e}_r + R\ddot{\theta}\mathbf{e}_\theta$, $\sqrt{gL\sin(\alpha)\tan(\alpha)}$

186, (b) $mR\dot{\theta}^2 = mg\cos(\theta) - |\mathbf{N}|$, $R\ddot{\theta} = g\sin(\theta)$ 186(a). $d = g/\omega^2$, $|\mathbf{T}| = m\omega^2\ell$.

187. (ii) $|\mathbf{T}| = 3mg\cos(\theta) + mv_0^2/\ell - 2mg$, (iii) $\cos(\theta) = -\frac{1}{2}$, $\theta = 120°$.

188. (iii) $|\mathbf{N}| = 3mg(2\cos(\theta) - 1)/3$, (iv) $\theta = \pi/3$, speed = $\sqrt{\frac{ag}{2}}$.

189. (ii) $|\mathbf{T}_1| = m(\ell\omega^2 + \sqrt{2}g)$, $|\mathbf{T}_2| = m(\ell\omega^2 - \sqrt{2}g)$, $\sqrt{\frac{g\sqrt{2}}{\ell}}$ 190. C 191. A

192. $I = M\ell^2/3$ 193. D

194. (ii) $\dot{\theta}^2 = 3g(1 - \cos(\theta))/\ell$, $\sqrt{\frac{3g}{\ell}}$, (iii) $V = mg(1 - \frac{3}{2}\cos\theta(1 - \cos\theta) - \frac{3}{4}\sin^2\theta)$

195. (a) $I\omega\mathbf{k}$, (b) $\frac{I}{I+mR^2}\omega$, (c) $\frac{1}{2}.\frac{mR^2}{I+mR^2}\omega^2$

196. (i) $\frac{1}{2}Ma^2 + \frac{1}{2}m(R^2 + a^2)$, $(M + m)\ddot{x} = (M + m)\, g\sin\alpha - |\mathbf{F}|$, $x = R\theta$, $\ddot{x} = \frac{2(M+m)R^2 g\sin\alpha}{(2M+3m)R^2 + (M+m)a^2}$

197. $I = 1031mr^2$ 198. $x = a\theta$, $\ddot{x} = g\sin\alpha/2$ 199. $I = 8m\ell^2$, $2\pi/3$

200. $(2M + m)\ddot{x} = (2M + m)g - |\mathbf{T}|$, $(MR^2 + \frac{1}{2}mr^2)\ddot{\theta} = |\mathbf{T}|r$ $x = r\theta$.

201. (1, 1), 1/2 kg, (9/20, 3/20), 1/20 kg 202 (1, 1), 1/6 kg,, (9/16, 11/20), Topples