M500 MST210 Revision Questions

Differential Equations (Unit 1)

(1) The first-order differential equation

$$\frac{dy}{dx} = \tan x - y \tan x \quad (0 < x < \pi/2)$$

is to be solved by the integrating factor method. Which option gives the correct integrating factor?

Options

A $\cos x$ **B** $\sin x$ **C** $\sec x$ **D** $\cot x$

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(2) The second-order differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$$

has complementary function $y = (Ax + B)e^{-2x}$. Select the option that gives a suitable trial solution for a particular integral.

Options

A $y = Ce^{-2x}$ **B** $y = Cx^2e^{-2x}$ **C** $y = Cxe^{-2x}$ **D** $y = Cx^3e^{-2x}$

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(3) The first-order differential equation

$$x\frac{dy}{dx} = 2x^2y + 3x^4$$

is to be solved by the integrating factor method. Which option gives the correct integrating factor?

Options

A e^{2x} **B** e^{-2x} **C** e^{x^2} **D** e^{-x^2}

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(4) Select the option that gives an expression for the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$$

Options

A $y = Ae^{2x} + Be^{3x}$ **B** $y = Ae^{-2x} + Be^{3x}$

C $y = e^{5x/2}(A\cos\frac{1}{2}x + B\sin\frac{1}{2}x)$ **D** $y = e^{-x/2}(A\cos\frac{3}{2}x + B\sin\frac{3}{2}x)$

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(5) Consider the differential equation

$$\frac{dy}{dx} = x^3 + xy$$

Which of the following options is correct?

Options

A The differential equation may be solved using the separation of variables method but not the integrating factor method.

B The differential equation may be solved using the integrating factor method but not the separation of variables method.

C The differential equation may be solved using either the integrating factor method or the separation of variables method.

D The differential equation cannot be solved using either the integrating factor method or the separation of variables method.

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(6) Select the option that gives an expression for the general solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0.$

A $y = Ee^{\sqrt{2}x} + Fe^{-\sqrt{2}x}$ **B** $y = E\sin(\sqrt{2}x) + F\cos(\sqrt{2}x)$ **C** $y = Ee^{-2x} + F$ **D** $y = (Ex + F)\,e^{-2x}$.

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(7) Find the solution of the differential equation

$$x\frac{dy}{dx} = 6x^3 - 3y \quad (x \geq 1)$$

which satisfies the initial condition $y(1) = 0$.

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(8) Find the solution of the differential equation

$$\frac{dy}{dx} + 2xy = x$$

which satisfies the initial condition

$y = 0$ when $x = 0$.

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(9) Find the particular solution of the differential equation

$$x\frac{dy}{dx} + 8y = 5x^2 \quad \text{for } x > 0$$

which satisfies the initial condition $y(1) = 2$.

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(10) Find the solution to the differential equation

$$x\frac{dy}{dx} = \sqrt{x} - 2y, \qquad \text{where } x > 0,$$

which satisfies the condition

$y(1) = 0$.

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(11) Find the solution to the differential equation

$$\frac{dy}{dx} = \sqrt{\frac{y}{x^3}}, \qquad \text{where } x \geq 1,$$

which satisfies the initial condition $y(1) = 4$

(12) Consider the differential equation

$$\frac{dy}{dt} = \frac{1+y^2}{1+4t^2}.$$

Find the general solution of the differential equation, expressing y explicitly as a function of t. Hence find the particular solution of the differential equation that satisfies the initial condition $y(0) = 1$.

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(13) Consider the differential equation

$$\frac{dy}{dx} = y^2(1 - x^2) \qquad (y \neq 0).$$

Find the general solution of the differential equation, expressing y explicitly as a function of x. Hence find the particular solution of the differential equation that satisfies the initial condition $y(0) = 1$.

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(14) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 8y = 4x + 5.$$

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(15) Find the solution to the differential equation

$$\ddot{x} + 6\dot{x} + 9x = 1 + \sin 3t$$

which satisfies the initial conditions

$$x(0) = \dot{x}(0) = 0.$$

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(16) Find the solution to the differential equation

$$\ddot{x} + 2\dot{x} + 5x = 2\sin t$$

which satisfies the initial conditions

$$x(0) = \tfrac{9}{5} \text{ and } \dot{x}(0) = \tfrac{2}{5}.$$

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(17) Find the solution to the differential equation

$$\ddot{x} - 2\dot{x} + 5x = 25t - 4e^t$$

which satisfies the initial conditions

$$x(0) = 0 \quad \text{and} \quad \dot{x}(0) = 0.$$

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(18) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 3e^{2x}.$$

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(19)

This question is concerned with the use of Euler's method to find a numerical solution of the following initial-value problem

$$\frac{dy}{dx} = x^2 + y + 1, \qquad y(0) = 0;$$

where y(x) is the solution to the given initial-value problem.

Determine an estimate for y(1) using a step size of 0.5.