M500 NEWSLETTER No.14 June 1974

Editor: (Mrs) Marion Stubbs

A PREVIEW OF M331 - Allan Solomon

The M331 “Integration and Normed Spaces” course is a third level ½-credit Mathematics course due for first presentation in 1975

It is a course on the theory and application of the Lebesgue Integral, based on “*Lebesgue Integration and Measure*” by Alan J. Weir, Cambridge University Press, 1973, paperback edition £2.40. An elementary though rigorous treatment of the Lebesgue Integral on the Real Spaces $\mathbb{R}^k$ is given including the convergence and other properties. The Lebesgue Integral is an important generalization of the ordinary Riemann integral, familiar to all our students who have taken the recommended prerequisites, and is even simpler to construct. This integral occurs in Probability Theory and Continuous Group Theory. A discussion of the L^p spaces, which include Hilbert Space, completes the course. Linear spaces, such as Hilbert space, are playing an increasingly important role in both Pure Mathematics and Modern Physics, and are a necessary ingredient in the equipment of most pure analysts and all modern theoretical physicists.

Prerequisites (recommended strongly) are M231 and M201.

The course is designed for those students who wish to take an Honours Course with special interest in Pure and/or Applied Mathematics and/or Theoretical Physics and/or Statistics.

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BOOKS FROM WALTON HALL - Graham Flegg

In M500/9 you asked for notes on books written by members of the OU Maths Faculty. Students may be interested therefore to hear of the TRANSWORLD STUDENT LIBRARY (Maths and Science) of which I am the General Editor. There are several titles in the series either written by or translated by members of the Faculty.

Theoretical Statistics - Stanley Collings
Calculus via Numerical Analysis - Alex Graham and Graham Flegg
Basic Mathematical Structures - Norman Gowar and Graham Flegg
Boolean Algebra - Graham Flegg
Points And Arrows - A. Kaufmann (trans. H.G.F.)
Reliability - A. Kaufmann (trans. A.G.)
Meteorology - H.-J. Tanck (trans. Ralph Smith)

There are several other basic mathematical works in the series, including a book inspired by Stanley Moses’ lectures to M100 Summer Schools on Problem Solving: *The Art of Problem Solving* - Stan Moses. More titles are on the way, including a history of maths title translated from the German by Maxim Bruckheimer, and two of the set books for AM289.

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KLEIN BOTTLE - Lytton Jarman’s Mum

As a mathematician please could you answer the following question: how can I start with an even number (30 sts on 3 needles), decrease 6 sts every other row and end up with an odd number (9 sts) on each needle?

Ed: Original pattern used 3 needles, not 4 (Impossible!)

Does that help a bit???

THE MATHEMATICS OF CRYSTALLOGRAPHY - Michael Gregory

This is not intended to replace a course unit, but rather to interest non-scientists in an application of mathematics.

Defining a crystal as a solid having a periodic arrangement of atoms in three dimensions, the ‘intuitive’ crystals such as diamond, emerald, ice and sugar are included. Modern techniques show that the metals and many materials whose individual particles are minute also satisfy the definition.

To understand the properties common to crystalline materials, imagine an origin (O) at a convenient point in the atomic array (fig.1), and draw three axes (OA, OB, OC, not co-planar) along directions of high concentration of atoms. At intervals *a*, *b*, *c* along OA, OB, OC the environment of the origin is repeated. The unit cell or ‘building block’ is defined by the lengths *a, b, c* and angles α, β, γ in the most general (triclinic) case. If the atomic arrangement is such that some lengths are equal or the angles have certain values, special cases arise, as shown in the table listing the six systems. A rhombohedral system is sometimes considered separately, but can always be treated as a hexagonal case.

We can refer to a set of equally-spaced parallel planes within the lattices by Miller indices *h k l* such that

$$\text{OA}' : \text{OB}' : \text{OC}' = \frac{a}{h} : \frac{b}{k} : \frac{c}{l},$$

where A′, B′, C′ are the intercepts with the three axes of the first plane from the origin (a corresponding plane always passes through the origin). The set of planes with *h k l* = 1 1 2 is shown in fig.2. The perpendicular distance d between adjacent *h k l* planes is important; in order to express *d* in terms of *h k l* and the cell constants consider fig.3 for the monoclinic case (the triclinic case is rather complicated). The intercepts are written A, B, C for simplicity, and OP and CR are construction lines as indicated.

△CRA is similar to △POA

Therefore $\dfrac{\frac{a}{h}}{\text{OP}} = \dfrac{\left(\frac{a}{h}\right) + \text{OR}}{\text{RC}}$

but $\text{OR} = \left(\frac{c}{l}\right)\cos(180° - \beta) = -\left(\frac{c}{l}\right)\cos\beta$

$\text{RC} = \left(\frac{c}{l}\right)\sin(180° - \beta) = -\left(\frac{c}{l}\right)\sin\beta$

therefore

$$\frac{1}{\text{OP}} = \frac{\left(\frac{a}{h}\right) - \left(\frac{c}{l}\right)\cos\beta}{\left(\frac{a}{h}\right)\left(\frac{c}{l}\right)\sin\beta} = \frac{l}{c\sin\beta} - \frac{h}{a\tan\beta}$$

where A, B, P are the intercepts of the plane on orthogonal axes.

[Note: 1 is the numeral, *l* the third Miller index]

The distance *d* is derived in coordinate geometry texts, or using methods of M201, by

$\frac{1}{d^2} = \frac{1}{\text{OA}^2} + \frac{1}{\text{OB}^2} + \frac{1}{\text{OP}^2}$ (this is easily verified for say OA zero)

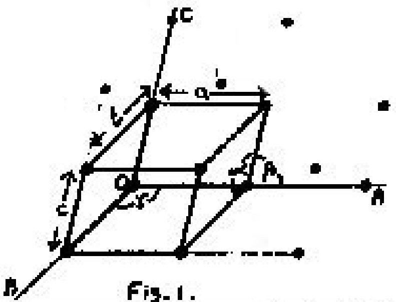


Fig. 1.

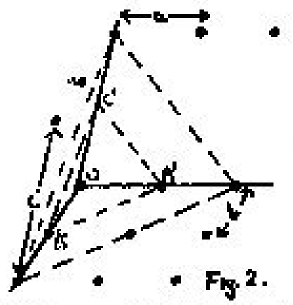


Fig. 2.

System	Cell Lengths	Angles (degrees) α β γ	$1/d^2$
Cubic	$a = b = c$	90 90 90	$\frac{(h^2+k^2+l^2)}{a^2}$
Tetragonal	$a = b \neq c$	90 90 90	$\frac{(h^2+k^2)}{a^2} + \left(\frac{l}{c}\right)^2$
Orthorhombic	$a \neq b \neq c$	90 90 90	$\left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 + \left(\frac{l}{c}\right)^2$
Hexagonal	$a = b \neq c$	90 90 120	$\frac{4}{3a^2}(h^2 + hk + k^2) + \left(\frac{l}{c}\right)^2$
Monoclinic	$a \neq b \neq c$	90 $\neq$90 90	$\frac{h^2}{a^2 \sin^2\beta} + \left(\frac{k}{b}\right)^2 + \frac{l^2}{c^2 \sin^2\beta} - \frac{2hl\cos\beta}{ac\sin^2\beta}$
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma$	rather complicated.

ON ⊥ plane ABC

Fig. 3.

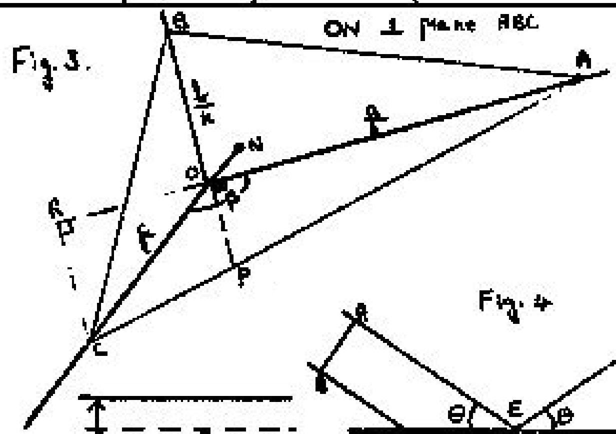
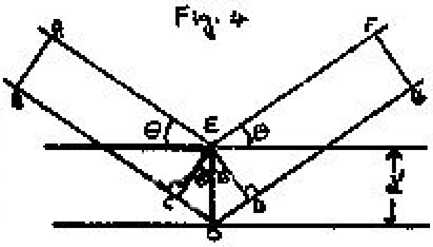


Fig. 4



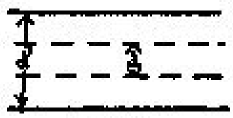


Fig. 5.

$$= \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2 \sin^2 \beta} - \frac{2hl}{ac \sin \beta \tan \beta} + \frac{h^2}{a^2 \tan^2 \beta}$$

adding the first and last terms, and modifying the fourth term using

$$1 + \frac{1}{\tan^2 \beta} = \frac{1}{sin^2 \beta} \quad \text{and} \quad \frac{1}{\sin \beta \tan \beta} = \frac{\cos \beta}{\sin^2 \beta}$$

gives $1/d^2$ for the monoclinic system as shown in the table.

One can derive the cubic, tetragonal and orthorhombic equations by substituting the special lengths and angles in the monoclinic one. The hexagonal form is derived by interchanging $b \leftrightarrow c$, $k \leftrightarrow l$, $\beta \leftrightarrow \gamma$, and substituting $a = b$, $\alpha = \beta = 90^o$ and $\gamma = 120^o$.

Almost all values of cell constants which are known have come from X-ray diffraction experiments. When a beam of X-rays of wavelength λ strikes a sample of crystalline material, beams A and B striking adjacent *h k l* planes of atoms will be partially reflected (fig.4). If the incident wavefront AB is to proceed along the new direction shown, the wavefront GE must be in phase, so the extra distance travelled along BOG compared with AEF must be a whole number of wavelengths $n\lambda$, $n \in \mathbb{Z}$. Thus, for reflection along the indicated path the extra distance = COD = $n\lambda$

but CO = OD $d' \sin \theta$

so the requirement is $2d' \sin \theta = n\lambda$ which is known as Bragg's law. If $n > 1$ it is convenient to imagine the extra planes equally spaced between those of fig.4, giving a spacing $d = d'/n$. Fig.5 illustrates the case with $n = 3$.

The Bragg equation becomes $2d \sin \theta = \lambda$.

Using the $1/d^2$ equations from the table and the last equation we can eliminate d to obtain $\sin^2 \theta$ equations for each system, e.g.

Cubic: $\sin^2 \theta = (\lambda^2/4a^2)(h^2 + k^2 + l^2)$;

Hexagonal: $\sin^2 \theta = (\lambda^2/3a^2)(h^2 + hk + k^2) + \lambda^2 l^2/4c^2$.

This is a convenient form for the analysis of X-ray diffraction photographs.

Tables of $\sin^2 \theta$ with θ in degrees and decimals (as usually obtained by calculation), and of $(h^2 + k^2 + l^2)$ and $(h^2 + hk + k^2)$ etc. for $h, k, l \in (\mathbb{Z}_0)^+$, are readily available.

The absence of solutions of $h^2 + k^2 + l^2 = 7$, and $h^2 + hk + k^2 = 2$ or 5 have significant results.

I hope to continue with more practical aspects of this subject in a later issue.

<u>Leslie Naylor</u>

I had a very helpful letter from Professor Bruckheimer which said there are to be courses on Mathematical Statistics (M341) and Computing (M351) in 1977 and 1976 respectively. Perhaps we could ask him to let M500 know what future plans are. This was the first I had heard about either of these courses.

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Yvonne Kedge

The arrival of the information that my only prospects for converting my pass degree into an honours lie in partial differential equations and numerical analysis (in 1976) means the end to my OU career at the end of this year.

How will other students ending this year cope with the withdrawal symptoms? It is an interesting and fearful prospect.

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Susan Davies

I am continuously astonished at the complete disregard shown by the OU for the needs of maths students at 3rd level. I see from our new Courses Handbook that by 1976, some 4 years after the OU's first graduates moved on to 3rd level, the Maths Faculty will finally produce the minimum of two full credits necessary to complete an honours degree in mathematical subjects.

By 1976 the Arts Faculty will be offering a choice of 9 full credits at 3rd and 4th level, some of which look rather more appealing than Numerical Computations and Partial Differential Equations. Doesn't the Maths Faculty care that most of its students are defecting to other Faculties at 3rd level?

I find it ironic that in the same mailing as the Courses Handbook I received a copy of *Sesame* containing a full-page advert on the joys of OU maths courses, complete with numerous ladders climbing up to 3rd and higher level. Shouldn't prospective maths students be warned that at present there is only half a ladder (M321) to 3rd level (and it appears to have a few rungs missing!)

I am willing to organise a petition via M500 readers and hope that students will send me the top part of the enclosed form as soon as possible. Then the bottom part can be used for collecting signatures at study centres/summer schools, with extra home-made sheets added as required, and can be returned later, but by (say) mid-September at the very latest.

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Marion Stubbs

As one in desperate straits, who cannot do calculus and hence incapable of M321, M331 and M332, I wrote to the A305 Course Team to ask if it is feasible to go straight into Arts 3rd level with no previous OU essay-writing courses. The answer was a qualified negative, although the Art History staff tutor thought that I personally might be able to tackle it, as a librarian in a College of Art.

It seems that, those of us with no second string to our bows are trapped in a cul-de-sac from which there is no normal escape. For me AM289 will be a 1975 course which sounds delicious in its own right and also offers a route into those 9 Arts credits.

COMPLETE PERMUTATIONS - David Asche, Staff Tutor

A permutation is said to be complete if each symbol is moved to a position different from what it had been before. These complete permutations, sometimes called derangements, occur in a variety of combinatorial problems. For example, suppose n men went to a party and each man handed in his hat to the cloakroom attendant. During the evening, the hats became mixed up and were returned at random to the guests. What is the probability that no one received his hat back? To answer this we would need to find the ratio of the number of complete permutations to the total number of permutations of n objects.

Let Dn be the number of complete permutations of n symbols. We can easily verify that $D_1 = 0$, $D_2 = 1$ and $D_3 = 2$. The interested reader may care to stop reading at this point and try to make the generalization himself.

The way I approached it was to try to discover a recurrence relation which would give D_n in terms of D_m's for $m < n$. This, in fact, worked so I will describe the method.

Let $I = \{1, 2, ..., n\}$ and let $f: I \to I$ be a complete permutation. This simply means that f is one-to-one and onto, and is such that $f(i) \neq i$ for $i \in I$. Starting with a particular symbol, for example 1, we can form the chain

$$1 \to f(1) \to f(f(1)) \to ...$$

where we know that $f(1) \neq 1$, $f(f(1)) \neq f(1)$ and so on. Since we are trying to reduce the number of symbols, let us define a new permutation which shortens this chain by jumping over $f(1)$. More explicitly, define $g: I - \{f(1)\} \to I - \{f(1)\}$ by

$$\begin{cases} g(1) = f(f(1), \\ g(x) = f(x) \text{ if } x \neq 1. \end{cases}$$

This looks promising, because g is a permutation of $n - 1$ symbols. The only snag is that it need not be a complete permutation. We have to distinguish two cases.

(i) $f(f(1)) \neq 1$, (ii) $f(f(1)) = 1$.

In case (i), g is a complete permutation of $n - 1$ symbols and there are precisely D_{n-1} of these.

In case (ii), we have $g(1) = 1$. But g acts as a complete permutation on the remaining $n - 2$ symbols so it follows that there are exactly D_{n-2} of these.

Thus the total number of such functions g equals $D_{n-1} + D_{n-2}$. Now note that $f(1)$ can take any of $n - 1$ possible values, so that we get the result $D_n = (n - 1)[D_{n-1} + D_{n-2}]$.

We now have our recurrence relation, so we can compute $D_4 = 3(2 + 1) = 9$, $D_5 = 4(9 + 2) = 44$ and so on. It would perhaps be nicer if we had an explicit formula for D_n in terms of n, so let's see if we can solve the recurrence relation. Firstly, be rearranging things, we can write it as

$$D_n = nD_{n-1} = -[D_{n-1} - (n - 1)D_{n-2}].$$

Now putting $C_n = D_n - nD_{n-1}$ $(n \geq 2)$, you will see that we get $C_n = -C_{n-1}$. Since $C_2 = D_2 - 2D_3 = 1$, we get $C_n = (-1)^n$. Hence we discover the simpler relation $D_n = nD_{n-1} + (-1)^n$. Now the trick is to put $P_n = D_n/n!$ and get $P_n = P_{n-1} + \frac{(-1)^n}{n!}$ from which we can write down $P_n = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots + \frac{(-1)^2}{n!}$ and so finally $D_n = n!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots + \frac{(-1)^n}{n!}\right]$.

So at last we have an explicit formula for D_n. It is not very compact and it doesn't really look as useful as the recurrence relation with which we started!

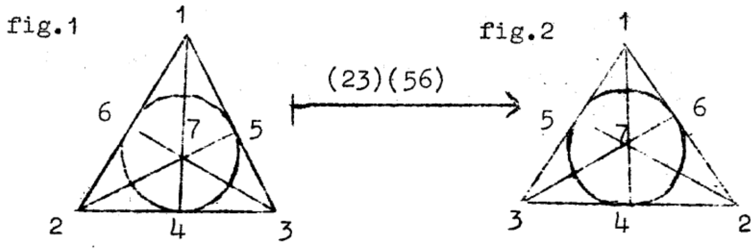
On the other hand, if we happen to know the infinite series for the function $x \to \exp(x)$, we will observe that P_n converges very rapidly to $\exp(-1)$.

Now, returning to our hat problem, the number P_n is the probability that no-one will get his own hat back. So the answer is very close to 0.3679, and is virtually independent of the number of people at the party, a rather curious result.

PROBLEM CORNER No.14

1. COLLINEATIONS - Bob Margolis

Consider fig.1 consisting of 7 points and 7 'lines', where 4-5-7 is also a 'line'. Certain permutations from S_7 have the property that they transform 'lines' to lines, i.e. are collineations. The permutation (23)(65) transforms fig.1 to fig.2, whereas the permutation (12) does not have this property. Denote the subset of S_7 which consists of collineations by G. (Emotive notation!) Investigate G. Is it a group? Finite? Order? Subgroups? Normal subgroups? Sylow subgroups? ... Conjugacy classes? ...



2. NETWORKS - Hugh McIntyre (solution unknown)

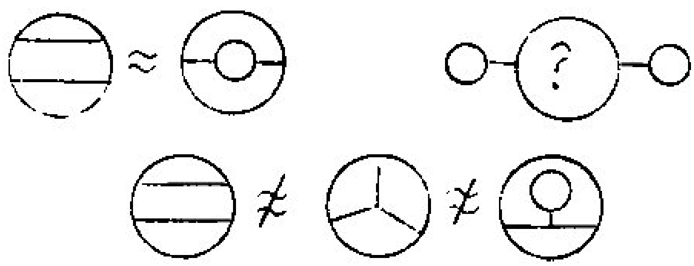
Consider the set of networks in the plane, each of which encloses precisely S spaces. In each network there are precisely I intersections of 3 lines, and none of higher order. Each network contains precisely L lines, a line being defined between 2 intersections.

$$\left.\begin{array}{l} I = 2(S-1) \\ L = S + N - 1 \end{array}\right\} \quad \text{for all networks.}$$

Two networks are distinct if they are not similar. They are similar if and only if there is a one-to-one correspondence pathwise between them, paths being defined only along lines, and not through spaces.

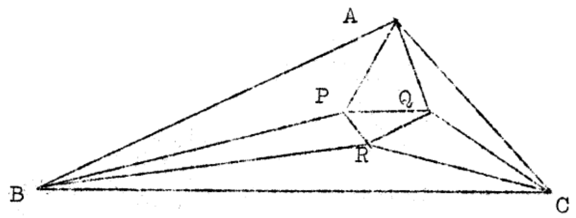
Question: For any S, How many DISTINCT networks exist?

for S = 3



3. TRADITIONAL TRIANGLE PROBLEM - Roger Claxton (sol. unknown)

Given any triangle ABC, trisect each angle. The points inside the triangle, where the trisections intersect, when joined together form an equilateral triangle. So in the diagram, PQR is equilateral. Prove.



<u>NOTE-NOTE-NOTE</u>: First solution to M202/9SAQ13 is by SUSAN DAVIES not by Hugh Tassell as stated. Papers mixed - no name on. Moral??? Sorry, Sue! Sorry, Hugh!

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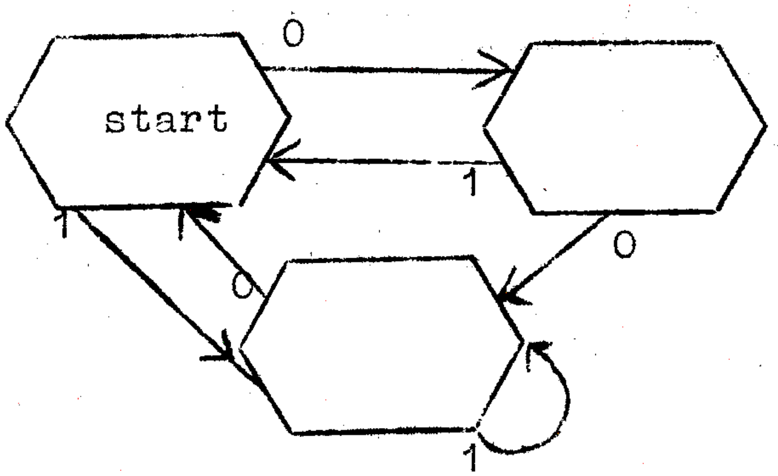
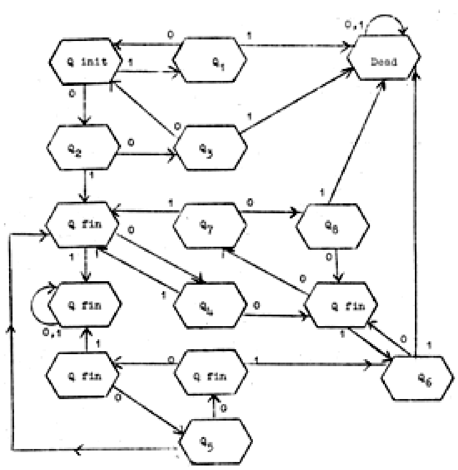
<u>SAQ 13 of M202/9</u>

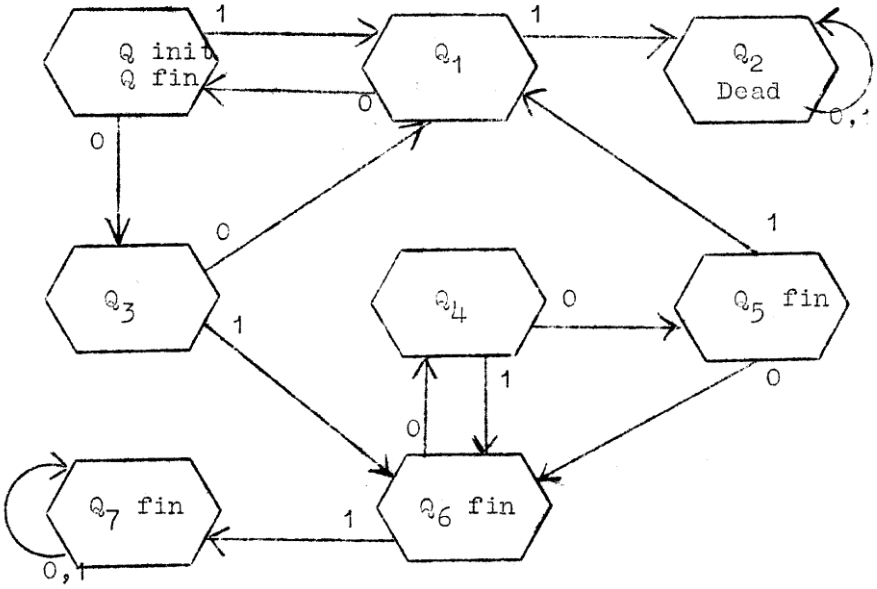
The problem is to find a regular expression which brings the machine back to its starting state, and then draw a ‘reverse’ state diagram. (The Course Team reported Minsky wrong and went out to drown their sorrows, advising us not to bother with it.)



<u>1. Hugh Tassell</u>



2. Peter Faulkner



Brief Explanation

I think it is clear that the machine will recognise all sequences (10 v 01✶00✶1)✶ but not so obvious that it will recognise only these. The problem is the state Q_7. However, considering any sequence which sends the machine into state Q_7, if you start at the (right hand) end of the sequence you will find that you can generate any number of 0's and 1's using just 10, 01✶00 and 01✶1 and the null sequence. Working backwards, eventually you will get to some sequence from the set 01(000)✶1 (which first sent the machine into state Q_7.) If you consider all the possibilities that can occur at this stage you will find that all can be generated using 10, 01✶00 and 01✶1.
(A more detailed explanation available if required.)

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Determinants, ref M500/13 - Dorothy Craggs

To factorise the determinant $D = \begin{vmatrix} e & R1 & R2 & S1 & S2 & S3 \\ R1 & R2 & e & S3 & S1 & S2 \\ R2 & e & R1 & S2 & S3 & S1 \\ S1 & S2 & S3 & e & R1 & R2 \\ S2 & S3 & S1 & R2 & e & R1 \\ S3 & S1 & S2 & R1 & R2 & e \end{vmatrix}$.

Let T1 = $e + R1 + R2 + S1 + S2 + S3$

T2 = $e + R1 + R2 - S1 - S2 - S3$

Perform row operations on D:

replace r1 by r1 + r2 + r3 + r4 + r5 + r6
r2 by r2 + r5
r3 by r3 + r4
r4 by r5 − r2
r5 by r4 − r3
r6 by r4 + r5 + r6 − (r1 + r2 + r3)

$$D = -\begin{vmatrix} T1 & T1 & T1 & T1 & T1 & T1 \\ R1 + S2 & R2 + S3 & c + S1 & R2 + S3 & c + S1 & R1 + S2 \\ R2 + S1 & c + S2 & R1 + S3 & c + S2 & R1 + S3 & R2 + S1 \\ S2 - R1 & S3 - R2 & S1 - c & R2 - S3 & c - S1 & R1 - S2 \\ S1 - R2 & S2 - c & S3 - R1 & c - S2 & R1 - S3 & R2 - S1 \\ -T2 & -T2 & -T2 & T2 & T2 & T2 \end{vmatrix}$$

Perform column operations:

replace c2 by c2 + c4 − c1 − c6
c3 by −c3 − c5 + c1 + c6
c4 by c4 − c2 + c1 − c6
c5 by −c5 + c3 + c6 − c1
c6 by c6 − c1

Then

$$D = -\begin{vmatrix} T1 & 0 & 0 & 0 & 0 & 0 \\ R1 + S2 & \begin{matrix} 2(R2 + S3) \\ -2(R1 + S2) \end{matrix} & \begin{matrix} 2(R1 + S2) \\ -2(c - S1) \end{matrix} & 0 & 0 & 0 \\ R2 + S1 & \begin{matrix} 2(c + S2) \\ -2(R2 + S1) \end{matrix} & \begin{matrix} 2(R2 + S1) \\ -2(R1 + S3) \end{matrix} & 0 & 0 & 0 \\ S2 - R1 & 0 & 0 & \begin{matrix} 2(R2-S3) \\ +2(S2-R1) \end{matrix} & \begin{matrix} 2(S1-c) \\ +2(R1-S2) \end{matrix} & 2(R1-S2) \\ S1 - R2 & 0 & 0 & \begin{matrix} 2(c-S2) \\ -2(S1-R2) \end{matrix} & \begin{matrix} 2(S3-R1) \\ +2(R2-S1) \end{matrix} & 2(R2-S2) \\ -T2 & 0 & 0 & 0 & 0 & 2T2 \end{vmatrix}$$

$$= -2^5\, T2.T1.D1.D2$$

where $D1 = \begin{vmatrix} (R2-R1)+(S3-S2) & (R1-c)+(S2-S1) \\ (c-R2)+(S2-S1) & (R2-R1)+(S1-S3) \end{vmatrix}$

and $D2 = \begin{vmatrix} (R2-R1)-(S3-S2) & (R1-c)-(S2-S1) \\ (c-R2)-(S2-S1) & (R2-R1)-(S1-S3) \end{vmatrix}$

Then D1 = P1 + P2 + P3 + P4, and D2 = P1 − P2 − P3 + P4

where $P1 = \begin{vmatrix} R2-R1 & R1-c \\ c-R2 & R2-R1 \end{vmatrix}$ $P2 = \begin{vmatrix} R2-R1 & R1-c \\ S2-S1 & S1-S3 \end{vmatrix}$

$P3 = \begin{vmatrix} S3-S2 & S2-S1 \\ c-R2 & R2-R1 \end{vmatrix}$

$P4 = \begin{vmatrix} S3-S2 & S2-S1 \\ S2-S1 & S1-S3 \end{vmatrix} = -\begin{vmatrix} S2-S1 & S3-S2 \\ S1-S3 & S2-S1 \end{vmatrix}$

In P3, replace col.1 by −c1−c2, then $-P3 = \begin{vmatrix} S1-S3 & S2-S1 \\ R1-c & R2-R1 \end{vmatrix}$

By interchanging first the rows and then the columns, it is evident that P3 = −P2.

Hence $D1 = D2 = P1 + P4$, and $D = -32T1.T2(P1 + P4)^2$.

By replacing $c2$ by $-c1 - c2$, $P1$ becomes $-\begin{vmatrix} R2 - R1 & e - R2 \\ e - R2 & R1 - e \end{vmatrix}$.

Alternatively, by replacing $r2$ by $-r1 - r2$, $P1$ becomes $\begin{vmatrix} R2 - R1 & R1 - e \\ R1 - e & e - R2 \end{vmatrix}$.

$$\text{Hence } P1 = \frac{-1}{2}\left\{\begin{vmatrix} R2 - R1 & e - R2 \\ e - R2 & R1 - e \end{vmatrix} + \begin{vmatrix} R2 - R1 & R1 - e \\ R1 - e & e - R2 \end{vmatrix}\right\}$$

$$= -½\,\{(R2 - R1)(R1 - e + e - R2)(e - R2)^2 - (R1 - e)^2\}$$

$$= ½\,\{(R1 - e)^2 + (R2 - R1)^2 + (e - R2)^2\}$$

$$= e^2 + R1^2 + R2^2 - eR1 - R1.R2 - R2e.$$

$-P4$ is isomorphic to $P1$, hence by a similar calculation

$$P4 = -\{S1^2 + S2^2 + S3^2 - S1.S2 - S2.S3 + S3.S1\}.$$

Hence $D = -32(e + R1 + R2 + S1 + S2 + S3)$

$$(e + R1 + R2 - S1 - S2 - S3)(e^2 + R1^2 + R2^2 - S2^2 - eR1 - R1.R2 - R2e + S1.S2 + S2.S2 + S3.S1)^2.$$

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Peter Hartley - Course Tutor, M231 (Nuneaton)

As a course tutor for M231 I have a financial allowance this year for personal tuition, using telephone, interchange of letters or 'house calls'. Although my students are aware of this they make virtually no demands, despite the fact that some of them would clearly benefit by a push in the right direction when they are stuck on problems. (Their TMAs show this.) Can anyone explain why? If it's just me perhaps one of my students should let me know. If anyone is finding the course time-consuming because of the intricacy of proofs, spare a sympathetic thought for the tutors who have to sort out whether your proof is logically sound or not. I find marking time considerably longer than M201, say.

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Margaret Corbett

Would any student of MDT241 who has previously done M201 like to discuss whether expectation (as used in finding the mean of a distribution) can be regarded as a linear transformation?

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CONDITIONAL REGISTRATION TIME - FIRST OFFER OF SET BOOKS FOR SALE

Sarah Kettlewell

I have the M251 books in mint condition. (The local book sale was held, I believe, early last November and was totally unadvertised.)

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Math-Quote - Ron Davidson

The true meaning of a term is to be found by observing what a man does with it, not what he says about it!

H. Bridgeman - The Logic of Modern Physics.

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Now for Sinbad - several thousand words have been submitted. All have been coded in order of arrival, and it is intended that all will reach publication eventually. Further replies to Sinbad's argument must be restricted to 300 words maximum or risk rejection.

ARE WE TYPICAL? GOOD HEAVENS! NO, WE ARE UNIQUE. - Datta Gumaste

There are at least two ways to study, investigate any situation. Sinbad (M500/13) has suggested one. This can he called 'empirical/inferential'.

Having had a corrupting (!) influence of M202 I am inclined to attack a problem via an axiomatic approach. The question posed by Sinbad is: 'What are the observable characteristics that all good mathematicians possess?'. Here is one answer.

Let GM = Good Mathematician.

We shall set up four axioms and deduce the results in the hope that they provide a good approximation to the reality.

Any person is a GM if he satisfies at least one of the following (axiomatic) conditions:

A GM
(1) Grows beard.
(2) Never grows old.
(3) Knows that he does not know.
(4) Does not know that nobody knows.

The flood of the results that follow use these axioms and some well-known facts about mathematics.

Lemma 1: A GM is ignorant.

Proof : A person is ignorant if he does not know.
By Axiom (3) a GM does not know, hence GM is ignorant.

Lemma 2: A GM is innocent.

Proof : If not, he would know that nobody knows.
But this contradicts axiom (4) and hence the lemma.

Lemma 3: A GM is an idiot.

Proof : This is immediate from Lemma 2. Oxford Dictionary defines innocent = ignorant.

Lemma 4: Every female is a GM.

Proof : Females never grow old, so they satisfy axiom (2).

SAQ 1 : Show that every female is either innocent or ignorant.
(WARNING: Do not use Lemma 3).

Lemma 5: A GM is a lateral thinker.

Proof : Observe the following equalities:

vertical thinking = logical thinking = yes/no thinking = true/false thinking.

Computer specialises in true/false thinking. Indeed computer is the emperor of vertical thinkers. But computer does not know that it does not know. In fact we have the equivalence: v is a vertical thinker if and only if v does not know that it does not know. By axiom (3) GM does know that he does not know, so GM cannot be a vertical thinker. Hence he is a lateral thinker.

Lemma 6: A GM has good sense of humour.

Proof : Let S = {x : x is a GM and x has a good sense of humour}.

We claim that Archimedes $\in S$.

Archimedes was a great mathematician. He was also a bit of a clown, for if not, he would not have run in the street without clothes. But clowns have good sense of humour. So Archimedes $\in S$.

S is not only non-empty but quite a distinguished set. Now let *t* be some mathematician. Ask him if he belongs to *S*. Unless he is insane he would die to be a life member of *S*. After all, who would not like to be in the company of Archimedes? We have shown that $t \in S$ whenever *t* is a GM.

SAQ 2 : Prove Lemma 6 by using one or more axioms.

Lemma 7: Only a GM can communicate.

Proof : Conmunication is possible only when those who desire it agree about the meanings to he assigned to the symbols they use and accept the rules of inference. Only mathematicians adopt such a procedure almost to the point of being incomprehensible. But GM does mathematics, and hence he, and he alone, can communicate

Lemma 8: A GM has a real contact with the real world.

Proof : The most scientific explanation of the real world is offered by physics. But the world so described is totally different from the world we see. If physicis true then the real world is an illusion. But physics cannot be false.

A mathematician by axiom (3) does not know. So he CREATES his universe. All the mathematical objects are his own creations. Further, everything that exists in this universe is well defined and crystal clear, but then such a universe must be real.

We have proved so many lemmas but not a single theorem; let us therefore prove

THE FIRST PROFOUND THEOREM: A good mathematician is GOD.

Proof: Whatever else God may be, God is the creator of the universe. But so is GM by lemma 8. Indeed, lemma 8 asserts more. GM's universe is the real thing. There is no illusion about it. And we are done.

SAQ 3: Show that the set *S* defined in lemma 6 is a singleton.

Corollary: The First Profound Theorem is the only theorem in our theory.

The proof is obvious and is left to the reader. In short, a GM is what you make of him.

SAQ 4: Prove at least 10 lemmas using axiom (1).

(Hint: Grow beard.)

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Willem van der Eyken

Do mathematicians have particular characteristics? Indeed, indeed, as Eysenk would say, are they stable introverts?

Have you tested for the four Group axioms? To begin with, ‘closure’. If we have a set *S* of students, and we apply an operation (say, a mathematical assignment) on the set, then we would expect the operation to be closed - i.e. mathematics students do not discuss their work with each other. To prove this assertion I think would need to carry out a survey among tutorial sessions, but am prepared to believe that *S* and the operation on M are indeed closed.

Secondly, associativity... . I won't pursue the matter through inverses and identity elements, but I think that with a little work (!) best left to the student (as they say in all the best text books) we could show that we have a group here which, as you suggest, is distinctive and has interesting properties. Moreover, there are sub-groups, some of whom will contain elements of the new History of Mathematics course, others technology or more applied courses.

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Ed: Assertion of para 2 herewith disproved. Substitute assertion only M100 and M201 do not discuss assignments; M202 and ex-M202 do?

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(1 = numeral 'one', l = lower-case letter l)

<u>'N-ary's Logarthmic Calculus'</u> - Henry Jones, M100

To me, and perhaps to other students of mathematics, nothing engenders more frustration than when given a theorem whose proof is too difficult and involved to merit our attention. This happens so often, even in OU mathematics, that it may well be the reason why so many have a distaste for the subject. A typical example is the familiar theorem:

$$\text{If } y \text{ represents } \exp x\text{, i.e. } e^x\text{, then } \lim_{n \text{ large}} \left(1+\frac{x}{n}\right)^n = y,$$

so that $x = \log_e y$. M100/7 shows us how difficult is its proof and comforts us with the knowledge that we are not to be examined on it. From the above theoren,

$$\lim_{n \text{ large}} n\left(y^{1/n}-1\right) = \log_e y.$$

Hence, given a number we can, in theory, thereby calculate an approximation to its logarithm. (I do not know how to do this without using 'common' logarithms in the calculations and nervously confronting the destructive rounding-off demon.)

By reversing the process, i.e. by starting with the number y, we can derive

$$\lim_{n \text{ large}} n\left(y^{1/n}-1\right)$$

directly and very simply. To this end we may use a simple theorem in 'N-ary's' logarithmic calculus, namely:

$$\log_e y = \int 1 \cdot ly \text{ (reminiscent of } \log_e y = \int \frac{1}{y}\, dy\text{) where } ly \text{ is the logarithmic complement of } dy.$$

You will notice that I give the indefinite instead of the definite integral. That is because I don't yet know how to express the latter logarithmically. Then, adopting the device of n equal divisions θy, where $\theta y = \frac{\Delta y}{y}$, we equate $\log_e y$ to $\lim_{n \text{ large}} n \cdot \theta y$, this being $n(y^{1/n} - 1)$ when n is large. Therefore $\log_e y = \lim_{n \text{ large}} n\left(y^{1/n}\right)$.

Incidentally, in the process we immediately derive expressions which, I fancy, throw light on Fermat's Last Theorem, but whilst being hopeful, I'm not so naive as to expect success. I'm a little concerned about the OU symbolism of the definite integral by $\int_a^b x \to f(x)$ ($x \in \mathbb{R}$ (say)), which, to me, seems inhibitory in the sense that, restricted thereby to Newtonian increments, we may be burying a truly promising sphere of mathematical and scientific knowledge. Can someone reassure me to the contrary? The above notes may give an inkling as to why the submission of a student's own proofs has not always been encouraged in the past. I well remember once losing marks by invoking a process whereby labour in evaluating maximum values, minimum values and points of inflection could often be reduced. It was based on another of ‘N-ary’s’ theorems: If $f(x) = \frac{g_1(x) g_2(x) \ldots g_r(x)}{h_1(x) h_2(x) \ldots h_n(x)}$ (multiplying the $g(x)$’s) then

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(a) when $g_1(x) = 0$ and all other $g(x)$ and $h(x) \neq 0$,

$$f'(x) = g_1'(x)\left(\frac{g_2(x) g_3(x) \ldots g_r(x)}{h_1(x)\, h_2(x) \ldots h_s(x)}\right)$$

(b) when more than one $g(x) = 0$ and all other $g(x)$ and $h(x) \neq 0$ then
$f'(x) = 0$.

A simple example

Let $f(x) = \pm\, (b/a)\sqrt{(a^2 - x^2)}$
then $f'(x) = \pm\, (b/a)(x/\sqrt{(a^2 - x^2)})$
therefore, when $x = 0$, $f'(x) = 0$, and from theorem (a),
$f''(x) = \pm\, (b/a)\sqrt{(a^2)} = \pm b/a^2$.
Therefore maximum and minimum values, b and $-b$ respectively, occur at $f(0)$.

You will see where the reduction in labour occurs. In more complicated expressions, and especially when finding points of inflection, the saving in labour can be considerable.

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John Carter - M100

I’m an M100 student with my back to the wall. I’d like the course to be nearer my personal standard, say M50. Since I’m committed and determined to finish the course, MOUTHS seems a good opportunity to get to know other students and maybe get some help out of the treacle.

Which brings me to an interesting topic. We’ve been told that M100 TMAs and CMAs have been too easy in past years (lucky pre-1974 students). Consequently this year we’re being examined to a higher standard, but at what cost? It all stems from the fact that in previous years M100 students have scored too many high marks in continuous assesment papers, as a result of which it’s been difficult to sort the wheat from the chaff until the final examination. This year, by setting more difficult TMAs and CMAs a wider spread of marks is hoped for.

My personal view is that mathematics students, being the perfectionists they are, will just spend longer on the assignments to ensure a good mark. Trouble is that if you spend a long time on your assignments you can’t be progressing in your studies - I wonder if other M100 students feel the same as I do? Incidentally my solution to the OU’s problem is not to have more difficult papers but simply to mark to a higher standard.

Just how do I lay my hands on the exam papers and Summer School ‘texts’ that Willem van der Eyken (M500/13) speaks of? I don’t mind paying for them.

Many congratulations for M500 and MOUTHS, It's a splendid idea and I can't wait to get my June copy hot from the press. I'm a trifle shy of ringing somebody up with my problems.

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Ed: Somebody not-so-shy please ring John, then???

Motto: If shy, refuse to conform to the 'Sinbad' Image. RING A TOTAL STRANGER THIS WEEK - pref. Not me, as my telephone is red hot, and others want calls, too. For shy ones, from my experience, the normal Opening Gambit is: 'You don't know me but...'. other gambits may be submitted to M500 for publication.

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MATHEMATICS DAY SCHOOLS IN CAMBRIDGE - David Asche

A number of Saturday Schools for students in Region 6 will take place in June. It may be that some students from other regions would like to come along as well. If so, would they please write to the Regional Office ... saying that they intend to be present and also indicating whether they want lunch provided (about 60p.). The dates and locations are as follows:

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SOLUTIONS TO PROBLEM CORNER No. 13

2. $194^4 + 291^4 = 97^5$ (Sue Davies, Bill Shannon.)

   Bill says: Rearrange to $\left(\frac{a}{c}\right)^4 + \left(\frac{b}{c}\right)^4 = c$. Then put $a/c = 2$, $b/c = 3$ (or any other integers)

   Sue says: There is a whole class of solutions of the form $(m(m^4 + n^4))^4 + (n(m^4 + n^4))^4 = (m^4 + n^4)^5$ for any integers $m, n \neq 1, m \neq n$.

3. The quartic is $a^8 - 2200a^6 + 1630000a^4 - 454000000a^2 + 38500000000$.
   Solution is approx. 12.31 ft. "About a page of working using Taylor's Theorem", says Eddie Kent, who did it.

4. {1, 2, 6, 7, 9, 14, 15, 18, 20} Sue Davies

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PRODUCTION NOTES - Editor

This is the maximum number of sheets of (cheap) duplicator paper which can be posted for the minimum rate. In fact it is slightly overweight! I hope that some better (more expensive, heavier) paper will soon arrive, to eliminate the show-through - but then you get less sheets postable for 3p. This issue seems to mark the final defeat of the joint problems of diagram and untypeable symbols, though I'm still heavy-handed with the special tool. When sending diagrams, please keep them as small as feasible, and set your own standard of quality - I may TRACE them. If poss., use a coin for circles. Sheets of diagrams for major articles, e.g. Crystallography, must still be in black ink or black biro on plain white paper, for Photostat and then thermal stencil for the carbon deposit on the Photostat. The result is always rather fuzzy. Or you can provide your own electronic stencil, if available and you want better quality.

No. 15 will be the Summer Issue, (July, August) to allow for editorial and your holidays. In hand are John Peters on Some Problems in Euclidean Space, Leslie Naylor on Squares, Tom Venis on answers and solutions to problems in books, more replies to Sinbad, several items from Datta (!), a dozen or more problems and some oddments. Keep it coming. It is

good to have stocks of such varied and interesting contributions waiting for publication, and makes it much easier to spread the typing over a longer period. Many thanks to all who have helped to fill this issue, which seems to me to be the best ever, and apologies for errors - I'm only a two-finger typist, really!

NB: Elsewhere in this issue I said "Pref. not me", referring to telephone. I meant this strictly in the context of experimental ringing of complete strangers by shy ones. All calls to me are welcome, really.

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To fill up 10 lines, consider the imminent Summer Schools. If you show M500 around, and anyone shows interest, please stress that I MUST have a stamped addressed envelope PLUS a loose, unused 3p stamp for sample. Sesame readers frequently omitted the extra stamp. Also on subject of summer schools, I suggest MOUTHS members who want to be identified by other MOUTHS members should wear a plain white circular 'badge' (buy them in WH Smith or make your own.) It should be possible to spot this from a distance, and enable you to make useful personal contacts. If anyone wants to design a badge or symbol for MOUTHS, please do!

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