M500 NEWSLETTER No. 15 JULY/AUGUST 1974

Editor: (Mrs) Marion Stubbs, Southampton

SQUARES - Leslie Naylor

If ever you are stuck with a blank sheet of paper and nothing better to do, why not try constructing squares! Not the 1, 4, 9, ... variety, but Latin, Graeco–Latin or magic squares.

A Latin square is an $n \times n$ array of letters, in which each letter occurs once only in each row or column. n is called the order of the square. Here are squares of orders 1, 2 and 3:

$$A \qquad \begin{matrix} A & B \\ B & A \end{matrix} \qquad \begin{matrix} A & B & C \\ B & C & A \\ C & A & B \end{matrix} \qquad \begin{matrix} A & B & C \\ C & A & B \\ B & C & A \end{matrix}.$$

The easiest way to generate one is to write the letters in order in the first column, and then complete each line in cyclic order.

A Graeco–Latin square is a Latin square combined with a Greek square of the same order, such that no pair of letters is repeated. Using lower-case Latin letters instead of Greek, two possible Graeco–Latin Squares are:

$$\begin{matrix} Aa & Bb & Cc \\ Cb & Ac & Ba \\ Bc & Ca & Ab \end{matrix} \qquad \begin{matrix} Aa & Bd & Cb & Dc \\ Db & Cc & Ba & Ad \\ Bc & Ab & Dd & Ca \\ Cd & Da & Ac & Bb \end{matrix}.$$

In the square of order 4, replace A, B, C, D with Jack, Queen, King, Ace and a, b, c, d with Clubs, Diamonds, Hearts and Spades and you have a solution to the problem of putting the sixteen picture cards in an array so that no picture or suite is repeated in any row or column.

Alternatively, replace A, B, C, D with 0, 4, 8, 12 and a, b, c, d with 0, 1, 2, 3 and add the pair in each position to give the magic square:

$$\begin{matrix} 0 & 7 & 9 & 14 \\ 13 & 10 & 4 & 3 \\ 6 & 1 & 15 & 8 \\ 11 & 12 & 2 & 5 \end{matrix}.$$

All rows and columns add up to 30, so does any 2 × 2 subsequence or diagonal, including 'split' diagonals such as 9 + 3 + 6 + 12 = 30, and 'split' subsquares such as 0 + 13 + 14 + 3 = 30. For a different total, add a constant to each number in the square. The substitutions can be any numbers, but the number of ways of obtaining the fixed total may be reduced.

You can make squares by several methods, depending on how many Credits you have, e.g.

(a) Trial, error and more trial (M100 students)
(b) Modular arithmetic (M100 unit on remainder classes)
(c) Solving simultaneous equations (M201)
(d) Permutations of a group (M202)
(e) By computer (M251)
(f) By looking in a book on randomised experiments for statistical analysis (MDT241)

Hence much, if not all Open University Maths is based on, connected with, or relevant to squares.

There has even been a design for a patchwork quilt based on a 10 × 10 Graeco–Latin square in glorious hand-stitched colour.

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<u>ANSWERS AND SOLUTIONS</u> - Tom Venis

How often have you desired an answer to a book problem, or even better, a partial/hint/complete solution after batting out one's brains?

There are available from <u>some</u> publishers Answer Books and sometimes Solutions Manuals for some textbooks that line the bookshelves. (I'm invariably found in Foyle's, Dillon's etc having been book-hooked many years ago.) Generally, these manuals are available only to those teaching, and I have acquired a few, but would like to know about others that exist. Would any lecturer/tutor/A.N. Other pass on titles that they may have obtained?

Those I have are listed below and I am willing to pass on answers/solutions over the telephone, through this newsletter or post (send s.a.e. please.) No guarantee of author's correctness and the Copyright Law prevents Xerox or other copying!

Bell and others. Modern university calculus. Holden-Day. (A.H.S.)
Allendorfer and Oakley. Principles of mathematics. McGraw-Hill. (A)
Brainerd and others. Topics in mathematics. 4 vols
  Functions
  Differential calculus
  Combinatorial maths
  Vectors and matrices. Frederick Warne. (A)
Fraleigh. Calculus: a linear approach, vol. 1. Addison-Wesley. (A.S)
Durst. The grammar of mathematics. Addison-Wesley. (A)
Marsden. Basic complex analysis. Freeman. (S)
Meriam. Statics and dynamics. 1st and 2nd edns. Wiley. (S)
Shames. Engineering mechanics, vols. 1 & 2. Prentice-Hall. (S)
Huang. ʺ ʺ Addison-Wesley. (S)
Halliday & Resnick. Physics. Wiley. (A)
ʺ Fundamentals of physics. Wiley. (A)
  Key: A = Answers H = Hints S = Solutions

If I am not at home, quote as above: author, book, page number and exercise/problem number. Note: not all problems are necessarily answered in these manuals.

I wonder if students who do not have any easy access to bookshops or specialist bookshops would like details of 'Worked Examples' books that are generally available. There are plenty of good ones, e.g. the Schaum (McGraw-Hill) series.

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MATH-QUOTE - Ron Davidson

The idea that = is a very simple relation is probably due to the fact that the discovery of such a relation teaches us that instead of two objects we have only one, so that simplifies our conception of the universe.

Charles Sanders Pierce

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Some Problems in Euclidean Space

It is often thought that the only new problems in mathematics are in areas which require years of undergraduate and postgraduate training. I should like to describe some problems in ordinary Euclidean space which are easy to understand, yet which are unfamiliar to all my mathematical friends, even if they are not entirely new. I am most grateful to Dr R. M. W. Wood, of Manchester University, for communicating these problems to me and allowing me to give an account of them here.

Suppose we are given two points A and B in the plane, which we shall suppose has fixed origin O. Then if angle AOB is acute, we can obviously choose rectangular axes, with the same origin O, so that A and B are in the positive quadrant. This means that A and B will both have positive coordinates with respect to these axes. We can generalise this a bit: if we have any number n of points $A_1, ..., A_n$ in the plane, and if all the angles A_iOA_j are acute, then we can still choose axes so that all the points A_i are in the positive quadrant. Problem 1 is to prove this, and it's pretty easy.

Now let's generalise to higher dimensions. Suppose we have three points A, B, C in three dimensional space and the three angles AOB, BOC, COA are all acute, where O is again a (fixed) origin. Can we find rectangular axes, with origin O of course, so that A, B, and C all lie in the positive "quadrant" (octant?), that is, so that all the coordinates of these points are positive? Problem 2 is to prove that this is so, and is not quite as easy as problem 1.

You might ask whether this can be generalised to more than three points in space. This is not the case. You can find 4 points A_1, A_2, A_3, A_4, so that all six angles A_iOA_j $(i \neq j)$ are acute, but, no matter how you choose coordinate axes through O, one of the points will always be outside the positive quadrant (equivalently, at least one of the coordinates of one of the points has to be negative). Problem 3 is to find four such points.

So for further generalisations we must turn to higher dimensions still. Suppose we have n points $A_1, ..., A_n$ in n-dimensional space such that the angles A_iOA_j are all acute. Can we find axes so that the A_i are in the positive quadrant? (You may prefer to state this in vector space language: given n vectors a_i, ..., a_n in Euclidean n-space $\mathbb{R}^n$ with the property that the inner products $a_i \cdot a_j$ are all positive, is there an orthogonal basis of $\mathbb{R}^n$ so that all the a_i's have all their coordinates positive? You will probably have to think in these terms to prove results in dimensions greater than three, and probably in the three dimensional case as well.)

The hard problem that I should like to suggest is: prove the result in four dimensions. Dr Wood has proved this and also that the analogous result is false on $\mathbb{R}^n$ for $n \geq 5$. There are many related questions that can be asked but they must wait for a later article, when I hope to include a roundup of responses to this article and some proofs.

JOHN PETERS
Staff Tutor in Maths, Open University, Yorkshire Region.

A PERSONAL VIEW ON M500 - Ian Ketley (Course Tutor M231 and OU Counsellor)

I feel that M500 is getting completely out of hand. The initial aim of being a course newsletter (M202) was well fulfilled - circulated thus to a limited number the format, style, grumbles and personal opinions were the order of the day. Most of the readers were aware of the points and of a similar mathematical standing, and hence could understand and follow the points raised. The subsequent expansion to a 'mass' readership has led to a loss of relevance for many and to the inclusion of, to put it mildly, dubious material. Carried on in this manner it is likely to damage the credibility of the OU mathematics student in the eyes of the mathematical community at large

I find it difficult to give a constructive criticism in a few words, but to try to convey my feelings I will refer to issue No. 14. The points I will raise are by no means exhaustive but simply symptomatic.

Firstly, the OU student as portrayed in M500 is too introspective. He looks into the OU system to find the answers and to develop his thoughts. Surely he should be aware of what is going on outside and read non-OU mathematical publications. I am sure the writer of the article on Crystallography would not claim all the credit for the theory given therein -

so why not give some references and suggest further reading rather than promise us a sequel. If ‘N-ary’ feels persecuted and stifled by OU notations he needs counselling or tutorial guidance, for his kind of mathematics is unsound (—and shouldn’t get past the editor (!!)) The impression is even given that books written or translated by the OU staff are the final word!

Secondly, it is time that many contributors realised that simply being on an OU mathematics course does not qualify them as being expert mathematicians who can see what courses are needed and how mathematics develops. Of course we all have our own ideas on what we’d like to see on the basis of what we’ve been exposed to - but mathematically speaking the OU hasn’t exposed very much. In truth, the exposure is even more limited than one realises - the text of a unit is often regurgitated in TMAs, CMAs and examinations (ensuring ultimate ‘success’) but with little appreciation or lasting effect to the extent that knowledge which should have been acquired from past courses is ‘unknown’ when it has to be used for a current course. It is perhaps unfortunate that the credit exemption system has led to students having inadequate springboards for further broad-based studies and to repeated requests for ‘higher level mathematics specialities’, whereas respectable courses (from an outsider’s point of view) meet with severe criticism from enlightened OU students.

I am in complete agreement (surprisingly!) with the Maths Faculty policy of providing glimpses of the basic fundamental areas of mathematics - the detail and order of some courses is unfortunate in some respects. It is perhaps the OU Honours structure that is not correct for mathematics - in fact I do not believe a common OU policy for all Faculties works to the advantage of mathematics students. Unfortunately, specialisation has become the accepted structure rather than the exception.

I think it is time to review the aim of M500 and the type of article to be published. This will, hopefully, result in a shorter publication of value to more people. Where pieces of mathematics are ‘developed’ or commented upon it seems desirable to print correct and accurate versions - should we expect the editor to do the vetting and/or correcting?

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(Ian is a mathematics lecturer at Southampton University, and has been involved with the OU since the early days, in various capacities.)

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Ed: Dr. Ketley is my Course Tutor, but is not a subscriber to M500.

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ON ‘SINBAD’ - Beth Taylor (A202 and Organiser of CHANNELS)

I’m fascinated by the thing about mathematical stereotypes, as it came the day after I’d been to a self-help group and been mowed down by other students for holding opinions they felt were at variance with the spirit of the Arts generally. I was so fed up. Sometimes I think I don’t like other Arts students as they wamble. Particularly the literary ones. Historians and philosophers are alright. I thought I fitted the suggested mathematical type more than the Arts type except that I have unquestionably a sense of humour, and in fact tend to hilarity very easily. However I’m not numerate. I got maths to O level but I only understood Euclidean geometry really. I cannot count. I don’t wish to do maths, but I love reading M500; it appeals to me more than the facetious unacademic *Sesame*. I may not understand what it says, but I get the feeling that I understand what it <u>means</u>, if that is acceptable to a mathematician.

I am really interested in philosophy, actually, and had a lovely time doing the logic units of A100. I only mention these things because I am the only all-Arts student who subscribes to M500, and I think the search for personality traits of all faculty students is interesting and, as Sinbad says, not frivolous. It seems to me a very valid system to work backwards from what people have chosen to do to find out who else might choose to do this. It’s a sort of Bloch historical regressive method.

I will look forward to the next M500. Perhaps hundreds of M500 mathematicians will write and say they are witty, tall, feminine, blonde lateral thinkers. Have you raised the question of music? It might be useful to discover if they are musical, on the well-known association of music with mathematics, and the apparently established fact that child-prodigies only appear in music, mathematics and chess.

We are all musical here, and my husband is an excellent mathematician. My young son shows signs of being very competent but quite indifferent about it.

Incidentally, if M500 is anything to go by, mathematicians express themselves very stylishly and economically. More than you can say for me, alas.

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Ed: Beth's CHANNELS was reported alongside MOUTHS in March *Sesame*. For details, send her a stamped envelope.

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Yvonne Kedge

I rise Sinbad. Not to the bait of the correlation between unfemininity and mathematical ability but to any aspersion cast against the wit and humour of mathematicians which has been the sustaining delight in my excursions with the OU and not least M500.

The Mathematics Faculty itself admittedly failed to sustain such dizzy virtues upon certain auspicious occasions – Minsky! – but I happily need not dwell on the past. How sad if the Mathematicians' flights of fancy were not to be tempered with humour.

(At what stage does one acquire that revered and coveted title?)

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Hugh Tassell

I was fascinated by Sinbad's article on mathematical stereotypes, even though it was a mixture of horse-sense and humbug – or am I one of the mathematicians without humour?

As for point 1, I do no more than quote *Mathematical Spectrum*, vol. 2 no. 1, in an article on "Mathematical Competitions" by Margaret Hayman: 'Even in countries where schools are completely co-educational and where there is no social prejudice against intelligent or scientific women, girls do markedly worse in mathematics after the age of 14. In the four years of the British Olympiad there have never been more than 3 girls out of 60 competitors and only once has one done reasonably well. In the international competition in

Yugoslavia only one of the 108 competitors was a girl (from Bulgaria)!

Prospective entrants for second-level courses please note!

The second point. Surely Sinbad ought to know by now not to judge people by appearances? I doubt if Sinbad could sort out the mathematicians from the artists at a conventional university. Possibly he was thinking of his own circle of acquaintances at the study centre, who, like mine, are mostly quiet, inconspicuous people, irrespective of their subject.

The third point is patently absurd. At Summer School last year Graham Read and a visiting professor of mathematics from the USA besides a number of tutors all enlivened an enjoyable stay with their light-heartedness and humour. I find this typical of most mathematical gatherings I have attended, including tutorials, study groups, etc. and I doubt indeed if they are any different from Arts gatherings. I sympathise with Sinbad if he continuously finds such poor company – I suggest he takes a course in another faculty next year.

I am inclined to disagree with points 4a and 4b. Being the OU Faculty with the least number of students, it seems natural that we would be the ‘least’ represented. The rapid growth of M500 and MOUTHS, with more members joining every issue, is another factor against this contention. I am not sure what Sinbad is trying to say in 4b anyway – a communications problem, perhaps.

For what it may be worth, I give my own idea of a mathematical stereotype. Besides being male

(1) He must be a problem solver. This means he must possess the qualities of persistence, concentration an open mind, creativity and above all, a dogged determination never to give up.

(2) He must be versatile. Geniuses like Newton, Leibniz, Gauss and Einstein did all sorts of things besides mathematics. Similarly, the modern mathematician must also be a combination of a philosopher and logician and have a smattering of the natural sciences, particularly physics, and the Greek, Hebrew and Russian (and a few more) alphabets.

(3) Good mathematicians tend to be good chess players - those who have

bothered to learn. In the Open University I have met an ex-Scottish Junior champion and an ex-London Schools champion, plus a lecturer who regularly writes to chess journals. There is, incidentally, a great deal of research going into the problem of programming a computer to play chess, and I would be interested to hear from anyone with ideas on the subject, or any other matter concerning chess.

(4) A tutor has pointed out that the most creative and productive mathematicians are usually very young. However, I would naturally not apply this criterion to applicants for mathematics in the Open University, which, after all, is what this article is all about.

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On THE PETITION - Bill Shannon

While I sympathise with the writers in their pleas for more 3rd level courses, I for one cannot support them.

Presumably course-team resources are limited and any increase in 3rd level courses could only be at the expense of 2nd level courses. It seems reasonable to suppose that there will be more ordinary degree students than Honours (although I hope to be among the latter) and that therefore proportionally more resources should be devoted to the 2nd level courses in order to give an adequate choice to ordinary degree students, who of course have to gain 4 credits at 2nd level as against the 2 credits at 3rd level for Honours students.

Arts students might well complain about their limited choice of 2nd level courses - only 4 credits by 1976. I think what is being overlooked also is the academic content of 2nd and 3rd level courses. Apart from the difference in standard expected, the Course Handbook makes it clear that 2nd level courses are designed for students in search of breadth, whereas 3rd level courses tend to be more narrowly specialised. In my view, it is the Arts Faculty who have got their balance wrong; either that or possibly there is little difference between their 2nd and 3rd levels except in the standard expected.

It is hard luck on those who do not like the limited choice of 3rd-level Maths courses, but I cannot see that an Honours degree, mostly in maths, would be appreciably degraded by the inclusion of one 3rd level credit in another faculty's courses. There may well be eccentric mathematicians who actually enjoy partial differential equations!

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Ed: Bill seems not to have read the front page on the Kettle Report in June *Sesame*. For those who have not received it, the official allocation of resources will be divided as follows, producing a total of 84 credits in the next 10 years:

Arts 16, Social Sciences 16, Science 15, Technology 15, Education 8, Mathematics 8, and ‘U’ 6.

If we of M500, with at least some sort of Faculty ‘identity’, do not protest, we are doing future OU mathematicians a disservice.

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<u>Tony Brooks</u>

I read with interest your note in M500/14 about doing 3rd level Arts courses. Since I did precisely what you have been advised not to do, namely to do a 3rd level Arts course without any previous experience in essay writing, my experience to date may be of interest.

It is difficult to find a way of directly comparing Arts and Mathematics courses. The only useful measure that occurs to me is the degree of sophistication of thought required to do a TMA. I think that anyone who has passed M202 is more than capable of dealing with the requirements of any 3rd level Arts course. I am finding A303 easier than M202 despite the fact that I have not written essays before. Of the OU courses I have done or am doing, my personal grading in terms of ‘sophistication of thought’ index from hardest to easiest would be:

(1) M202
(2) A303
(3) M231
(4) AMST283
(5) M100

I am also finding that in terms of time spent on each course I am doing this year, A303, M231 and AMST283 are taking roughly equal amounts of time.

I realise that 4 months on one 3rd level Arts course may not be the best basis on which to judge the rest of 3rd level Arts courses, however, my advice would be to go ahead and do A305.

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Ed: Thanks for this very useful, factual information. But I don't WANT to do Arts – I'm only choosing the one which looks as though it could be reasonably digestible. After all, there is some mathematical beauty in Architecture. I cannot see anything else in its favour, unless someone wants to spend public money on making me 'cultured'.

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Yvonne Keates - M100 Leeds

There only appear to be about 5 MOUTHS members within 100 miles of here! My previous course was just A100, because exactly half-way through D100 I withdrew because I didn't find it worthwhile – full of sociological banalities. M100 is indubitably worthwhile but incomprehensible, to one who last did very elementary maths for School Certificate 38 years ago! I fully expect to drop out of M100 very shortly, but if so, I'll be repeating it next year. I really belong to the M50 subset of M500 mentioned on M500/14.

Having previously met Arts and Social Science students (and being by background one of that group) I'm in a very good position to assess the peculiarities of Maths students and the comparative standards of the courses. The difficulty of M100 for an 'open' student compared with A100 or D100 is quite amazing. An utterly uneducated person possessing grit and determination (I've known one such) can pass D100; it seems to me M100 is only meant for the 'A' level types.

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Ed: M100 students are strongly advised not to drop out at this stage. All A, B and C year students of M100 have felt precisely the same. Summer School tidies things up.

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<u>MORPHISIA - a mathematician's disease</u> - Datta Gumaste

This is a disease exclusively found in mathematicians. Those who enjoy playing the game of mathematics – as opposed to those who find joy in using it – are more likely to catch this disease.

Morphisia was first introduced openly in the U.K. by the OU, and those unfortunate ones who decided to study M100 were the first victims.

Unit 3 of M100 is responsible for inflicting it, and M202 for its continuous dangerous spread. It is becoming clear that no previous history of mathematical sophistication is necessary for a person to get afflicted by this mental disorder (although there is no sufficient evidence to support this statement).

Some of the observable symptoms are:

(a) Blindness – i.e. inability to see things as they are.
(b) Sleepless nights and restless days.
(c) Tendency to use poetry as a normal medium of communication.
(d) Obsession to draw triangles and rectangles most of the time.

Some experts feel that it is difficult to distinguish between morphisians and algebra cists.

Morphisians see morphisms where there are none. They waste their valuable time in interpreting perfectly normal situations in terms of morphisms. For example, a morphisian is not satisfied with usual definition of continuity of functions in terms of epsilons and deltas; he defines continuity in terms of morphisms. Consider the following commutative diagram:

$$\begin{tikzcd}
a_n \arrow[r] \arrow[d, "f"'] & \mathbf{Lim}(a_n) \arrow[d, "f"] \\
f(a_n) \arrow[r] & \mathbf{lim}(f(a_n) = f(\mathbf{lim}(a_n))
\end{tikzcd}$$

In the diagram, $a \in A$ where A is the set of convergent sequences, 'lim' is a unary operation on A which sends every $a_n \in A$ to its limit, $\lim(a_n)$ which belongs to $\mathbb{R}$, the set of all reals. f is a function on A which maps (a_n) to $f(a_n)$.

$$\text{Thus we have } \lim(f(a_n)) = f(\lim(a_n)). \qquad (1)$$

The morphism says that f is continuous if and only if f is a morphism on $(A, \lim)$ to $(f(A), \lim)$, which satisfies the equation (1).

Some very distinguished writers of textbooks are chronic morphisians. See, for example, *Algebra* by Birkhoff and Maclane. This delightful book is full of morphisms and commutative diagrams. The writers even define subgroups in terms of morphisms. This book is a real treat for a morphisian and compulsory reading for a non-morphisian(!)

Once afflicted a patient is likely to be a long life sufferer. Surprisingly, he does not want to get rid of this malady.

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A morphisian is reported to have claimed the idea of a morphism together with that of a set is sufficient to construct most of mathematics. This is how the argument goes:

The axiomatic set theory of M202 has two primitive notions – ‘set’ and the relation of ‘belonging’. These notions and a few axioms are sufficient to build the set theory, and the rest of mathematics follows.

Morphisian claims that $\in$ (i.e. ‘belonging to’ or ‘membership of’) is a morphism. That $\in$ is a function is easy to show. Let A be some non-empty set. Then $\in$ is defined on A to $\{A\}$ as follows:

$$\in : x \in A, x \in A.$$

That is, $\in$ sends every x in A to A. $\in$ is an onto function, the image set being $\{A\}$.

Now let o be some closed binary operation on A. Then

$$x, y \in A \Rightarrow (x \text{ o } y) \in A.$$

To prove the claim that $\in$ is a morphism it is necessary to show that

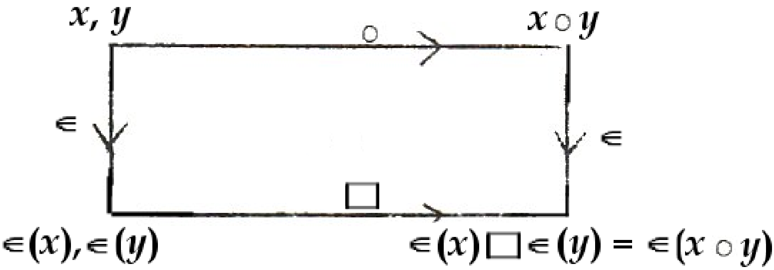
$\in(x \text{ o } y) = \in(x) \square \in(y)$, where $\square$ is some operation defined on $\{A\}$.

Let □ be defined on *A* as follows:

$A \,\square\, A = \{a: a = x \circ y,\ x, y \in A\}$.

Now watch: $\in(x \circ y) = \in(x) = \in(y) = A = A \,\square\, A = \in(x) \,\square\, \in(y)$.

We have seen that $\in$ is a morphism.



(note that 'o' is any operation and '□' is its image)

Here we have a result which says that morphisms are at the very root of mathematics, and yet they flower in every branch of mathematics.

In non-medical terms, morphisia means falling in love with with morphisms. It's a most lovely fall, worth having again and again.

LONG LIVE MORPHISMS!

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FOR SALE

Dr. R. C. Kumar (ex-tutor, M100)

I was, some time ago, a tutor with the OU and was sent all the mailings for the M100 course. I have, at the moment, a complete set of the entire material for the M100 course, i.e. all the books and associated units that are sent to a tutor. I would like to dispose of this entire material as soon as possible.

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<u>SOLUTIONS TO PROBLEM CORNER No. 14</u>

Only one, actually. Editorially, I would appreciate comments on the

Problems – too difficult, too easy, too many, too few, etc. I cannot control what problems readers submit, and the most successful do seem to have been the most difficult, which is odd, but at least, if I have any choice, it would be nice to know what sort you prefer. (OK – like me, you haven't time!) – Ed.

Collimations - Roger Claxton

Note: I have assumed '4-5-7' in the question should have read '4-5-6'. (Bob Margolis confirms this, and apologises! Ed.)

Collimations aside, the complete set of permutations on fig. 1 (equivalently on S_7) produces 5040 diagrams all equivalent to within isomorphism. As we require the collinear property to be preserved by the permutation, we must look for a subset of S_7. A preliminary investigation showed that if we consider the rotations and reflections of the triangle we get S_3 (to within isomorphism). Another source (a puzzle) suggested that the approach was as follows: the first number (say 1) can be permuted to any one of 7 positions. Once this is done, choose any of the triples radiating from 1 (say 1 6 2). 6 & 2 can be placed on any one of the three lines radiating from 1 and in either order – i.e. 1 6 2 and 1 2 6 are valid. Thus 6 & 2 can be placed in 6 ways. If we consider 7 & 4 next, again we can arrange these in either order on two radicals – i.e. there are 4 possibilities. So we are now up to $7 \times 6 \times 4 = 168$ possibilities. In fact the placings so far will fix the last two positions, as a quick check will show, and so we are left with a result of 168 collinear solutions. This implies 5040/168 = 30 'different', i.e. no collinear.

Now since we have a complete set of collimations, a composite operation can only make another collimation of the same type. If we note that the Identity is included within our 168 we discover a subgroup of S_7 of order 168.

Now $168 = 2^3 \times 7 \times 3$ ... which is not too helpful. And this is where I sign off – end of part I!

(Sorry about the poor ending!)

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PROBLEM CORNER No. 15

1. SUNSHINE – Richard Ahrens

“Every point on the earth’s surface is in sunshine for the same amount of time”.

Prove this statement under the following assumptions and approximations:

(1) The atmosphere including clouds does not exist.
(2) We are considering a sufficiently long time – say a year.
(3) The length of a day is small compared with the length of a year.
(4) The earth’s orbit is circular.
(5) The sun is sufficiently far away to allow us to regard sunlight as parallel.

In case you feel that you should know the angle between the earth’s axis and the plane of its orbit, it is 66½º, but it is not necessary to know this.

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2. TOILET ROLL - Bob Margolis

A lunatic student at Q.M.C. (London) once taped the end of a roll of toilet paper to the side of the tower block there. He then allowed the thing to unroll itself down the wall.

At what point is the total energy the same as if the whole roll had simply been dropped from the same starting point?

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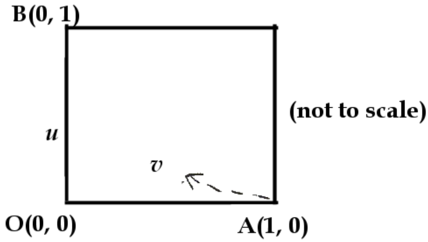
3. Datta Gumaste

Prove or disprove: for any positive integer k,

$k^n - (k - 1)$ is not divisible by any natural number $n > 1$.

4. Hoary and non-hoary RABBITS - Peter Hartley (M231 C-T)



A rabbit leaves O and travels along OB at constant speed u. A dog leaves A at constant speed v at the instant the rabbit leaves O and chases the rabbit. At what time and position does the dog catch the rabbit, or, failing that, how far is the dog from B when the rabbit reaches B

(a) when $v > u$ (fairly straightforward)

(b) when $v < u$ (somewhat harder, i.e. I'd like to know how to do it.)

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MATHEMATICAL PERIODICALS

This list was supplied by Tony Brooks from information given in the '*Bulletin'* of the Institute of Mathematics and its Applications, September 1973. Parker & Son Ltd. will handle subscriptions, and further enquiries should be sent to their periodicals department. Publishers generally accept subscriptions paid on a calendar year basis, but it seems to take several months to organise an American computer to put you on its mailing list, so worth starting now.

Abbreviations: AIA = And Its Applications   Int = International
Jnl = Journal Maths = Mathematics   Mathl = Mathematical

Acta Arithmetica (1936 -)
Algebra Universalis (1971 -)
Annals of Mathl Logic (1970 -)
Annals of Probability (1973 -)
Communications in Algebra (1973 -)
Discrete Mathematics (1971 -)
Fibonacci Quarterly (1963 -)
Functional Analysis AIA (1967 -)
trans. from Russian
General Topology AIA (1971 -)
Geometriae Dedicata (1972 -)
Int Jnl of Computer Maths (1964 -)
Int Jnl of Game Theory (1971 -)
Jnl of Algebra (1964 -)
Jnl of Approximation Theory (1968 -)
Jnl of Combinatorial Theory (1966 -)
Jnl of Differential Eqns. (1965 -)
Jnl of Differential Geometry (1967 -)
Jnl of Functional Analysis (1967 -)
Jnl of Mathl Analysis (1960 -)
Jnl of Number Theory (1969 -)
Jnl of Pure and Applied Algebra (1971 -)
Jnl of Symbolic Logic (1936 -)
Linear Algebra AIA (1968 -)
Maths of Computation (1960 -)
Semigroup Forum (1970 -)
Studia Logica (1953 -)
Tensor (1938 -)
Theory of Probability AIA (1956 -)
trans. from Russian
Topology (1962 -)

To these may be added:
American Mathematical Monthly - £7.60 p.a.
Jnl of Recreational Maths - £7.15 p.a.
Manifold (from Univ. of Warwick) - 80p p.a.
Mathematical Spectrum (for Sixth Forms and undergraduates) - 50p p.a.
from The Editor, Univ. of Sheffield

Further information welcomed, especially from anyone with access to a University Library.

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A GROUP FOR M202 TO PLAY WITH - contributed by Phil Goble, of Portsmouth College of Education Compiled by students of Portsmouth College of Education

(1st)

	I	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
I	I	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
A	A	B	C	I	W	X	U	V	E	F	G	D	P	M	N	O	K	L	H	J	T	Q	R	S
B	B	C	I	A	R	S	T	Q	X	U	V	W	O	P	M	N	G	D	E	F	J	K	L	H
C	C	I	A	B	L	H	K	J	S	T	Q	R	N	O	P	M	V	W	X	U	F	G	D	E
D	D	E	F	G	I	A	B	C	U	V	W	X	Q	R	S	T	M	N	O	P	H	J	K	L
E	E	F	G	D	K	L	H	J	A	B	C	I	T	Q	R	S	W	X	U	V	P	M	N	O
F	F	G	D	E	N	O	P	M	L	H	J	K	S	T	Q	R	C	I	A	B	V	W	X	U
G	G	D	E	F	X	U	V	W	O	P	M	N	R	S	T	Q	J	K	L	H	B	C	I	A
H	H	J	K	L	Q	R	S	T	I	A	B	C	U	V	W	X	D	E	F	G	M	N	O	P
J	J	K	L	H	O	P	M	N	R	S	T	Q	X	U	V	W	B	C	I	A	G	D	E	F
K	K	L	H	J	E	F	G	D	P	M	N	O	W	X	U	V	T	Q	R	S	A	B	C	I
L	L	H	J	K	C	I	A	B	F	G	D	E	V	W	X	U	N	O	P	M	S	T	Q	R
M	M	N	O	P	U	V	W	X	Q	R	S	T	I	A	B	C	H	J	K	L	D	E	F	G
N	N	O	P	M	F	G	D	E	V	W	X	U	C	I	A	B	S	T	Q	R	L	H	J	K
O	O	P	M	N	J	K	L	H	G	D	E	F	B	C	I	A	X	U	V	W	R	S	T	Q
P	P	M	N	O	T	Q	R	S	K	L	H	J	A	B	C	I	E	F	G	D	W	X	U	V
Q	Q	R	S	T	H	J	K	L	M	N	O	P	D	E	F	G	U	V	W	X	I	A	B	C
R	R	S	T	Q	B	C	I	A	J	K	L	H	G	D	E	F	O	P	M	N	X	U	V	W
S	S	T	Q	R	V	W	X	U	C	I	A	B	F	G	D	E	L	H	J	K	N	O	P	M
T	T	Q	R	S	P	M	N	O	W	X	U	V	E	F	G	D	A	B	C	I	K	L	H	J
U	U	V	W	X	M	N	O	P	D	E	F	G	H	J	K	L	I	A	B	C	Q	R	S	T
V	V	W	X	U	S	T	Q	R	N	O	P	M	L	H	J	K	F	G	D	E	C	I	A	B
W	W	X	U	V	A	B	C	I	T	Q	R	S	K	L	H	J	P	M	N	O	E	F	G	D
X	X	U	V	W	G	D	E	F	B	C	I	A	J	K	L	H	R	S	T	Q	O	P	M	N

Key: the permutations are

I = $\begin{matrix}1234\\1234\end{matrix}$ D = $\begin{matrix}1234\\1243\end{matrix}$ H = $\begin{matrix}1234\\1324\end{matrix}$ M = $\begin{matrix}1234\\1432\end{matrix}$ Q = $\begin{matrix}1234\\1342\end{matrix}$ U = $\begin{matrix}1234\\1423\end{matrix}$

A = $\begin{matrix}1234\\2341\end{matrix}$ E = $\begin{matrix}1234\\2431\end{matrix}$ J = $\begin{matrix}1234\\3241\end{matrix}$ N = $\begin{matrix}1234\\4321\end{matrix}$ R = $\begin{matrix}1234\\3421\end{matrix}$ V = $\begin{matrix}1234\\4231\end{matrix}$

$B = \begin{matrix}1234\\3412\end{matrix}$ $F = \begin{matrix}1234\\4312\end{matrix}$ $K = \begin{matrix}1234\\2413\end{matrix}$ $O = \begin{matrix}1234\\3214\end{matrix}$ $S = \begin{matrix}1234\\4213\end{matrix}$ $W = \begin{matrix}1234\\2314\end{matrix}$

$C = \begin{matrix}1234\\4123\end{matrix}$ $G = \begin{matrix}1234\\3124\end{matrix}$ $L = \begin{matrix}1234\\4132\end{matrix}$ $P = \begin{matrix}1234\\2143\end{matrix}$ $T = \begin{matrix}1234\\2134\end{matrix}$ $X = \begin{matrix}1234\\3142\end{matrix}$

(e. & o.e.)

EDITORIAL

One line left to say very little is now on hand for No.16 (September). Richard Ahrens with More Hoops, and a Crossword from Michael Gregory and a few problems, info from Tony Brooks on the I.M.A., and still several pieces from Datta. Letters, maths, comments, crits wanted.