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THE RULES OF THE GAME - Richard Ahrens, "Maintenance Man, M202"

If ever I found myself obliged (presumably at the point of a gun) to say what it is that distinguishes mathematics from other forms of intellectual activity I would probably begin by saying something like:- Mathematics is characterised by the fact that its practitioners operate in a state of perfect knowledge of the assumptions they are making (axioms), the rules of inference they allow themselves (logic), and all relevant facts (theorems). Now while the sheer arrogance and pomposity of such a statement is probably sufficient grounds for dismissing it, a more serious charge is that it simply isn't true of the activity that the O.U. Maths Faculty and its students are engaged in. We are all concerned with the communication of mathematics, and here the rules of the game are far from precise.

It is tempting to argue that mathematics presents the opportunity of saying exactly what you mean without any ambiguity, but a serious attempt in this direction is doomed to failure. The point you are trying to get across becomes buried in a mass of notational detail. (Try reading a page or two of Russell and Whitehead's 'Principia Mathematica'). In an attempt to overcome this problem, mathematicians resort to the following device. They write in a slightly modified form of English and adopt a number of conventions about the use of this language that they assume are well understood by the reader. These conventions are rarely written down explicitly - everyone learns to use them by some trial and error process. It is very much like teaching people to play football by simply putting them on a football field with an umpire and telling them that the object of the game is to get the ball in the goal. It is likely that after a large number of penalties some of the participants will learn to abide by the rules of the game even though they might remain unable to state these rules precisely.

It is high time I provided a mathematical illustration. When I was learning algebra at school I met sentences like:

" $x^2 + 3x + 2 = 0$" which we called equations,

and sentences like: "$(x + 1)^2 = x^2 + 2x + 1$" that looked like equations but were called identities. Sometimes we wrote $\equiv$ instead of $=$ in the latter case. I learned

to live with these expressions but never quite understood what was going on, until some time later I learned about quantifiers. It was just as if the umpire gave in for a moment and explained the offside rule. "For all x" and "There exists an x" are important phrases frequently used by mathematicians, but they don't always bother to write them down. ("For all x" is omitted much more frequently than "There exists an x"). " $x^2 + 3x + 2 = 0$" and "$(x + 1)^2 = x^2 + 2x + 1$" are both equations, probably satisfied by some values of x and not satisfied by other values. It just happens that every x satisfies the second equation. "For all x, $(x + 1)^2 = x^2 + 2x + 1$" is an entirely different sort of statement. The fact that this is

true tells us nothing about x but does say something about polynomial functions. It now seems that neither I nor my schoolteacher understood a convention that expects you to write in a quantifier if the statement needs one.

Exactly the same problem has arisen in connection with OU CMA questions: Each OU maths course has felt obliged to issue a Stop Press with a note along the following lines: "When answering TRUE/FALSE questions, only answer TRUE if the statement is always true. If the statement is sometimes true and sometimes false, answer FALSE". When the OU asks: "Is the following statement true

If $x < y$ then $\dfrac{1}{x} > \dfrac{1}{y}$ $(x, y \in \mathbb{R}, x, y \neq 0)$",

students protest that the question is meaningless. Some choices of x and y give a true result - others do not. They are more likely to protest if they have read Unit 17 of M100) and now recognise this as an open sentence and have been told that questions of truth are meaningless when applied to such a sentence. An alternative to (I think an improvement on) the Stop Press message might be: "If you recognise a question as nonsense it is probably because someone has left a quantifier out in the hope that you would insert it for yourself". Try put ting "For all $x, y \in \mathbb{R}$" in front of the example above. I haven't been able to state exactly the rule that operates here, and perhaps some reader can clarify things further. Could M500 begin a collection of rules for this game? Some suggestions: Can anyone provide a list of possible uses of the word 'define'? Is there a general rule that allows us to write "$2 < x < 6$" but forbids "$6 < x < 2$" when we mean "$x > 6$ or $x < 2$"? When a paragraph begins "Let V be a vector space" does this always mean "We are now beginning a discussion that applies to all vector spaces"? (it does *not* usually mean "Take a particular vector space of your choice"!)

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PROBLEM CORNER No.9

<u>1 . - from R. Seton-Browne, Tatsfield, Kent</u>

Determine a series of consecutive integers which does not contain a single prime.

<u>2 . - from John Bennett, Romsey, Hampshire</u>

If a hoop is a set with a binary operation (H, o) such that for all $x, y, z \in H$

$x \mathrm{\,o\,} x = x$
$x \mathrm{\,o\,} (y \mathrm{\,o\,} z) = (x \mathrm{\,o\,} y) \mathrm{\,o\,} (x \mathrm{\,o\,} z)$
$(x \mathrm{\,o\,} y) \mathrm{\,o\,} z = (z \mathrm{\,o\,} z) \mathrm{\,o\,} (y \mathrm{\,o\,} z)$
$x \mathrm{\,o\,} y = x \mathrm{\,o\,} z$ implies $y = z$
$x \mathrm{\,o\,} y = z \mathrm{\,o\,} y$ implies $x = z$

Does there exist a finite hoop of order 10? (Answer unknown).

<u>SOLUTIONS TO No. 8</u>

Car-washing: $58/78 = \frac{1}{2} + (\frac{1}{2} \times 19/39)$

Crossnumber (rows across): 65121 / 16807 / 70322 / 51255 / 74613

<center>9 page 3</center>

This issue of M500 is, of course, dominated by second-level students simply because no M100 students have written anything. Since M100 students may feel "left out", or "overpowered", this page reprints some of the M100 contributions from 1973 issues, and we will hope for some articles, views, letters or plain cries of 'HELP!' from M100/1974 as soon as possible.

<u>Diane Miles</u> - Uttoxeter, Staffordshire (reprinted from No.8, Oct.73)

Thank you for your copy of M500. I wish I had known about you earlier. I think MOUTHS and self-help is a wonderful idea. The main problem with being an isolated student is that time and effort are used most uneconomically. When I come to a 'stone-wall', as I frequently do, the two courses open to me are:

1. spend hours bashing away at it, reading and re-reading, flipping between the glossary and the text, trying to extract meaning from the definitions, searching for a line read too hastily which will prove the key to the mystery.

2. To give up until I can contact and get a response from a staff tutor or

Bletchley. This may take hours or days. Meanwhile, progress grinds to a halt and good study opportunities may be lost. (Good study opportunity: children in bed and mind still able to function!)

For me, the ideal solutions would be:

1. A beautifully clearly written text, with lots of exercises like a high school book. Then if the student could not deduce the meaning from the text, then he or she might be able to gather the meaning by induction (not mathematical induction!) from the questions and answers. In the M100 text I find I have to 'waste' a lot of exercises by using them as examples to try to understand what the question meant, and yet I would like to do a lot of exercises to try to etch the matter into my brain.

2. A telephone line with a mathematical mind at the end of it - run on the lines of a fire-station.

Perhaps MOUTHS provides the nearest practicable approach to (2). Does it? Do they (or we) phone each other up and say things like 'Do you understand about kernels?' or 'Can you translate the notation in book so-and-so, page so-and-so?' I think MOUTHS is a worth-while experiment, and I hope it succeeds.

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Ed: Yes, we do - telephone each other in the way described. We also argue about CMA questions, since they are short, interesting and relevant - one learns to argue to the death when one's CMA result depends on it, and this is mathematically beneficial. Sometimes we just want a fellow victim to cheer us up after a low grade, because even the most sympathetic family/friends do not talk our precise language, really.

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9 page 4

Ronald Davidson - Southampton (reprinted from No.8, Oct.73)

The Summer School was just fantastic. I'm sure you can imagine my reaction. One big surprise was meeting Geoff Yates at the top of a 130 ft. hill behind Stirling's campus one afternoon. One of the tutors decided that an afternoon's relaxation would do us all a power of good, and it seems that someone in Geoff's course had the same idea. Peter Weir was also at Stirling the week I was there. You get a great feeling out of this! My studies got seriously out of line over the past month, what with work, moving (which was rather protracted for various

reasons) and a holiday. There were times in the wee small hours when the candle was getting low and my spirits were following it as I wrestled with a TMA. I thought "I must be mad to go on like this!" - however, I've persevered (and got 9.67 for the TMA!)

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After BA, a new job? - M100 student, Poole, Dorset
(reprinted from No.7 Sept. 73)

I am a professional applied mathematician employed in industry, taking M100 and M251 this year as an updating exercise in 'modern' maths. Until recently I used to be one of Harry Hitch's problem solvers at the British Aircraft Corporation (ref. M100 TV programme). But last year I switched to the Systems Development Group of the same company at Hurn Airport, where I administer the section of analyst/programmers.

This company is one of the few places outside the academic world where mathematics graduates can obtain relevant employment. Furthermore, this employment is of a visibly productive nature where the end result flaps its fin and takes off for Majorca or wherever.

Consequently, I would recommend, expect or hope that OU graduates will be taking their places among the newcomers to this group in the due course of time. Also, I hope they will find some comfort in the thought that at least someone here knows something of the self-inflicted torture required to achieve the illusive credits of the B.A. (OU).

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And finally, a letter from a Counsellor, which may encourage M100 students who are not very sure what counselling is all about:

Charles Green - M100 Counsellor, Jersey

We are very isolated here - only the M100 students who are doing well contact me from Guernsey - I then find that the others have withdrawn. The Open University are now realising that Jersey and Guernsey are separate islands - a fact which incidentally has only just penetrated to the Kennel Club.

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9 page 5

UNPROVABLE FACTS - Peter Weir, Coventry

I have an arrogant approach to unprovable facts in mathematics: there should not be any!

If we set up a mathematical system by strict axioms, then we should be able to prove any fact that arises as a result of the axioms. Boolean algebra (M100) is a simple example of such a system, and as far as I know there are no unprovable facts in it.

But when it comes to a system involving the natural numbers (1, 2, 3, ..., etc.) and a few elementary operations (+, −, etc.) then it is an entirely different question. The system can be set up axiomatically in more than one way - Peano's axioms (M202) are usually involved. Yet there are unprovable facts!

One example will suffice to make a lot of people waste a lot of time and get nowhere. Take any positive integer N ($\neq 0$). To get a new, improved N, take $N/2$ if N even, or $(3N + 1)/2$ if N odd.

Repeat ad nauseum. The result is always 1, sooner or later, 'always' being intuitive, as someone has yet to prove it. But someone has yet to disprove it - you try any number and see.

It looks an easy problem to solve, but you try it. I wasted hours on it at Summer School (where I purloined it), and got nowhere. You can pick up a few results, try binary trees, etc.

Here I must start to wax philosophical. I must take mathematics back to the time of Bishop Berkeley (1685-1753), Bishop Cloyne. Those of us old enough to remember will recall that Bishop Berkeley was unhappy about mathematicians letting dx tend to zero and dividing by it, yet getting the right results. It was not until the time of Weierstrass in the second half of the 19th century that mathematicians stopped building castles on marshy ground, and derived a rigorous definition of a limit (M100).

Although my analogy with the good Bishop will be incomprehensible to all but myself, I think I am roughly saying what he said. We do not know our own backyard (the natural numbers) well enough yet. Should we venture further, and if so are we on safe ground? (The last two sentences were somewhat overacted, but you get the idea.)

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A SELECTION OF MATHEMATICS BOOKS - Michael J. Gregory

Here are some mathematics books which I have found interesting, and in some

cases useful in studying Open University courses. The prices are only approximate.

'The fascination of groups' (F.J. Budden, C.U.P., 1971. 596 pp.$18.50) contains numerous examples with good illustrations of group manipulations. It is complementary to the groups units of M202, covering similar ground to the course without stressing the course. There are chapters on groups in geometry, music and campanology, and an interesting and illuminating book is obtained.

The special feature of 'An elementary treatise on curve tracing' (P. Frost, Chelsea Pub. Co., 1960. 210pp. $5.50) is the comprehensive collection of 270 curves up to degree 8, with their equations. Calculus is not used, and some of the techniques applied in the detailed analyses of the curves seem rather eccentric; the book was first published in 1872.

For anyone who enjoys mathematical puzzles, 'Barlow's tables of squares, cubes, square roots, cube roots and reciprocals of all integers up to 12,500' (ed. L. J. Comrie, Spon, 1968. 258pp. £2) will prove a great time-saver, despite the title. It also has higher powers of the lower integers, for instance 99^{10} and 9^{20}. Comrie's remarkable record of publishing almost error-free tables was discussed in M100 S4 RC.

I found 'Introduction to difference equations' (S. Goldberg, WileyScience Eds., 1958. 260pp. £1.75) interesting and helpful for both M100 and M201. It contains examples mainly from the social sciences, dealing very clearly with a wide range of recurrence formulae.

As a link between the Arts and Mathematics, you will probably enjoy 'Symmetry' H. Weyl, Princeton U.P., 1952. 168pp. $5) Illustrations of symmetry are taken from painting, sculpture, architecture and natural objects as the iris, sea shells and crystals; these are explained in terms of the group theory of M100.

'Mathematical models' (H. M. Cundy and A. P. Rollett, OUP, 1961. £1.50) appeals to me because it covers a wide range of models. It includes chapters on plane geometrical models, polyhedra (with nets for cutting out on card), ruled surfaces, Möbius strips, as well as mechanical and logical models. It is a practical book with some theoretical background, and has something for everyone to make.

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(no one ever seems to write anything controversial which should please Walton Hall! Still, it's nice to know who our new members of MOUTHS are, and what makes them tick.)

Susan Major M202, Coventry

I was introduced to M500 through the good works of Peter Weir at the Coventry Study Centre. I have just finished M100, and have applied to study M202 in 1974, but everyone seems to be doing their best to put me off, so I shall probably need some sympathy. I happened to notice in an old copy of Sesame (Dec.72 - Ed.) that you are a librarian - this cheered me up as I am too, and mathematical librarians seem few and far between. I am a librarian at the Warwickshire Postgraduate Medical Centre here in Coventry, but have worked in all departments of public libraries, and in a Government Research library for about 8 years in all. Maths has always been interesting to me (I'm hopeless at writing essays) so find this course quite a challenge. I have filled in your questionnaire as I would like to receive any future copies of your newsletter to keep in touch.

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 Ed: Someone has said that wherever you go in the OU, you find someone with an unexpected specialisation. More in this vein would be welcomed - we might usefully pool our many abilities?

9 page 7

Margaret Cotton - Leamington Spa

I am a full time teacher and at the moment have the typical end of term outlook - just driving ourselves through the last few days.

To turn to OU maths, I honestly think we would all have handled the exam (M202) better if we hadn't got into such a state by the end of the course. There simply wasn't time to feel confident in all that abstract structure. I know, in general, students did find it an interesting course, but it was far too much to take in. Anyway, all we can do now is see if we managed to scrape through. I had two other exams as well, and although they were certainly fair enough papers I found the workload of $1 + \frac{1}{2} + \frac{1}{2}$ credits such that I don't know whether I passed any of them.

The Leamington Maths Group will be meeting on Monday evenings at Leamington Study Centre, 7.00 - 9.00 once the OU year starts. Our regular

threesome went to the new students' induction and generally tried to be friendly. We are making it open for all maths courses and just hope that we can encourage a general mingling and discussions of problems. We may structure it more later if there seems a need. We ran an M202 Saturday school last year and shall certainly keep the idea of such a day in mind for this year if it is possible. Anyone interested can either attend the Mid-Warwickshire College of Further Education on a Monday, or may ring me for further details. I really think we value this student contact, and certainly working together we clarify things for each other.

Thanks for starting MOUTHS/M500. Let's hope it does help some students.

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Michael Gregory - Farnham

Although the problems of obtaining both financial and written contributions to M500 rest mainly with the editor, I feel that many OU mathematicians would suffer if it collapsed. It has been spreading ideas and opinions on diverse mathematical topics which cannot be covered by course units and set books, and also helps people with a common interest to contact each other.

I realise that many students have little spare time after their studies, and that between November and January neglected jobs must be completed. I suspect that many do not realise how easy it is to write an article of say 200-500 words about some aspect of mathematics which has aroused their interest. There must be surveyors, engineers, programmers and market researchers etc. who could recount anecdotes or problems of mathematical interest.

I wonder whether prizes for correct solutions to the problems would encourage more of us to submit solutions. Do you think this would have the required affect? If we contribute to the cost and the written material, and let the editor know that the newsletter is read and appreciated (in a form which she can publish!) then M500 should flourish. If this does not happen, A in Carlisle may never know what B in Bristol thinks about the Pell equation or the snub cube.

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9 page 8

Lytton Jarman, Rugby

One of the enjoyable things about being an OU student is the surprises it offers. Whilst trying to find out what was happening amidst a tangled mass of vibrating

M201 springs I had to decide what to do in 1973. A course entitled "Topics in Pure Mathematics" sounded as though I would get away from those springs, so I went to the 1972 BA Degree Handbook to read the précis. 202 would "explore one of the mathematical islands". Ah! So that's what it's all about. O.K. I signed on.

Surprises? Yes. Early in January I landed on that Island and with a local study centre with no other members of that Island. I was to remain an OU Robinson Crusoe. Each week a surprise envelope was dropped on the Island, requesting almost impossible tasks. It was the amount and complexity of the work that completely prevented enjoyment of life.

However, the exam. came as to escape in a self-made boat. About 100 yards out it sank miserably. Luckily I was picked up by a passing Counsellor and shall now be able to spend 1974 on two OU half Islands. I hope life will be easier there. If it is as bad as 202-land I shall not bother to build a boat.

<p style="text-align:center">++++++</p>

Yvonne Kedge, Sonning Common, Oxon.

Dear Tony Brooks, As a member of the 'vociferous minority' I too don't object to the TMAs. It is he devious wrigglings of the CMAs and examination questions that I object to, that seem to test low cunning mostly. May the most crafty and aggressive win.

Dear Dr. Mason, Dr. Dey, etc., Not being a masochist I've moaned about you during the year, but oh to have you back - you were at least an excuse for avoiding the housework.

Please, before next year (i.e. 1974, Ed.) could someone explain to me how to study more selectively and in more detail, as we are advised, in a style comparable to that of conventional (immature?) students and yet still be able to answer CMAs?

<p style="text-align:center">++++++</p>

Peter Weir, Coventry

Am I the only person doing M202 in 1974?

I tried to get the book mentioned by Patrick Sharkey in M500/8. The bookshop, being thick, had never heard of it. I later found it on the shelves. They may possibly be excused because the book is in fact 'Background to set and group

theory' by D. E. Mansfield and M. Bruckheimer (note the order - important in alphabetical filing!!) Chatto & Windus, 1971. £2 hardback. Seems good.

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Ed: Could someone at Walton Hall please compile a complete list for M500 of books written by *all* of our mentors, pref. with brief comments by the authors? Many thanks in anticipation.

<center>9 page 9</center>

EDITORIAL This issue of M500 is being sent as a sample to all who have read or heard about us. We hope you will like the general idea, and will wish to join in. M500 can only be as good as its readers make it, and its chief aim is to encourage inter-student and student/staff contact and exchange of ideas. Criticisms, ideas and comments are always welcomed, but positive action is always better than criticism so send along anything of any interest to OU mathematics students which YOU can write. Contributions can be as informal or as rigorous as you choose (length rarely more than 600 words, please.)

M500 is student-operated and student-"owned", and is strictly non-profit-making, but it must be self-supporting, paying for its own materials, postage and labour costs. A team of ladies is paid to do the routine work, leaving the ed. free to edit (and to do OU units from time to time), but we are still totally dependent upon an office duplicator, and ought to save up to buy our own machine. The subscription has been fixed at £1 for ten issues, because most of the 1973 founder subscribers tucked a £1 note in an envelope when asked to subscribe whatever they thought M500 was worth. For each 10p issue, 3.33p is for the stamp and envelope, about 2p is for paper and stencils, about 3p covers the labour costs, and 1.67p appears to be profit, but is available for sundry expenses. Any surplus will be used to save for the duplicator or to adjust the next subscription.

Do let's hear from you all. We even print advertisements - 'small' ads are free (at the moment), but you have to expect to pay for the paper if your ad is large. Staff can help by keeping us informed of societies we could join, conferences and short courses we could attend, periodicals and books we should enjoy, and of anything else which conventional students learn about by constant contact with with mathematicians, libraries, notice boards and each other. (Remember

that many of us have no mathematical contact other than the contents of units and set books, and have often never heard of things which conventional students take for granted.)

Letters to the Editor will be regarded as potential material for publication (edited if necessary, though without censorship of opinions) unless marked 'Personal'.

9 page 10

M.O.U.T.H.S.   LIST No. 9 December 1973

MOUTHS = Mathematics OU students Telephone self-Help Scheme, and has been operating since January 1973. The following is a complete list of members with their telephone numbers (or addresses) and courses. Addresses are also available but are omitted from this list 9 to save space. The list is roughly regionalised, but the area codes are defined by the counties and do not necessarily correspond to OU regional codes. Members are urged to use the scheme frequently and not to wait until they run into special difficulties. During 1973 many complete strangers became good friends over the telephone. Those without home telephones can join the scheme, using public call boxes. . . .

9 pages (11, 12,) 13

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NOTICE: Open lecture 'How to grow trees' - an intro. to graph theory - by Dr. Robin Wilson (Walton Hall) on Thurs Jan 31st, 7.30pm, at the Adult Education Centre Annexe, St Faith St., Maidstone, Kent. Admission 10p. Contact Bob Davies (Sevenoaks) for details.