M500
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M500 is a student-operated and student-owned newsletter for Open University mathematics students and staff. It is designed to alleviate student academic isolation by providing a forum for public) discussion of individuals' mathematical interests. Articles and solutions are not necessarily correct, but invite criticism.

MOUTHS is a list of names, addresses, courses and telephone numbers of voluntary members, by means of which private contacts may be made by any who wish to share OU and general mathematical experiences or who wish to form telephone or correspondence self-help groups.

DESIGN-A-COVER COMPETITION

The front cover of M500 can provide space for the mathematical arts, hitherto neglected by this publication. Designs will be approved or rejected by the editor, advised by professional Graphic Designers where necessary, and can take any mathematical art form which is reproducible on a stencil. Effectively this means that there cannot be large areas of black ink. The finished designs must be in black ink or black biro or very black Photostat on plain white paper, and should measure not more than $10'' \times 6''$ printed area, This leaves a white border on both foolscap and A4 paper. Alternatively, designs may be submitted in final form already cut on an infra-red (thermal), electronic or typing stencil with Gestetner or Roneo heading after acceptance in rough form.

By 'mathematical art form' is meant any two-dimensional design which involves mathematical aesthetic appreciation.

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The maze on the cover of this issue is reproduced by permission of Latimer New Dimensions Ltd., Publishers, from *Greg Bright's Maze Book* available in hardback at £1.75 and paperback at 90p.

It was reproduced by means of carbon-deposit Photostats of both maze and lettering independently. These were then pasted on to white paper and finally fed into a thermal stencil-cutting machine, which reacts only to carbon images. It is essential to use Cow Gum and to keep the gum well away from design areas. Failure to do so results in wrinkling of the design - cf. M500/17 November 1974.

Start at the middle of the maze, and attempt to work your way to the outside. There is no time limit and no solution given.

AN ATTEMPT TO CLIMB MOUNT PEN-GELLI

This mountain is in a very remote area of central Wales and is one of the Six Brothers of Waltall. It was in 1971 that this range was discovered and climbers have since essayed to scale their heights. The other notable summits are, of course, Mt Ferguson, Mt Pentz, Mt Learmouth, Mt Sparkes and Mt James. Base camps have been set up on all these except the latter, which is accessible from the others. There are other camps at higher levels and by now some intrepid climbers have reached the summits. Would-be climbers will be disappointed if they consult their "Poucher" as to the best route, the range having been so recently discovered. Nor, alas, is there an amusing and informative description by Mr Wainwright to stuff in one's pocket on the journey. No; here climbers must rely on their own wits.

Pen-gelli is, of course, a Welsh name. (Ignorant Englishmen pronounce it as though it rhymed with "jelly", but a nearer approximation to the correct pronunciation would be "gethly".) Its meaning is "top of the grove" and probably refers back to the days when the Forest of Calculus reached to the summit. Now, however, the craggy top is bare and its jagged rocks are outlined against the blue sky. Snow lies in the gullies and "phisms" throughout the year and the ascent of the mountain, which is steep at all levels, is acknowledged to be of uncommon difficulty. Many are those who try and fail but the devotees of Mt Pen-gelli are renowned for their persistence.

I am no climber; I am really only a rambler, and to attempt to climb Mt Pen-gelli even just to base camp was, no doubt, very foolish, especially as my only footwear was my old, very traditional, School Certificate sandals bought in 1936! Naturally these were cracked and full of holes. However, they had taken me without mishap to the base camp on Mt Ferguson, along a route that was very clearly marked (no map was necessary), through scenery that resembled parkland, with decorative statuary at intervals, and accompanied by the sound of birds trilling their sweet bach-song. Nor had the incline been too steep. When I scan the higher reaches of Mt Ferguson through my binoculars they look to me very attractive (no wonder the climbers are so many), but I was determined to reach base camp on another mountain before returning by the chair lift to Mt Ferguson's base camp. I essayed Mt Learmonth for a time; this was a good economic journey, but I withdrew as I found the ground too arid for my taste and I grew irritated with hacking my way through the dense patches of sociological verbiage.

It was in February 1974 that I met my fellow climbers at the base of Mt Pen-gelli to obtain a briefing on the route from three very helpful local counsellors. I noticed that most climbers were far better shod than I; many wore A-level or HNC model boots, while some even boasted the superb Teacher's Certificate or Undergraduate boots, which are well adapted to the rocks on the highest slopes. However, there were some like myself in sandals, though of more recent make. Those of us in sandals soon lost sight of the well-booted brigade, who strode ahead, armed with maps and compasses.

I made very slow progress but I found the going at first clear, well-marked, and not too steep. This ground belonged to a local farmer called Gowar, and he had obviously been at

pains to signpost the route. The path lay alongside a charming brook, the R. Heimer, bordered with luxurient goatsbeard. This river has played no small part in forming the shape of Mt Pen-gelli - indeed, the mountain is sometimes referred to as Mt Heimer. Now, however, I gather this brook has disappeared, a not uncommon occurrence in limestone areas.

On leaving Mr Gowar's land I found the way grew very confined between narrow bounds, which made the going difficult. Indeed, at one point I became embarrassingly wedged in an extremely narrow gap-stile (meant for slimmers only) but my shouts for help were answered by a friend close by, who pulled me clear.

It was on the next section of the route that I first met with those curious "phisms" which abounded along the way. These are rectangular enclosures bordered by dry stone walls, the limestone of which sparkles in the sunlight; they make a pleasing pattern on the slopes of Mt Pen-gelli and, through my binoculars, I could clearly discern them on the higher reaches of the mountain. Though attractive to look at, they are irritating to negotiate. For instance, arrows point to the right and then to the right again round the periphery of these enclosures but, on arriving at the diagonally opposite corner, from which point the route continues, one finds the way barred by huge iron gates on which is a notice: "You may only proceed if you can find the way round by initially turning left." So, back to the beginning to begin the weary trudge by the arrow pointing to the left. Obviously one then needs to turn left again, but the path is not visible and one has to make one's way over a limestone pavement in which are deep grikes, or fissures. Finding somewhere to jump over these is a great problem, especially when the stone is wet and slippery (for, by now, drizzle had set in and the summit of the mountain was outlined against a rarely blue sky). It was with difficulty that I eventually reached the diagonally opposite corner to continue on my way. Later I found more phisms; they were distributed all over the mountain in associated groups. Though I became better adapted to negotiating them, these phisms always proved to be difficult and time-consuming obstacles to progress. On none of the other mountains in the range are these peculiar constructions found, and they give a unique aspect to the slopes of Mt Pen-gelli.

For some time after this the route presented no especial difficulties; in places the land was full of undulations of unequal height, and at one place I passed a ruin where there was a sign on which could still be discerned the name, "The Grange". The weather was becoming steadily worse and the summit crags were now outlined against a dark grey, stormy sky, but my spirits were kept up by the periodical meetings with my fellow climbers and the local counsellors (to say nothing of the cheer of the cuppa) provided.

Then the going became rather more difficult on account of the amount of private property and the many "Keep Out" notices, for here the public right of way was set between limits, and to find the boundary, or limit, of these properties and proceed along this was a lengthy and tedious business. It was about here too that I found a strange notice: "GET - THE DEMON". I turned round and there he was, by me. Not much like a demon, but square and steely-looking, with a price-tag, in dollars, still on him. He was quite polite at first and used to point the way and say "Please go ahead". If I found a difficulty he would say courteously, "Please try again". Quite a nice friendly demon, I thought at first.

By now I was approaching a wooded area which was actually an outlying section of the Forest of Calculus, but the trees here did not grow too closely together and the going was quite good. I came across a Forestry Commission bulldozer which was carving a passage out of the hillside. A little further on, when out of the wood, I had a pleasant surprise - a picnic spot,

where one could sit down comfortably and eat one's meal at a truthtable. Here the sun was shining again and this proved a pleasant interlude.

It was after this that things started to go wrong. Firstly, my relations with the demon were becoming soured. Whichever way I turned he would shout "Error" or "You are DIM". He kept telling me to run and then would immediately shout "Return". Sometimes he was a bit personal and told me to scratch. Whenever I was in difficulties he uttered no word to help me, hut would mutter something under his breath which made me think he would kill me, or else he would complain about me, saying, "Bad, bad list delimiter", which remark I could not understand. In the end he suddenly turned on me and, giving me a slip of paper, said, "You're on your own now. We're throwing you in off the end of the pier, and leaving you to swim. I'm not going to be around to help you when you meet Nat Logz. Bye." He then vanished and I was left holding the slip of paper which had written on it "??". Altogether he became very incomprehensible and I decided to forget about him.

So I journeyed on alone and found that I had to pass through a series of logic gates which had electrical connections, so that one had to be careful not to be short-circuited. At this time the sun was no longer shining and the air had become rather chill, but I became a good deal more chilled when, on crossing an old lane, I found a very wrecked car by the roadside and in it sat a ghost. This latter was busy writing figures with a transparent pen on transparent paper. I asked him what he was doing.

"I'm trying to find out how fast I was going at the instant when I crashed", he answered.

"Does it matter now?", I queried, but he merely replied with a ghost of a smile and continued his deliberations.

As I continued on my way I noticed that the summit was no longer visible because of the steepness of the sides of the mountain. I came to a signpost on which was written, "GOTO Penrose". This is the name of a smaller hill on the flanks of Mt Pen-gelli which must be climbed in order to reach base camp. I am not sure of the meaning of the name but it seems likely it was once "Pen-rhos", or "top of the moor", which description probably relates to the days prior to the seventeenth century, for, since then, it has been thickly planted with the Forest of Calculus. This is no mere wood; it is a huge Forest, composed of immense trees whose leaves intertwine to keep out the light. No doubt those who can see the wood for the trees would call it beautiful. It is now in the Forestry Commission hands, but they have allowed the undergrowth to become very dense indeed, so that progress is extremely difficult. Not that I managed to progress at all; I simply walked round in circles, for I lacked a compass to guide me. Furthermore, the many notices helped to mislead, for they were all alike. "TAN", they said, in fiery letters against a background of flames. ("Tan" is Welsh for fire). Everywhere I turned I saw the same notice, "TAN"; I suppose they were necessary cos people are so careless.

Before I had gone very far I nearly tripped over two men kneeling down in the undergrowth and peering at something.

"What are you doing", I asked.

"Sh", replied one, loquaciously.

"Hold on a sec", said the other. Both appeared to be outdoor types and were well-tanned. I saw then that the first speaker was consulting what looked like a huge telephone directory, but which was entitled "Six-figure Tables". He was obviously a man of great integrity.

Then the second man turned to me and said, 'We're hunting the sine, of course". I just smiled pleasantly and went on my way, anxious not to display my ignorance of natural history, for I knew nothing of sines, let alone being able to differentiate between the sub-species. Actually I felt rather sorry for the poor little thing, whatever it might be. (But I have since gathered that there is no need to fear for the species, as they multiply quite readily.)

Later in my lengthy and aimless wanderings in the Forest of Calculus I met a curious and rather difficult individual, L. Nathaniel Logz by name. He is a great exponent of the need for the integration of all primitive functions, or so he told me. Personally, I didn't understand him at all and the directions he gave me confused me still further. I went backwards and forwards, substituting one path for another, but all to no avail. Sometimes, indeed I would just stand stationary at a point, unable to decide where to turn. In the centre of the forest I was surprised to discover an aqueduct, whose construction I could not understand. It was soon after this that I stumbled somehow out of the forest, only to find myself at the same place where I had entered it. How to avoid it and so progress? I did not know. But by now I was absolutely exhausted and so I lay down on the ground to sleep. It was a disturbed rest, for I was troubled by dreams; at first I thought I was running a race with a tortoise, but even here I could not win. Later a crowd of barbers appeared on the scene, all shaving each other, and then one started to shave off the goatsbeard that bordered the R. Heimer. ...

At this point I awoke and quickly remembered my unhappy position. I glanced up the mountain; the top was not visible, but I could see level upon level, all inaccessible. Obviously I would never make even the base camp and I started to consider ways of getting down the mountain. Only those who reach the camps are credited with a chair lift. Low in spirit, I decided there was nothing to do but become a drop-down. But that helpful MOUTH, George Dingley, cruising through the air in his man-powered machine, spotted my predicament and descended to take me aboard and then landed me at the base of the mountain, disappointed but unhurt.

For some time afterwards I scanned the slopes of Mt Sparkes through my binoculars. Perhaps there might be a train up - even an Advanced Passenger train. But, to raise my spirits, I filled a large glass with M500 and the effect of this so cheered me up that my thoughts returned to Mt Pen-gelli. I would have another try.

So I am now busy studying natural history and reading the biography of Mr L. Nat Logz in order to understand how his mind works. I am also trying to mend the holes in my sandals, as best I can. Indeed, I now look forward to the time when the mist which covers the whole range in the depth of winter will clear; then I know that the fine crags of Mt Pen-gelli will be outlined against the blue, blue sky, its phisms will glisten in the sunlight, the colours of the young leaves in the Forest of Calculus will add to the mountain's glory..........

<div align="right">Vera Keates, Leeds</div>

CONCERNING M500-I UNITS - Geoff Bennett

Somewhere in a previous M500 was announced a milli Helen - that unit of beauty which will launch just one ship. I have two follow-up offerings:

(1) Since 'mho' is used for conductance (as opposed to 'ohm' for resistance), what about a 'Neleh' to grade those with unbeautiful features?

(2) And opposition by wives to OU activities could surely be measured in 'Jezebels'?

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SO YOU'RE NOT ENJOYING THE PARTY? - Michael Gregory
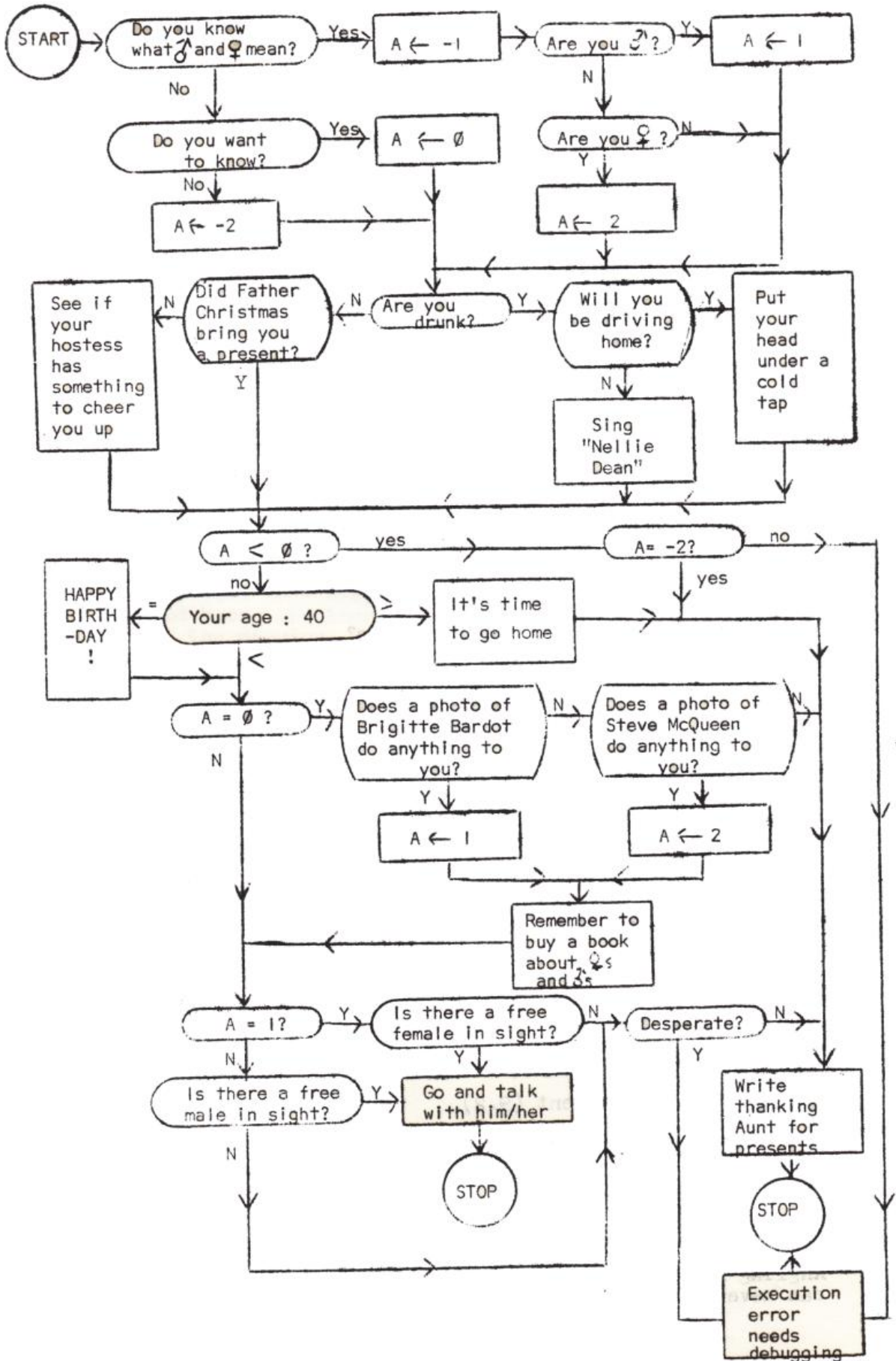
ALGEBRAIC SYSTEMS

CAN WE CONSTRUCT A NEW ALGEBRAIC SYSTEM FROM AN OLD ONE ?

1. Consider a pair (*S*, o) where *S* is a non-empty set and o is a binary operation which satisfies:

    (a) o is closed,

    (b) o is associative;

S is called a semi-group.

2. (*M*,o,*e*) is a triple where

    M is a non-empty set

    o is a closed binary operation

    o is associative

    e is the identity element

M is called a monoid.

Now,

(a) Is it possible to construct *M* from *S* in the same way we construct the set of integers from the set of natural numbers?

Could we, for example, define an equivalence relation on the Cartesian product of *C* with itself, where *C* is the class of semigroups, and produce equivalence classes which turn out to be monoids?

(b) If we can, what is the construction?

(c) If not, (i) can we prove the impossibility of such construction and (ii) is there any other method to construct the class of monoids from the class of semigroups?

(d) What about construction of groups either from the class of semigroups or that of monoids? I suspect that none of these constructions are possible, but I do not know if there is a proof of impossibility.

Does anyone know answers to one or more of the questions proposed?

(Note that we can always construct a field of quotients from an integral domain.)

<div align="right">Datta Gumaste</div>

MATH-QUOTES - Eddie Kent

All things began in order, so shall they end, and so shall they begin again; according to the ordainer of order and mystical mathematics of the city of heaven.

<div align="right">Sir Thomas Browne, *Cyrus' Garden*, ch. 5.</div>

Angling may be said to be like the mathematics, that it can never be fully learnt.

<div align="right">Isaak Walton, *Compleat Angler*: Epistle to the Reader.</div>

<div align="center">+++++++</div>

historia mathematica

The date of this document is uncertain. It seems to be a fragment of an ancient Arabic letter. In this translation words or letters enclosed in parentheses are not in the original manuscript, but have been added to assist the reader. (?) indicates that part of the text is indecipherable

"In the name of God, the Merciful the Compassionate; may God bless and save Aham Re Ad and his companions.

I found, may God lengthen the life of the noble Shaykh my lord and my ruler, in the handwriting of Muhammad bin al-Rowl (?) upon the black wall a treatise (risala) on the inequalities of triangles (giving a method) different from what was in the treatise (risala) of the Shaykh Micha El Spiv Ak, the Associate (?) of the Commander of the Faithful, may God support him.

He (Rowl) said, after the conditions were mentioned:

1. $a \leq |a|$ and $b \leq |b|$ which is evident and so by adding together, $a + b \leq |a| + |b|$,

2. $-a \leq |a|$ and $-b \leq |b|$ which is again evident and so by adding together, $(-a) + (-b) = -(a + b) \leq |a| + |b|$.

3. It must be that $|a + b|$ is either equal to $a + b$ or it is equal to $-(a + b)$ as the noble Shaykh Micha El, may God bless and increase him, has said in his definition of the modulus.

4. We put $|a + b| = a + b$ and so it will be true that $|a + b| \leq |a| + |b|$ as we found (in (1)).

5. We put $|a + b| = -(a + b)$ and thus it will again be true that $|a + b| \leq |a| + |b|$ as he (Rowl) says (in (2)).

6. We have found that $|a + b| \leq |a| + |b|$ in both cases, which is as the Shaykh set out to prove.

The difference between this (method) and that which is in (the work on) the analysis of (?) by the noble Shaykh El Spiv Ak, may God support him, is that El Micha considers four cases which has no utility and is hard to comprehend, but here we take only two cases and it is easy.

The proof of what Muhammad bin al-Rowl has said is evident, and so I wanted to inform my lord the Shaykh and his companions in order to put it in writing that he (?they) have it at hand. Peace (be upon you)."

Marion Stubbs

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Remember 1971/2 ? - J. E. C.

A Foundation student at Bangor,

At four in the morning, felt anger.

"The tutors" he said,

"Have all gone to bed;

The OU should prohibit such languor."

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and a partridge in a pear tree.

On the 5th day of Christmas my true love sent to me

       5 gold rings
       4 calling birds
       3 French hens
       2 turtle-doves
    AND a partridge in a pear tree.

Total $= 5 + 4 + 3 + 2 + 1$ gifts.

Similarly, on the 4th day, 10 gifts are received,
  and on the 6th day, 21 gifts are received.

So after 5 days a total of $15 + 10 + 6 + 3 + 1 = 35$ gifts are received. It follows that on the nth day the True Love sent something after the nature of

    n lords a-leaping
    $n - 1$ pipers whistling
    $n - 2$ cows a-calving
    …
    and a partridge in a pear tree.

There is an interesting distribution of objects received.
Distribution after 12 or 11 days:

	$n = 12$	$n = 11$
Pear trees	12	11
Doves	22	20
Hens	30	27
Birds	36	32
Rings	40	35
Geese	42	36
Swans	42	35
Maids	40	32
Pipers	36	27
Ladies	30	20
Lords	22	11
Drummers	12	—

  12   22   30   36   40   42
     10   8   6   4   2   1st diff.
       2   2   2   2   2nd diff.

2nd difference constant $\Rightarrow$ square function.

Out of 364 gifts received during 12 days there are:

    12 trees   3.30%,
    172 birds 47.25%,
    40 rings   10.99%,
    140 people 38.46%.

So one gets a preponderance of birds (nearly half). with people running a close second. 11% gold rings is reasonable. 3% pear trees is insignificant.

There is a preponderance of the nth gift after $2n$ days, although the rate of increase drops off sharply. I mean that after $2n$ days, the true love receives the greatest number of the nth (and $n + 1$)th if n is even). After that the amount drops off quickly.

To get 40 gold rings you need to take only 22 leaping lords, but to get 80 gold rings you would have to accept 110 lords. Increase the number of lords five times to double the number of rings.

I am sure that all sorts of problems could be worked out from this, but here are just two for starters:

1. How many gifts are sent on the nth day?

2. How many gifts are sent in total after n days?

Colin Davies, Garsington

MATH-STRACT

Intuition, that elusive vital agent, is always at work in creative mathematics, motivating and guiding even the most abstract thinking. In the most familiar manifestation, geometrical intuition, it has figured in many major recent advances that have occurred in or flowed from work in geometry. Yet there is a powerful compulsion in mathematics to reduce the visible role of intuition, or perhaps one may better say to buttress it, by precise and rigorous reasoning.

Richard Courant. Extract from "*Mathematics in the modern world: readings from Scientific American*" Folkestone: W.H. Freeman, (1968). £3.30

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𝔉𝔏𝔒𝔚𝔈ℜ𝔖

There once lived, in a small mining village, a Pitman named Edgar Swan. Now Edgar had two consuming passions in his life; One was the little garden behind his cottage, and the other was, strangely, maths.

He would spend many happy hours working in his garden and pondering over mathematical problems. Soon he began to relate his problems to his garden and he decided to try to combine his two passions.

His first big step was to move the little wicket gate from its position on the East of the garden, and after a morning's work he had his own Nor' gate. This so inspired him that he cut down some of his trees and made a set of log tables.

He soon took to inviting relations round to view his domain, and puzzling them by inviting them to draw coloured balls out of a large Grecian urn. His neighbours got quite worried about Edgar when he took to shouting instructions at his fruit trees, but he would explain that he was only trying to produce an ordered pair.

Edgar's great triumph came, however, quite unexpectedly. He was digging up some carrots one morning when he found one with an unusual shape. He was so delighted that he took it straight round to his tutor, and the two of them were thus the first people to actually see the square root of miner Swan.

<div align="right">Integer</div>

𝔹 𝔾 𝕊

The British Computer Society, which is an organisation for those interested in computing in this country, will accept OU Maths students as student members at an annual subscription of £4. For this one gets:

1. Quarterly issues of the *Computer Journal*, the most important journal dealing with computer science in this country.

2. Weekly issues of the Society's newspaper 'Computing'.

3. Reduced subscriptions for the foremost American computing journals - the *Communications of the Association for Computing Machinery*, the *Journal of the Association for Computing Machinery* (a more specialised journal) and *Computer Surveys* (which publishes mainly tutorial papers on computer science subjects).

4. Whatever activities your local branch of the BCS may arrange. These include visits to installations, lectures and social evenings.

The address to write to is: The British Computer Society Portland Place London

<div align="right">Andrew Arblaster Sheffield</div>

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<div align="center">
At Durham a student named Kath

Pondered aloud in her bath

"Why does administration

Insist I'm Foundation?

When I'm actually Linear Math."
</div>

<div align="right">J E C</div>

<div align="center">+++++++</div>

BITS AND BUCKETS

... Neither do I like the invasion of 'proper' maths, which I think is what Dr Ketley wanted. For instance, don't you think things like 'Inversion of Hilbert Matrices' and 'Crystallography' could be left to the course units or any textbooks?

<div align="right">Hugh Tassell</div>

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... All I had to say really was that I wished you'ld keep to mathematics. I would hate you to change the name to 'Open Set' or anything equally clever-clever... What I mean is, I don't care very much what you call it as long as it's mathematics inside.

<div align="right">Eddie Kent</div>

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... I like your artistic embellishments, and look forward to receiving M500 each month. Much of it still seems above my head, but I enjoy trying to understand the M202-type articles (a course I have not done, nor is it planned in the future). I have also spent several happy hours trying to solve some of the problems when by rights I should have been doing TMAs or CMAs or the like.

<div align="right">Annette Walsh (M321)</div>

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(Darn it - I've lost the one that said, approximately:
"Why waste time/money/paper on having a cover?" - Ed.)

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... You ask for comments on the new cover and the headings inside. Both are improvements. The cover, in particular, is worthy of praise.
Quote of the year: M500/17 p.15 "M100 doesn't have a high maths content" (Russell Brass). Could have fooled me. Hugh McIntyre

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M500/17 arrived in the post a.m. Thursday 7th November, just as I was reaching a peak of agitation over the MST282 exam only a couple of hours away. It was most relaxing. I still made a hash of the exam but calmly.

I was rather surprised to see from Russell Brass that M100 doesn't have a high mathematical content. I was under the, obviously mistaken, impression that I had done rather a lot of maths in that course and had even found it rather difficult.

<div align="right">Sidney Silverstone</div>

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Keep up the good work, Marion; as W. de la Mare said in "Off the Ground" - 'Here's me paying my forty shilling... and it comes most willing... '.   John Marks

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... With regards to your enquiry in respect of cover for Open University Student Kits, we would ask your advices as to the make-up of these kits on top of the Mini Computer and also whether these kits will be kept at your home or at the University and your earliest advices in this respect will be appreciated.   LLH Ltd. Insurance Brokers

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I was studying M100 this year, hut unfortunately I had to withdraw circa Unit 18. I was posted out of London to live in Cheltenham at very short notice, and I had to undertake several programming courses in order to be trained for my new posting. It was a case of deciding between OU and career - and career won.

I was bitterly disappointed, as I was thoroughly enjoying M100 up to the time I was posted out of London. I have now come to terms with it and am now eager to start again in 1975 on M100 (if the OU machinery works and I am re-allocated for M100!)

I am really writing to ask if there are any other '74 M100 'drop-outs' who are preparing to start again in January 75. We may have fallen by the wayside this year, but we will get there next year. Presumably there are two types who are going to re-start M100 - those who had to drop out because of outside commitments and those who found M100 too tough and fell by the way.

If there are any MOUTHS who did finally pass M100 after withdrawing the previous year I am sure that their advice will make us feel part of the OU system rather than banished failures or drop-outs trying to get back in the system. This is a particular problem rather than a general one I am writing about, but the advice may help me and a few others.

Dave Windsor, Cheltenham

SUE ON THE SENATE

... I finally sent the '3rd Level Petition' signatures to Ian Dey as he appealed in *Sesame* for student opinions. There were 90 signatures eventually, though a lot of these were from Study Centres and Summer Schools. Hope it has some effect.

Incidentally, I have been elected as student representative to the Senate, so I may have chance to forward the cause of Maths students there.

Sue Davies, Allestree

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WANTED

A copy of HILTON, H., *Plane algebraic curves*. London: OUP, 1932. 2nd edition or later. Please contact Michael Gregory, Farnham.

Michael adds: When I previously tried to obtain an out-of-print book - I advertised for 'Mathematical curves and their properties' in *Exchange and Mart* - I had two replies. One was from someone in a mental home in Jamaica offering me 'Teach yourself algebra' and the other was an offer of 'The mathematics of beauty' from a bookseller with salacious hints as to the contents. I did not follow up either - maybe I would now be knowing more maths.

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Many of the points in Ian Ketley's article (M500/15) were not directly related to M500 as a publication. I particularly refer to paragraph 4 and as I am totally unqualified to comment on that, I shall confine this reply to specifically M500 points.

I am not aware of the initial aims of M500 since I only got to know of it at the beginning of this year through the Maths Faculty Stop Presses. It appealed to me then as a good means of achieving albeit intermittent communication with other OU students. One has to remember that M500 is also the vehicle for publishing the names and addresses of MOUTHS so it serves a dual communications role. We all accept that M500 is informal and does not necessarily claim erudition; we would be foolish if we did not. Given the drop-out rate of many OU maths courses I feel that anything which increases the possibility of communication "between students is a good thing. If the students do not like the contributions or think them irrelevant then I would expect them to complain and state what they wish to see instead.

I agree with Ian that the OU student should 'know what is going on outside'. M500 provides information for the individual student on what other students are doing and what the OU is doing or proposing to do. He must look to other publications for a wider view. M500/15 provided a good starting list. Other information on publications and developments could be provided by contributions from Tutorial and Academic Staff.

Any publication is only as good as the contributions it receives. A selection policy can only be adopted where a surfeit of material is available and even then it may not be realistic or relevant. We are all capable of acceptance and rejection (I hope!) so I prefer to see a free publication where the onus is on the reader to assess a contribution rather than force such a role on an already overworked editor.

<div align="right">Roger Claxton</div>

<div align="center">+++++++</div>

I fully endorse all the comments made by the anti-Ketley camp. The only point I want to make is what is Dr Ketley's definition of 'unsound'? Without disputing his verdict, I think there is danger in dismissing anything new merely because it apparently lacks rigorous justification. It is a matter of historical record that on occasions the mathematical 'establishment' has wound up with egg all over its face, by its too-ready rejection of new ideas. For example:

Maths-Quote 1: from BASIC MATHEMATICS FOR RADIO AND ELECTRONICS
        by F M Colebrook and J W Head.
        Ch. 8 - Heaviside's Technique for Solving More Difficult Problems.

'This technique was used by Heaviside in a heuristic or exploratory manner, and full mathematical justification was not available for twenty years or so... Heaviside was a very extraordinary and unusual man, but it is important to note that he was a practical engineer, not a professional mathematician. Indeed, for many years he was despised and rejected by contemporary Cambridge mathematicians. Heaviside's calculus came into being because he wanted to solve practical problems, although an advanced modern textbook on the subject may look like the work of a pure mathematician.'

Maths-Quote 2: from T291 Unit 8 - Fourier Series:

'This idea is not easy to accept. Indeed, it was met with incredulity when Joseph Fourier put it forward in 1807.'

As a student with one foot on the 'applied' side, I reject the idea that a new mathematical

technique should be thrown away if the 'pure' ones cannot find a rigorous justification for it. Either the technique gives the right answers or it doesn't. If it does, then rigorous justification must exist.

In any case surely one of the aims of Maths education is to teach us how to distinguish sound from unsound Mathematics? And how are we going to learn this art if we never see any unsound Maths?

No, as Lytton says, let's have more unsound maths; the experience of pin-pointing the flaws is likely to be more beneficial than merely regurgitating textbook material.

<div align="right">Bill Shannon</div>

<div align="center">+++++++</div>

I was delighted to see Dr Ketley's article providing proof of Sinbad's assertion that mathematicians have no sense of humour. I am amazed that anyone could take M500 so seriously. It is not a serious mathematical journal for the publication of learned articles; it is simply a means of communication for OU maths students to express their personal opinions, mathematical theories and outright flights of fancy. In fact, the sort of mathematical gossip which students of a 'normal' university can indulge in bars, common rooms etc. but which OU students can only carry out at a distance via the printed word.

Inevitably, as mathematical novices, some of the material will he 'dubious', some of the mathematics 'unsound' (although I had assumed that N-ary's piece was written with tongue-in-cheek, but perhaps as a female, hence a non-mathematician, I have an excessive sense of humour). I fail to see how our contributions, however erroneous, can damage the standing of the OU student in the eyes of the outside world, since M500 readership is restricted to OU students and staff. All articles printed are open to comment and criticism from readers (even non-subscribers!) and I am sure all contributors would welcome having our errors pointed out to us. So by all means criticise our opinions and theories, Dr Ketley, but please don't criticise our right to have them.

<div align="right">Sue Davies</div>

<div align="center">+++++++</div>

Ed: This finishes the 'Replies to Dr Ketley' saga, apart from those from Henry Jones and Michael Gregory. I am not really wanting to publish these, since it seems undignified to me to defend ourselves - 'we' being the three most violently attacked. I think that the general readership has defended us, M500 and the OU magnificently, and thank everyone who has replied for their support and correct analysis of the original and continuing aims of M500.

<div align="center">+++++++</div>

FOR SALE

2 Schaum Series textbooks, in perfect condition, which provide invaluable help with M202:

General Topology - £1.75 (£2.15) - contains 650 worked examples;
Modern Algebra - £1.50 (£1.90) - contains 425 worked examples.

<div align="right">Contact J. M. Hutton</div>

<div align="center">+++++++</div>

Complete set of books for AMST 283 for sale, only £3.50 inc. postage.

<div align="right">Tony Brooks</div>

<div align="center">+++++++</div>

SOLUTIONS

(Most sent full-TMA-length solutions, but there isn't room here).

10.1 <u>Mastermind</u> Minimum number of tries at Mastermind which guarantees a solution is 5. David Wells, Puzzles Editor, 'Games & Puzzles'. David is our very first non-OU subscriber, and says he has been reading M500 with great interest. He is surely very welcome, as a member of 'the mathematical community at large' who actually likes us enough to part with £1.75!

16 <u>Diners and Dinners</u> 7 diners, 7 dinners, 21 meals, 3 chairs.
Bob Escolme. Full solution later when space permits.

17.2 <u>Telephones</u> Room 2178.... Telephone 8712   R. Seton-Browne (9), by computer.

17.4 <u>Fishermen</u> $27n - 2$ (Hugh MclIntyre, Eddie Kent, Marjorie Brew).
                    $27n + 25$ (Sue Davies).
   No maximum. Minimum is 25.

17.5 <u>Hypotenuse</u> 21_{12} (number system base 12). (Sue Davies, Eddie Kent).

16.2 <u>Goat Grazing</u>

(1) Let $R = 1$ and tether the goat at the obvious place. Then the function goat: goat$(x) = \sqrt{L^2 - x^2}$ adequately describes the boundary of the goat's dinner plate, were the fence around the field absent.

Since the goat is tethered on the perimeter of the field, field: field$(x) = \sqrt{1 - (x - 1)^2}$ adequately describes the field. (I always did wonder why f and g were used as function names...) . Now if the goat grazes half the field, the shaded areas are equal, i.e.:

$$\int_a^L g \;\; = \;\; \int_a^1 f$$
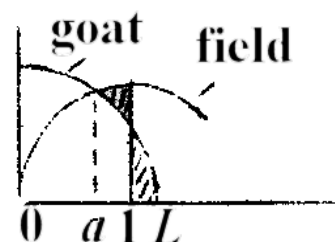
Also, $f(x) = g(x)$ at $x = a \Rightarrow a = L^2/2$. Clearly, $1 < L < \sqrt{2}$

At this point I depart from mathematics and commence to use PARKER'S SPECIAL SLOPPY METHOD.

Using the calculating engine kindly provided by the OU plus a flow-chart, cribbed from M251 set book, which integrates, I choose a value for L, integrate both functions and compare values. If different, I choose another value for L.

Result: 1.158725 ± 0.000005, and ignoring round-off error inherent in the machinery!

Incidentally, SLOPPY METHOD was devised through an inability to apply M100 Unit 2 iterative techniques. I'd be interested to learn how it should be done!

How much fence does the farmer require to contain the tethered goat?

John Parker

(2) $L/R = 2 \cos x$, where x is the solution to

$$\frac{\pi}{4} + x \cos 2x - \tfrac{1}{2} \sin 2x \; = \; 0.$$

Using a programmable calculator, $x = 0.952848$ ($54.59°$) and $L/R = 1.15873$.

<u>Method</u>: (See diagram). It helps if only a semi-circle is used, and R is equated to 1.

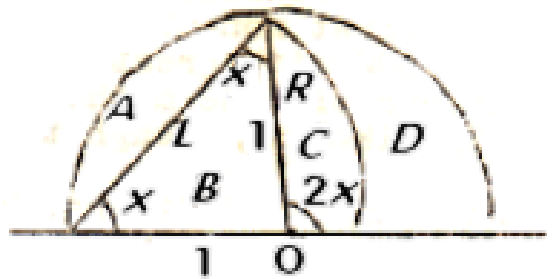Required: Area $A + B + C =$ Area D.

$A + B = \dfrac{\pi}{2} - x$

$B + C = \dfrac{xL^2}{2}$

$B = \dfrac{L}{2}\sqrt{\left(1 - \left(\dfrac{L}{2}\right)^2\right)}$

$A + B + C = \dfrac{\pi}{2} - \left(1 - \dfrac{L^2}{2}\right)x - \dfrac{L}{2}\sqrt{1 - \left(\dfrac{L}{2}\right)^2} = \dfrac{\pi}{4}$

In triangle B, $\cos x = L/2$. Substituting, and using trig. identities

$$\frac{\pi}{4} + x \cos 2x - \frac{1}{2}\sin 2x = 0.$$

<div align="right">Bill Shannon
(I hope - but no name on it - Ed.)</div>

<div align="center">+++++++++</div>

15.4 <u>Rabbits</u> - Peter Hartley

If dog's co-ordinates are $(x(t), y(t))$ at time t, then its path is defined by

$$\left.\begin{aligned}\frac{dy}{dx} &= \frac{-(ut-y)}{x} \\ v^2 &= \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\end{aligned}\right\} \text{ with } y = \frac{dy}{dx} = 0 \text{ when } x = 1.$$

Substituting $T = vt$ and $s = u/v$, T can be eliminated to give

$$x = \frac{d^2 y}{dx^2} = s\sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

Solving for dy/dx and then integrating the result:

$$y = \frac{1}{2}\left(\frac{x^{1+s}}{1+s} - \frac{x^{1-s}}{1-s}\right) + \frac{s}{1-s^2}\ (s \neq 1).$$

Cases: (1) $0 \le s \le 1$ (i.e. $u < v$), $y = \dfrac{s}{1-s^2} = \dfrac{uv}{v^2 - u^2}$ when $x = 0$. If $v^2 - u^2 \le uv$ (i.e. $u \le \dfrac{\sqrt{5}-1}{2}v$)

the dog catches the rabbit at $y = \dfrac{uv}{v^2 - u^2}$ at time $t = \dfrac{y}{u} = \dfrac{v}{v^2 - u^2}$.

If $uv < v^2 - u^2$ (i.e. $\dfrac{\sqrt{5}-1}{2}v < u < v$) the dog fails to catch the rabbit.

(2) $s > 1$ (i.e. $u > v$). The path $(x(t), y(t))$ never crosses the y-axis. The dog fails to catch the rabbit.

In each case that the dog fails to catch the rabbit we require the co-ordinates $(x(1/u), y(l/u))$, since $t = 1/u$ when the rabbit reaches B. One can find

$$t = \frac{1}{v}\left\{\frac{1}{1-s^2} - \frac{1}{2}\left(\frac{x^{1+s}}{1+s}\right)\right\} \tag{1}$$

but there seems no way to find x or y explicitly in terms of t. (The problem can be solved numerically by solving the equation (1) for x with $t = 1/u$, then substitute for y.)

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Hugh McIntyre writes: ...I must tender apologies for lumbering you with an incorrect solution... the rabbit, the dog and A are not, in general, co-linear. This, changes things.

Richard Ahrens asked if I believed each day has equal quantities of light and dark. Answer: no. In my solution to his 'Sunshine' problem I was taking full advantage of his special offer to ignore the tilt of the earth's axis. He did not actually offer this, I know, but that is the way I read it. It did make the problem much easier. My solution was slightly tongue-in-cheek.
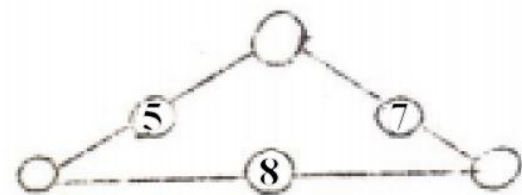
P R O B L E M S 1 8

1.

FORTY (Hugh McIntyre)	SEND (Tom Dale)	Solve in the
TEN	MORE	usual way -
TEN	MONEY	integers =
SIXTY		letters

2. A yoyo is lying on its side on a horizontal table. The string is pulled gently but with increasing force. What happens? Please state any assumptions, conditions, etc.

Bob Margolis.

3.

ARITHMOGRAMS

Find numbers such that the sums taken in pairs are 5,7,8 respectively. Generalise. Is the solution unique?

Now consider this diagram (left). Is the solution unique? What is the kernel?

Theorem It is only possible to construct a quadrilateral with four given points as midpoints of the sides if the points are vertices of a parallelogram, but in that case it is possible to draw an unlimited number of such quadrilaterals including skew ones.

What about other polygons? Bob Davis (new Staff MOUTH)

4. Find the next two terms and a rule for generating the sequences:

(a) 1, 2, 4, 8, 18, 52, 206, 1080, 6994, ... , (b) 3, 3, 5, 4, 4, 3, 5, 5, 4, 3, 6, 6, ...

Marion Stubbs

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A MERRY CHRISTMAS EDITORIAL

As we go to 'press' (6/12/74) 92 of the 230 expiring subscriptions have been renewed, and of these 28 have sent *more* than the suggested 25p. donation for the duplicator. Nearly everybody has sent at least 25p., but the number of cheques for £2.50, £3, £4 and even £5 has been astounding. The duplicator fund now stands at £53, with £44 still needed. It has all been very warming and encouraging, and one wonders if any other OU society could expect 2/5ths of its membership to renew a doubled subscription two months early. Subs continue to arrive at the rate of two or three each day, and it is easily manageable. I am most grateful for this kind of help.

However, let's not laugh too soon. Information that paper had doubled in price arrived after I had written the No. 17 appeal. (We use about 7 reams per issue, and fortunately I still have some in stock. I will change to A4 to please the metric moaners IF it is available next time I order, but there was absolutely none in Southampton last January.) I thought £1.75 sounded a bit steep, but obviously it will all be needed, especially when the postage increases again.

The Grand Opening 1975 MOUTHS List is promised with M500/20, early in February. Please be sure to send in your 1975 course details before mid-January. This applies particularly to new members whose subscriptions are not yet expiring. I still need your details please, if you want to be on the G.O.M.L. though no cash required!

With regard to the title, 'M500' was proposed by Peter Weir in 1973 and accepted by me, as despotic ruler of this publication. A month or so later John Bennett thought of 'Open Set'. At that time the whole future of M500 was in the balance, so the title was hardly worth discussing. Now we have enough members to make a vote significant, although many of you seemed to object to this democratic system. At the moment, 'M500' is strongly winning:

M500	-	55 votes
Open Set	-	20 votes
Abstained	-	17

The Weekend-Work-In looks feasible, from present numbers, and further information is being sent to interested members only, to avoid boring others. Write, enclosing stamped envelope, if you want to be kept informed and haven't received anything.

M500/19 - January 1975

A quantity of material is waiting - overflow from 18. But there is still room for more, .and of course No. 20 is waiting to be filled. Please try to keep Solutions to problems short?

MOUTHS 18 - full details with G.O.M.L. 1975

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MERRY CHRISTMAS !

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