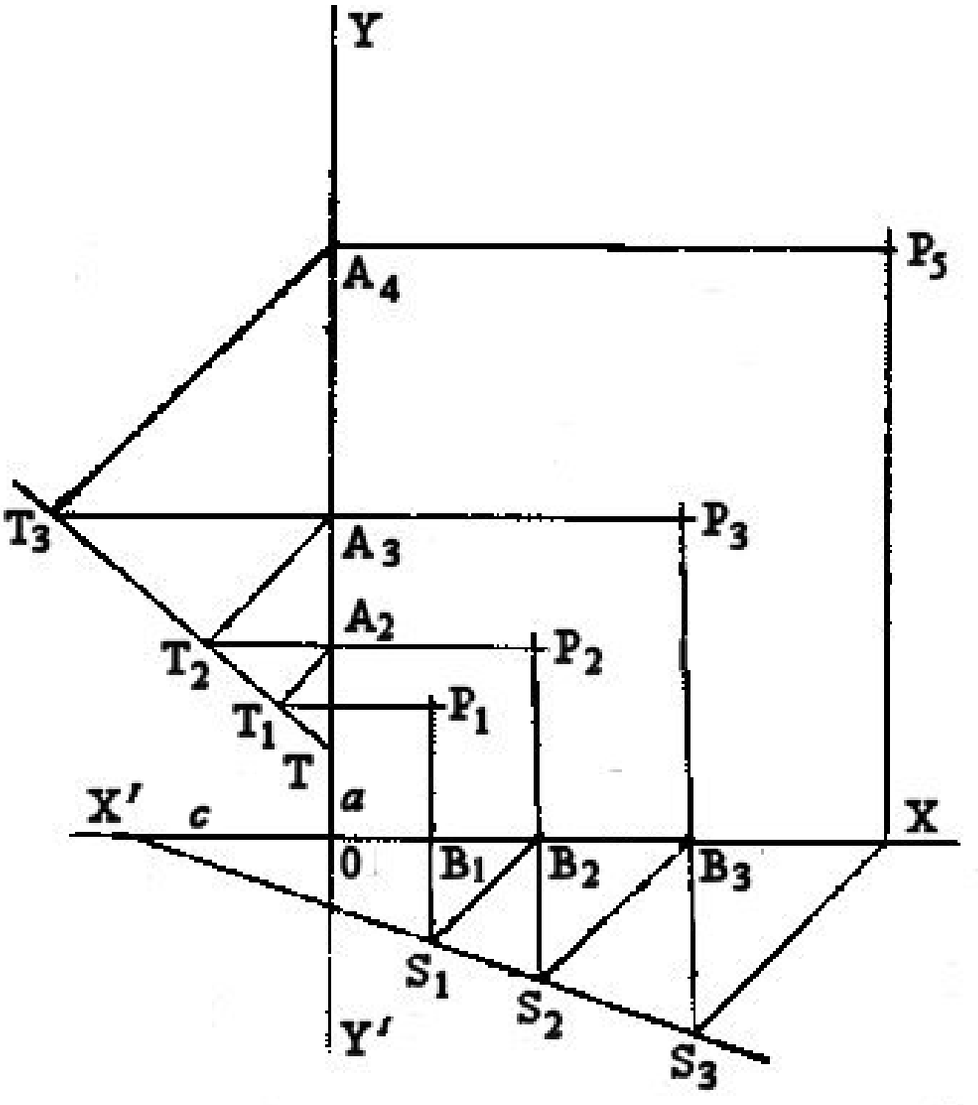

Geometrical Construction for the graph of $y = a + b(x + c)^n$

M500 is a student-operated and student-owned magazine for Open University Mathematics students and staff. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct, but invite criticism and comment.

MOUTHS is a list of names addresses and telephones, together with previous and present courses of voluntary members, by means of which private contacts may be made by any who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned, and do not necessarily represent those of either the editor or the Open University.

The construction on this month's cover was given by Professor Perry in *Nature* December 3 1903. It is described on page 6.

M500 is published by Marion Stubbs.

Contributions for M500 should be sent to the editor - Eddie Kent.

Subscriptions, address changes, MOUTHS data, and membership details of any kind to the membership secretary - Peter Weir.

Material not covered by the above to Marion.

Please note that anything sent to *any* of the above will be regarded as potential matter for publication unless marked PERSONAL.

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EDITORIAL

The Editorial for issue 27 appears, unusually and uniquely, at the front because M500 has arrived at a turning point and everyone ought to know.

In the first place the cost must go up I am as appalled and horrified as you but the reasons are given on another page. Marion can't really be expected to go on turning the handle of the duplicating machine 8000 times a month if it can be avoided. And anyway we have inflation. So would everyone very kindly part with some money right away please - even if your subscription doesn't run out yet?

Next, the weekend workin has intervened since M500 26 appeared and the opportunity was taken to hold a meeting. This was to discuss the formation of an M500 society with a constitution and officers.

Some progress was made. It was agreed by about 30 members that a constitution would be helpful. With their blessing a smaller group later met to get down to particulars. These were simple enough: Marion Stubbs and Eddie Kent will publish and edit M500; the society will have a membership secretary in the person of Peter Weir; their shall also be a treasurer (for which post applications are invited); there will be no restrictions on the membership of either M500 or the committee. It was also felt for some technical reason that the society's cheques should have two signatures (out of three registered with the bank). But no doubt our treasurer will advise on this.

Since The M500 Society is to be democratic most decisions will be voted on through the leisurely pages of M500. And of course all the posts are elective.

Any comments please?

Very little material is on hand for issue 28: come on M100, let's hear from you. 28 will be the Christmas issue. Fun and mathematical games welcomed.

EK

WANTED Surplus copies of M500 24 Special Issue. Please do not throw them away - either send them to M500 or at least pass them on to another student interested in maths but not doing an M-course this year. If you don't know anyone in this set and think your counsellor would know of someone, hand it/them over to him.

Meanwhile there are M500 subscribers who are actually asking me for extra copies for friends and we can probably get rid of as many copies as are available, given time. Next year's M100 students would like them for a start. Please send them to Peter Weir - our new 'Membership Secretary'.

Marion Stubbs

MATHEMATICS WEEKEND WORK-IN 1975

I would like to thank all the staff for their superhuman labours during the MWWI. All students present will certainly wish to join me in this vote of thanks.

I expect (or hope) that someone other than me has written some sort of report for M500 which is elsewhere in this issue. As ‘Conference Organiser’ may I say that the many thanks which I personally received during the Weekend made everything very worth while. I was noting odd things like students in general staying up until 1 a.m. or 2.30 a.m. - which to me proved that they were not exactly bored with things! One of the bridge schools apparently went to bed at 4 a.m. on Saturday night. I could not help noticing that the staff, who might well have been totally exhausted after some 12 hours non-stop teaching, stayed with it - even until 4 a.m. About the only things I criticised myself were the Rum Babas, to which I had been looking forward for 6 months, and the difficulty in finding Aston hall porters in the evenings. I even enjoyed the totally unexpected Wagnerian accompaniment to some tutorial sessions.

There will presumably be some demand for a repeat performance of the Weekend, if not of the Wagner, in 1976, and plans are already afoot. We are looking for host institutions in the Midlands - although not thereby rejecting Aston, which has the supreme virtue of being very flexible about bookings, not objecting when I ‘only’ produced 91 persons instead of 120-150 as originally reserved, extending the absolute deadline right up until Thursday 4th September. Few institutions would be so generous. Assuming students do want such a weekend in 1976, it would help such a lot if you would BOOK EARLY. I spent most of the year really worried sick, and vowing I would never do it again, with a personal £50 deposit at stake for a minimum guarantee of 50 persons which I made and yet had only about 35 advance bookings until midsummer. Nobody bothered to read expensive Sesame ads and attempts to obtain publicity in Open Forum and Stop Presses failed dismally. Then, after the proper deadline, the numbers virtually doubled. When M100 and M321 received the belated Special Issue, telephone bookings were coming in all through the final week, and this again was a constant worry since there was very little time to send out the documentation to these late bookings or to receive their payments. So, as Organiser, I do ask people to look at Sesame ads, since they are the only means by which students can communicate with *all* others; and to respond in good time.

Finally, to the eight students who did not turn up, either without explanation or with only a letter which arrived actually on the 5th September, may I say - DO USE YOUR TELEPHONE next time. There was no need for anyone to lose any cash if only they had been urgent about cancelling. As it is, 8 people have lost

£17, but at least 8 new people booked in during that final week and might have been waiting desperately for cancellations. I'd be glad to have any advance bookings now for 1976! How about a £5 deposit, refundable if the date doesn't fit your programme? Please state dates on which you already know you will not be free to attend. Also please state whether you would prefer Easter or Early September or both - at least one student asks for TWO weekends! I would guess that the final price is likely to be around £25 and I thought this even before the news that OU Summer School fees will be £50. (Our £17 weekend was about half the price of a Summer School.)

Marion Stubbs

DEAR MARION . . .

Thank you for all the work organising the Maths Weekend. : Not only did everything appear to be well organised and run smoothly but the weekend was certainly worth it from my point of view. Exhausting though it was, I have now regained my confidence and hope to finish M321 this year. A week ago I was on the point of withdrawing from the course due to being well behind. I am still not up to date. But I have been mathematically regenerated at Aston.

Elsie Gleadhill

Sorry to add to your already overwhelming correspondence but I am just writing to thank you for organising the math's weekend. I found it very useful and hope that it will be possible to repeat it in future years.

Reg Naulis

Just writing to thank you for organising the MWWI - I not only enjoyed it very much but it was extremely instructive and has been a great help to me. If I don't pass now I am just dim. I am sure you went home feeling it was a success - I hope we all convinced you of that. You did a marvellous job.

Miek Warden

A short note of appreciation for the tremendous amount of work that you must have put in to make the Birmingham weekend the success it undoubtedly was. I went along determined to conquer M231; and was disappointed in this respect but with such a choice of other subjects I was well able to find enough to do and was very impressed by Richard Shreeve on M251.

Everyone I spoke to was enthusiastic about the weekend and hope that it can be repeated although, I hope, with the responsibility more evenly divided next time for your sake!

Ann Jamieson

Congratulations on a very successful and enjoyable weekend.

Hillary Randall (M100)

Perhaps I could put on record my sincere appreciation of the staff members who took part in the maths week-end organised by our one and only Marion at Birmingham. Had I been Peter Hartley I would have been hoarse after the first evening session but Peter was there bang on time both on Saturday and Sunday, and plugged on the whole of the week-end, scattering sympathetic murmurs to dolts like myself who kept staring up at his hieroglyphics and chewing their pens... If it's any encouragement o him and his colleagues I went back home determined, to do rather better than I had performed at his sessions, and benefited enormously from his kind tutelage.

Willem van der Eyken

SITUATION VACANT

The M500 S0ciety urgently needs a treasurer; a chartered accountant preferred. Will someone kindly volunteer - or sneakily put forward a friend's name?

All(?) he(?) will need to do will be to specify how the money shall be paid in and keep track of it, and DEAL WITH INLAND REVENUE DEMANDS!

U3 (= U2 = U1)

PLEASE put your MOUTHS number on everything addressed to me, Eddie Kent or Peter Weir. We have to look it up EVERY TIME, and it is jolly aggravating. If you can't find yourself on a list how do you expect anyone else to - including Peter, Eddie and me?

Marion Stubbs

PRINTING DETAILS M500

1. Current price

4.0p	per copy for 'admin' (mainly postage and telephone)
1.0	envelope
0.5	ink
5.5	stamp
1.5	labour (stapling and putting in envelope)
5.0	material (paper and stencils, &c)
17.5p	per copy = £1.75 for 10 issues.

2. As above but with 8.5p postage - no other inflation costs

17.5
3.0
20.5p or £2.05 for 10 + inflation.

3. Quotation for printing from editor's prepared material

4.0p per copy 'admin'
1.0 envelope
8.5 stamp
12.5 printing stapling and folding
26.0p → £2.60, say £3 sub to cover other inflation.

oOo

As you can see from the above, costs are going up. Those of us delegated with the authority have decided that method 3 will do us for 1976 (since the membership is growing too large for Marion to keep turning the handle of the duplicating machine. There is another method suggested by the printer which will probably look prettier but it will cost more and take longer to produce; We can perhaps think about this for 1977.

Even if we don't go in for the printing (and if you think we shouldn't please say so immediately) you will not get away with the £2.05 quoted above since there are other inflation costs to be taken into account. The suggestion is that the subscription goes up to £2.50 to retain the format exactly as it is. Vote for that if you want to drive Marion into an early grave.

Note from handle-turner: Hey, I've been on EVERY page so far! *Did you notice EDDIE KENT is editor*? *He done it, not me. M.*

CONSTRUCTION of $y = a + b(x + c)n$: see cover.

Draw a pair of rectangular axes X'OX and Y'OY. Cut off OS from OX' = c and OT from OY = a. (The figure is drawn for positive values of a and c.) Choose any two values for x, calculate their corresponding y-values and plot P_1 and P_2. Through P_1, P_2 draw P_1A_1, P_2A_2 parallel to XO, meeting OY in A_1, A_2. Through A_2 draw A_2T_1 at 45° to OY meeting P_1A_1 produced in T_1. Similarly draw P_1B_1 and P_2B_2, then B_2S_1 at 45° to OX, meeting P_1B_1 produced. Join TT_1, SS_1 and produce.

The other points P_i are produced as follows: produce P_2A_2 to T_2. Draw T_2A_3 parallel to T_1A_1, meeting OY in A_3; OX in B_3. The point P_3 on the curve has coordinates (B_3, A_3) Similarly the point P_j is at (B_j, A_j).

Clearly $1 + \tan\theta = (1 + \tan\phi)\, n$, where $\angle YTT_1 = \theta$ and $\angle XSS_1 = \phi$ and so the curve could be drawn by choosing a value for ϕ, calculating θ and working from a single point P_1.

Eddie Kent

THEOREM THEOREM

Get your brains round this one, logic freaks. I thought of it while washing my hair; it's a bit thin perhaps, but then so is the hair:

Theorem: No theorem is always true.

Proof: If the theorem were untrue, a counterexample would exist. Which would show that the theorem was not always true. But then it would be an example, not a counterexample. The contradiction proves the theorem. But it's still not always true.

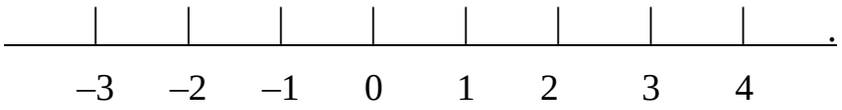
Roger Bridgman

New ideas are born out of a state of confusion - A N Whitehead.

THE AXIOM OF ARCHIMEDES

There seems to be some misunderstanding surrounding Archimedes's Axiom.

The axiom is an axiom about the real number system (whatever that means), not about the integers. The formal statement is "Given any real number *x* however large (positive) there exists an integer *n* which is ≥ *x*." Your previous correspondents seem to think the axiom says that there are infinitely many integers, but it is not saying this at all; what it is saying is that there exist arbitrarily large numbers. If one pictures the real number system as represented by the points of a line with the integers marked off at equal intervals along it



Archimedes says there are integer points at arbitrarily large distances away from zero. It is easy to mark off an infinity of points

$$x_1, x_2, x_3, \ldots, x_n, \ldots$$

on the line which do not have this property, for example,

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots, \frac{n}{n+1}, \ldots$$

all of which are < 1. So Archimedes's Axiom is a much stronger statement than the mere assertion that there are infinitely many integers.

John Reade (Staff)

oOo

ESSAYS

In S100 I had to write one on "The misuse of Biology". This in 500 words. I couldn't write such a thing in so few. Not an essay, though I could churn out 500 words and call it one. Anyway, back it came with an F attached. I had, it was claimed, written it on "The avoidance of the Misuse of Biology". Fair enough, I suppose, for my thesis was that "misuse" is hard to define, let alone in 500 words.

Hugh Mclntyre

ooo

It was long ago shown by Galileo that in order to boil eggs by whirling them round your head in a sling, as the Babylonians did, it is necessary to be a Babylonian.

A S Housman

SKEWES' NUMBER

I wondered if any M500 readers could perhaps supply some information. I am trying to find out some more details concerning Skewes' Number. All I have so far is that it is reputed to be the largest finite number that has ever shown up in a mathematical proof. I believe it concerns prime numbers and was part of a task set to S Skewes by a British mathematician, John Edensor Littlewood, in 1914. The only other information I have is that Skewes published a paper in 1955 recalculating the number; his original assumption was based on the fact that the 'Riemann Hypothesis' was true (whatever that is!) and the 1955 paper recalculated this number assuming the hypothesis *not* to be true. The numbers are $10^{10^{10^{10^{1.53}}}}$ assuming the hypothesis and $10^{10^{10^{10^{3}}}}$ assuming the hypothesis not to be true. The sheer size of the numbers is staggering - I would love to know what the proof is all about, as I am sure would many M500 readers.

Barry McNaughton

Ed - My book has it that E Skewes published in 1933 a paper which showed that

$$\exists x < 10^{10^{10^{34}}} : \pi(x) > \mathrm{li}(x)$$

where $\pi(x)$ is the number of primes $< x$ and $\mathrm{li}(x)$ is the logarithmic integral function.

Any advance?

00O00

Dear Marion, Thanks for your reminder - I shall be delighted to receive another years issues of M500; I am going to use some of the ideas I have filched from them in Puzzles when I find time to look them over. Yours David Wells (Games & Puzzles)

Ed - David is one of our two non-OU subscribers and is delightedly renewing. This letter must be worth recording.

oooooooo

OOH!

The ova l is a sort of circle with personality. When we look at an oval it does something to us; a circle leaves us cold, especially the Arctic Circle. There is something about an oval that is very refreshing. But it must be a new oval; a used, or secondhand oval is quite unexciting and has no particular use except for putting next to a similar one so people can say: "Look at those two old used ovals!"

One reason we are prone to like ovals better than circles is that all circles are precisely the same shape, while an oval can be different from all other ovals and still be oval. If you are unfamiliar with what an oval looks like, try sitting gingerly on a fully blown-up round toy balloon. What, you are sitting on is an oval, unless you sit down too hard. In that case, what you are sitting on is probably a floor.

In a recent nationwide survey, conducted clandestinely by the OOO*, it was found that ovals are preferred, 12 to 10 over any other known shape of anything. This may not be conclusive but it certainly shows a trend - a trend towards the establishment of a new law abolishing surveys.

Lytton Jarman

*O-u Oval-lovers Organisation.

ooooo

A GROUSE

Why always put the contributor's name at the *end* of his work? I like to know who wrote something before I read it, so I can allow my prejudice free reign - and 'real' mags do it, so why not ours?

Roger Bridgman

(Ed - OK I'11 try it out next issue; if I remember!)

ALICE'S ADVENTURES IN BRUMILAND

(with apologies to the only readable mathematician)

All in an Aston afternoon
Pull leisurely we glide;
When our slide rules with little skill
By little arms are plied:
While tutors' hands make vain pretence
Our wanderings to guide.

Alice was beginning to get very tired of sitting in Stafford Tower and having nothing to do; once or twice she had peeped into the units all the other students were reading, but they had no pictures or conversations like her Arts course units had, "and what is the use of a unit" thought Alice "without pictures or conversations?"

So she was considering whether the pleasures of a drink would be worth the trouble of getting up and finding the bar, when suddenly a group of Mouths, with bloodshot eyes rushed past her. Burning with curiosity she ran across the campus after them, and was just in time to see them disappear into the Main Building, saying "O dear, O dear, we shall be too late." Alice went in after them, never once considering how in the world she was to get out again.

Once in the entrance hall, she saw a door marked "lift". She entered and it immediately started going up, up, up – would her rise never come to an end? Suddenly it stopped with a bump and the door opened to reveal a long carpeted corridor, and from the distance the mumble of low voices. As she moved closer she could almost pick out words, but they seemed unintelligible. "Constitution versus quorum," she heard. "Oh dear," she thought, "have I dropped into D332 by mistake? or possibly with all that Latin it's A291."

At the end of the corridor was a long room, furnished with armchairs. There was also a bar, but it was closed. All the occupants of the room appeared to be arguing as to the probability of having a constitution without a quorum.

"P(C | -Q) > P(Q | -C)" said Alice.

"That's not quite right," said the Caterpillar; but at that very moment the White Rabbit rushed in.

"This isn't the Aston Suite," he said, and rushed out again.

"Then how do we get a drink?" asked Alice.

"Why," said the Dodo, "the easiest way to explain it is to do it," and immediately everyone got up and started rushing round the room, eventually all leaving along the corridor by which Alice had entered. There was no "One, two,

three, ..., *n*" like races at home, but they all began running when they liked, and left off when they liked, so it was not easy to know when the race was over.

As they tore along one corridor, Alice saw a big sign saying "Floor 8."

"But we haven't gone up any stairs, yet we started on floor six," cried Alice. "Even if the corridor is an inclined plane we should have passed through floor seven."

"Floor 7 - No Text," gasped a passing Mouth; but before Alice could query the logic of this she was astonished to see at the end of the corridor another large room just like the one they had left, but with the armchairs on the left instead of the right, and yet another closed bar.

"I must have passed through the looking-glass," she thought."I wonder if I should hold my units up to a mirror, whether the words would go the right way again?"

"Start thinking about things and you'll get left behind," grinned a large Cheshire Cat.

"Would you tell me, please, which way I ought to go from here?"

"That depends a good deal upon where you want to get to," said the Cat.

"I don't much care where –" said Alice.

"Then it doesn't matter which way you go," said the Cat.

"– so long as I get *somewhere*," Alice added.

"Oh you're sure to do that," said the Cat, "if you only walk long enough;" and vanished.

Finding herself alone, Alice ran back along the corridor, but clearly she must have taken a different turning, for there was in front of her four of the strangest lifts she had ever seen. Continuous belts of platforms just big enough to allow one person to stand on were moving continuously, two going up, two going down; and presumably to go any further she would have to step into one.

"I wonder what happens at the top?" she asked of the crowd around her. At this moment someone indeed stepped onto one of the upward moving platforms and disappeared from view.

"The platforms must pass over a big wheel at the top and come down the other side upside-down," suggested someone.

"That would be dangerous for anyone who did not know how many floors there are," argued another. "There must be a device for ejecting you at the top floor."

Other Mouths quickly joined in the discussion, but Alice, who had been assured that Mouths were the most logical of creatures, was surprised during this

display of deduction to see many other mathematicians, despairing of either a drink or a solution, were quietly stepping as if at random into lifts going either up or down, as if the only way to solve a problem was by the method of exhaustion.

“There are stairs as well, you know,” said the Mouse, and deductionists and cowards alike rushed gratefully down them.

“I’m glad my solution is non-assessed and non-examinable,” thought Alice, “but I'm beginning to wish the same were true of everything.”

The stairs down led to yet another hall, and there, at last were chairs, smiling faces, and an open bar.“At last,” cried Alice.

“The Court will assemble in the next room,” cried the Queen, and Alice found herself hurried out of the bar into yet another large room with no drinks.

“Are we to have a constitution?” asked the King. “Let the jury consider their verdict.”

“No, no,” said the Queen. “Sentence first, verdict afterwards.”

“Quorum, quorum, quorum,” growled the assembly.

“Stuff and nonsense,” cried Alice loudly.

“Off with her head,” the Queen whispered, having quite lost her voice. Nobody moved.

Alice quietly tiptoed out of the hall towards the bar, and later, refreshed, contemplated the strange adventures she had had that afternoon, and the strange companions she had found in Brumiland.

John Williams

xox

“In the proposition ‘the cat is on the mat’, ‘the cat ‘designates the cat, ‘on’ designates the relation of being on, and ‘the mat’ designates the mat: the proposition asserts that the first (the cat) and the third (the mat), in that order, are related by the second (the relation of being on). The fact that the cat is on the mat consists of the cat and the mat, related so that the former is on the latter.”

George Pitcher (ed.) *Truth*, p. 11.

P I

I am about to reveal my ignorance (despite Grade 2) of all things concerned with M231.

According to John Machin (1706) in M500/26 we can approximate π by the series:

$$\pi = 16 \arctan 1/5 - 4 \arctan 1/239.$$

Now my Prinztronic 3001M calculator works only in degrees, because its degrees – radians key has been used for a pre-programmed Stop Watch device, so I needed a value for π to use this formula involving arctans, since

$$16 \arctan 1/5 - 4 \arctan 1/239 - 179.99998 (\neq \pi). \qquad 1$$

Admittedly it is close to 180 = π (in degrees). Without a value for π I needed a value for 1 radian = $r = 180/\pi$, by definition.

I would like to know why, if one already has a value for *r*, one would wish to use this complicated approximation to π. If one can use trig series in radians at all, then surely it is simpler and more accurate to say $\pi = 4 \arctan 1$, which it is.

The odd thing is that, according to my Prinztronic 3001M,

$$4 \arctan 1 = 179.99996, \text{ and not } 180. \qquad 2$$

(Presumably 'they' used a value for π to obtain arctans and the rest?)

When I illicitly used my 'π' key to obtain a value for *r*, I obtained $180/\pi = 57.29578$, and then (1)/*r* and (2)/*r* both give π = 3.14159 to 5 decimal places, after which they differ. After more experimentation, mainly using Parker's Special Sloppy Method of M500/16 due to similar inability ever to understand M100 Unit 2, pretending that I recalled *r* = 'approx 57.3', I obtained (l)/57.3 = 3.1413609 and (2) 57.3 = 3.1413605. Both are correct to only 3 decimal places, not surprisingly.

So, John Machin, please write to M500 from Up There Where All Good Mathematicians Go and tell me how you were able, in 1706, with no electronic devices, to use arctan at all. If you could use arctan in radians you must have been able to know a value for π from the identity $\pi = 180/r$. And why did you *prefer* (1) to (2) since (2) actually defines π??

Maybe all you were saying, with the delight of a discoverer, was that $4 \arctan 1/5 - \arctan 1/239 = \arctan 1$? My 3001M would agree with an absolute error of 0.000006, which is pretty good, but surely there must be an infinite number of such arctan identities? Why leap into all the textbooks, let alone M500, with your approximation?

Marion Stubbs

ADVICE FOR NEW STUDENTS

Like Joyce Moore (M500/26) I did M100 21 years after School Certificate, with only some practical maths in between.

It was therefore a shock to open the first unit and come across strange words like ‘domain’ and ‘mapping’.

My advice is to forget previous maths and do not waste time and energy translating everything into a known framework.

Take the course as it comes and enjoy it. It may not all be crystal clear, but after all, it is a foundation course for future years, and a good one.

Brian Woodgate

SUMMER LIGHTNING

SM351 apart, no third level maths course offers a Summer-School component. My random proddings seem to suggest that the Maths Faculty has guessed that a Summer School component frightens intending students away from that course. I suspect, on the contrary, that maths students obtain sweetness (sometimes) and light (always) from their Summer Schools.

A joint school for M231 and M332 seems to me natural and for M321 and MST282, feasible. If MOUTHS readers agree that the Summer School boosts morale, acumen and possibly numbers, should they not now represent their feelings to the Faculty before it is too late, even for 1977?

Patrick Sharkey (Ml00 Cllr)

AND WHILE WE'RE DIGGING AT THE OU...

I saw an OU degree certificate which is extremely disappointing. It doesn't list the courses and is the plainest of documents. Certainly it is the poorest of any literature produced by the OU. Has anyone officially complained? You get a better certificate for a bronze medal in Ballroom dancing.

Lytton Jarman

A MILLION

How to see a million things all at once: make up a 1 metre square of millimetre graph paper. Very good for primary school children.

David Wells (Games and Puzzles)

SOLUTIONS

25.2 CUBIC HYPERCUBE

Prove the product of 4 consecutive integers is not a cube.

Assume $q^3 = (n-1)n(n+1) = n(n-1)(n^2+n-2) = n^2(n+1)^2 - 2n(n+1)$

$\therefore\ 1 + q^3 = (n(n+1)-1)^2 = (q+1)(q^2-q+1)$.

Now $q^2 - q + 1 = (q+1)^2 - 3(q+1) + 3)$ so that the only possible divisor of $q+1$ and $q^2 - q + 1$ is 3; but $3|q$ by assumption so the two have no common divisor. Since their product is a square we have $q + 1 = a^2$ and $q^2 - q + 1 = b^2$ (*) from which $a^4 - 3a^2 + 3 = b^2$ or $(2a^2 - 3)^2 + 3 = 4b^2$ which involves $b^2 = 1$, and from (*) $q = 1$ or $q = 0$. But $q^3 \geq 4!$ hence $q \geq 3$ and this contradiction falsifies the hypothesis.

Steve Murphy

26.1 FOUR FOURS II

$71 = \frac{4!+4.4}{.4} = \frac{4!\surd - \surd.4}{.4} = \frac{\dot{4}\sqrt[\dot{4}]{4} - .\dot{4}}{.4}$. Bill Shannon

$71 = (44 + 4! + \sqrt{4})^{+}$. (See M202). Eddie Kent

26.2 TOP PRIMES

Find the highest prime of the form $6k + 5$ (a), $+1$ (b), $+3$ (c).

There is a theorem attributed to the 19th century German mathematician LejeuneDirichlet, which states that, if the greatest common divisor of a and b is 1 then the set $\{an + b: n \in \mathbb{J}\}$ contains an infinite number of primes. (Note that the gcd of a and b must be 1 for there to be more than one prime in the set.)

Chris Rowley (Staff)

26.4 THE PIG

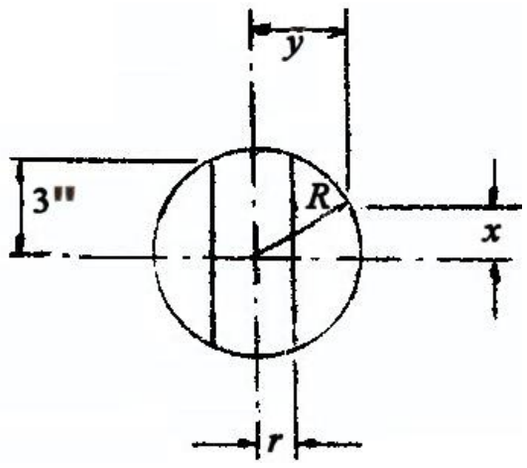
Show that if $10q + p$ is a square and q is odd, $p = 6$.

q odd implies $10q + p = 20d + 10 + p = (20d + 8) + 2 + p = n^2$.

Since every integer is of the form $2m$ or $2m + 1$, every square is congruent to 1 or 0 modulo 4 and so $n^2 \in \{1, 4, 5, 8, 9, ...\}$ and $p \in \{2, 3, 6, 7\} = A$. Also every integer is of the form $5m$, $5m \pm 1$, $5m \pm 2$ and each square is congruent to 0, 1 or 4 modulo 5. $\therefore\ p \in \{1, 4, 5, 6, 9\} = B$. $A \cap B = 6$.

Tom Dale

26.3 A BORING TEASER



Volume left in annulus, V, = $\pi(y^2 - r^2)dx$.

But $y^2 = R^2 - x^2$ and $r^2 = R^2 - 3^2$, so that $V = \pi(9 - x^2)$ which is independent of R.
Put $r = 0$ then $R = 3$ and $V = \frac{4}{3}\pi R^2 = 36\pi$ (cubic inches).

Dennis Hendley)

PROBLEMS

27.1 PERRY

a) Prove that he construction on page 6 works.

b) Prove the assertion $1 + \tan\theta = (1 + \tan\phi)^n$, where θ and ϕ are as given on page 6.

27.2 TWENTY QUESTIONS - Bill Shannon

If I think of a number which you are to determine by asking me no more than twenty questions, the answers being only yes or no, what is the largest number I can choose which can be determined in 20 questions or less?

27.3 FIND THE NEXT TERMS. Next two terms and rule wanted.

a) 1, 2, 4, 9, 11, 13, 29, ... (For M201 students) - Jeremy Humphries.

b) ¼, ½, 1, 3, 6, 12, ... - Tom Dale.

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INNER PRODUCT: I told our ed - who is EDDIE KENT in case you hadn't noticed! - that I needed 2 pages for MOUTHS, but indeed it turned out to be 1½, so room for news of the latest Stubbs creation.

DICE STAR TREK is a calculating Space wargame for ONE player. Must be fond of arithmetic or like playing with a calculator. Many readers must have played STAR TREK ($STTR1) on computer terminals, at disastrous expense to the OU. I asked *Games & Puzzles* magazine if a non-computer version existed. Not to their knowledge, they said, but send us yours this minute, please. 6 weeks later it is done. STAR TREK lasts for hours by computer; DICE STAR TREK lasts for days, weeks or perhaps months without one, every minute packed with +, –, × and ÷ and an occasional square root if you like that sort of complicated mathematics, but sq. roots are optional! And my price, folks, for this unique masterpiece of simulation is a modest 50p, which *includes* postage. The postage alone will be about 16p. I have run off 150 copies anticipating your habitual enthusiasm, and can produce more if necessary. You need little except dice (to produce random numbers) and a chessboard and a lot of paper. You can put it down and pick up whenever you like. It does NOT look like a computer program, but more like a book.

Marion Stubbs (address on back of cover, of course!)