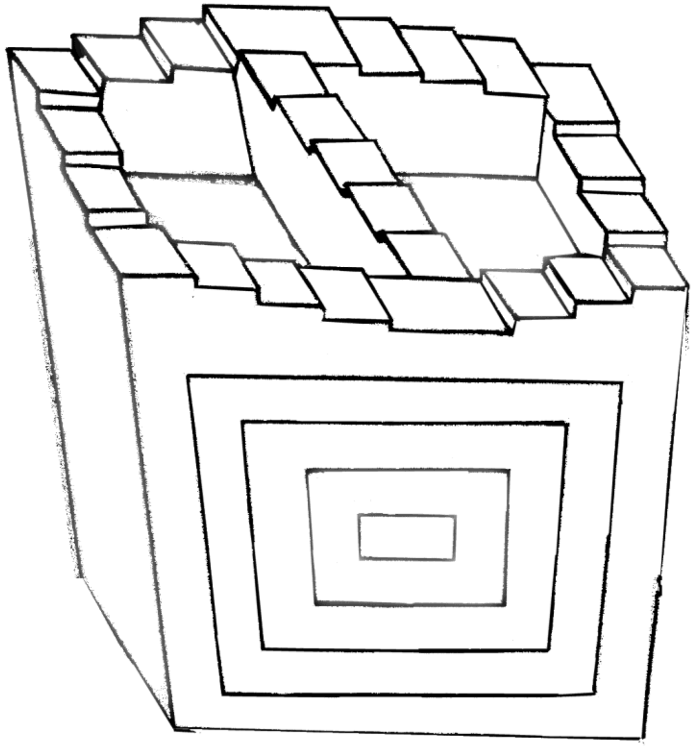

M500 30

M500 is a non-profit student-operated and student-owned magazine for Open University mathematics students and staff, and for any others who are interested. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct but invite criticism and comment. Articles submitted for publication should normally be less than 600 words in length, although subjects which require more space may be split by the authors into instalments.

MOUTHS is a list of names addresses and telephones, together with previous and present OU courses, of voluntary members; by means of which private contacts may be made by any who wish to share OU and. general mathematical interests or who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned, and do not necessarily represent those of either the editor or the Open University.

Tony Brooks: The cover design is based on the impossible staircase which first appeared in *The British Journal of Psychology* in 1958 and was used by the artist M. C. Escher in his work *Ascending And Descending*. My double version has some interesting properties. If one walks around the outside then it behaves like a normal staircase. But should one get tired of such a boring and conventional way two alternatives are possible. Using the centre steps and the right half of the staircase one can permanently ascend by going round anticlockwise; and then one can descend by walking anticlockwise around the left hand half.

M500 is published by Marion Stubbs.

Contributions for M500 should be sent to the editor - Eddie Kent.

Subscriptions, address changes, MOUTHS data, and membership details of any kind to the membership secretary - Peter Weir.

Material not covered by the above to Marion

PLEASE NOTE THAT ANYTHING NOT COVERED BY THE ABOVE WILL BE REGARDED AS POTENTIAL MATERIAL FOR PUBLICATION UNLESS MARKED "PERSONAL"

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LETTER FROM AMERICA

Lewis Johnson

In March 1975 I found myself on the campus of the University of Massachusetts (U-Mass to the initiates), rather to the detriment of a good start with SM351 and MTD241. I was, of all odd things, taken up by the Graduate Biology School and came away with the following problem

A population is defined as a set of Polymers $\{P_i\}$. P_i consists of i + 1 nodules connected by i links, rather like equispaced beads on a string. The population evolves in time from an initial state in which we have N similar polymers $P_{n.}$ These decay to polymers of lower order by severance of the links.

The probability of any link being severed in a given time is constant. In other words, given n links the number severed in one second is An where A is a constant. All links are equally likely to be ruptured.

No polymer suffers more than one break at the same instant.

After a time the population consists of a mixture of polymers P_i where i = 0, 1, 2, ..., n. The problem is to determine the distribution of P_i at time t, i.e.: x_i where $x_{i\,=}$ number of P_i at time t. I think I have a solution.

We would 1ike to estimate the population parameters A and n. nA is thought to be about 0.5 so that after the elapse of a reasonable time it is a rare event to see a polymer of order higher than P_4. The subsidiary problem is then to determine n from a count of P_0, P_1, P_2 and P_3 with t unknown.

Given nA = 0.5 (very tentative at the moment) then the time constant is 2 seconds and the higher polymers will virtually all have vanished after 10 seconds. The experimental times involved are much longer than this which explains why polymers of higher order than P_4 are rarely seen.

(Unless some nasty-minded reader falsifies my solution this will be published as part of a biological study, but that will be some time in the future and in the States.)

MATHEMATICAL PERIODICALS

John Warner

Since this is the only time I have managed to put pen to paper since starting your excellent magazine, I thought I may appeal to other readers for any information concerning other mathematics magazines which would supplement the enjoyment I derive from M500. I have fond childhood memories of “Mathematical Pie” and “Mathematics Students Journal” but have not heard of them for many years. Perhaps some other readers may like to give a short report on any magazines that they think may interest OU maths readers (or their children).

Ed - Back in M500 15 there was a list from Tony Brooks, with additions. It is below, with additions and alterations.

Acta Arithmetica
Algebra Universalis
Annals of Mathl Logic
Annals of Probability
Communications in Algebra
Discrete Mathematics
Fibonacci Quarterly
Functional Analysis aia (from the Russian)
General Topology aia
Geometriæ Dedicata
Int Jnl of Computer Maths
Int Jnl of Game Theory
Jnl of Algebra
Jnl of Approximation Theory
Jnl of Combinatorial Theory
Jnl of Differential Equations
Jnl of Differential Geometry
Jnl of Functional Analysis
Jnl of Mathl Analysis aia
Jnl of Number Theory
Jnl of Pure and Applied Algebra
Jnl of Symbolic Logic
Linear Algebra aia
Mathematics of Computation
Semigroup Forum
Studia Logica
Tensor
Theory of Probability aia (from the Russian)
Topology
(aia = and its applications)

plus

American Mathematical Monthly - £7.60 a year,
Journal of Recreational Mathematics - £7.15 a year,
Manifold (University of Warwick) - £1 a year,
Mathematical Spectrum (Hicks Building, University of Sheffield) - for sixth forms and undergraduates - 70p a year,
Creative Computing (see m500 25 15) - $8 a year.

Any information about any of the above, or any others, as well as about mathematical books, welcomed.

SOMETHING TO SAY? - Ian Turner

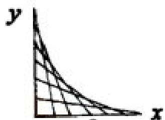
Stung by the last para in 29 but still really on the side of those - if you've nowt to say, shut up! - I will not bore you with a catalog of my OU prowess but say, I think, something fresh. I have *used* my OU M251 in considerable depth in computerising our personnel records at work in a system which I believe to be well balanced between ease of use, cost and efficiency of storage. OU-TSB was rapidly replaced by Basic Plus however. It became operational officially (1 January) and has only one remaining problem. “It” counts 1003 and the manual count is 1002 staff -only 0.1% error perhaps but 100% wrong for one unfortunate.

CONSTRUCTIONS II

Roger Claxton

Michael Gregory's “Constructions” article (M500 28) prompted me to investigate at closer quarters the principles that I have myself used for two-dimensional cotton threaded pictures. These have recently achieved some popularity in kit form (yachts, fish etc), but nine are usually abstract, hideous and constructed from my own sketches. The effects obtainable are numerous even with the basic shape in figure 1. The effect is obtained (figure 2) exactly as in MG's construction.

1



2

The result is a good approximation to a curve even with fairly substantial ‘steps’. There are obvious extensions to the basic shape:

1) x axis of greater or lesser length than y axis;
2) angle between axes greater than or less than 90°;
3) axes ceasing at some point other than $x = y = 0$.

These all have interesting effects on the resultant shape.

But what of the mathematics of the curves? My own abilities don't take me very far but what I have done is offered below.

I took the basic shape shown above with threads joining x = l0 and $y = 1$, $x = 9$ and $y = 2$, ..., $x = 1$ and $y = 10$. The resultant straight lines have equations e_0, e_1, ..., e_9 equal to $x + (9/2)y = 9$, ..., $x + (1/10)y = 1$; i.e.

$$e_n = x + \frac{q-n}{n+1} y = q - n,$$

where q is the initial value of x and n is the step value (absolute) down for x and up for y. The points on the curve are formed by the intersection of line e_n with line $e_n + 1$. The solutions of these equations gives

$$x = q - n - \frac{(q-n)(n+2)}{q+1}\,; \quad y = \frac{(n+1)(n+2)}{q+1}.$$

We can now calculate the intersections for each successive pair of lines to get a table of known points on our curve

n	x	y	n	x	y
0	90/11	2/11	4	30/11	30/11
1	72/11	6/11	5	20/11	42/11
2	56/11	12/11	6	12/11	56/11
3	42/11	20/11	7	6/11	72/11
			8	2/11	90/11

This is symmetric as we might expect, but what function is represented? Here I need help. The second differences of *x* and *y* separately are constant but the Gregory Newton formula (M100 Unit 4 page 44) doesn't help since the steps (the values of *x*) are not of constant length. The three simultaneous equations approach gave unpleasant values for the coefficients of the quadratic. What next?

ESSAYS

Harold Moulson

In M500 24 6, Peter Weir raised the question of mathematical essays in examinations. Having laboured long and filled many many sheets of paper with inane drivel in an attempt to produce a short contribution for M500, I heartily agree that we ought to be more literate than we are. I say “we” as, looking back through the pages to M500 9, a small group of contributors are keeping the pages full. It would appear that there are many like myself who have difficulty translating mathematical thoughts into the written word. M500 offers us the chance to practise without being subjected to too rigorous an examination of our grammatical structures (I think). As to the 15% marks that the question carried, I think that this was about right. Anything less and the question could be comfortably ignored, while anything greater would be a real imposition to those of us who have little practise at writing.

Earlier this year I discovered a comprehensive listing of all the branches of mathematics. This is on pages 700-725 of the Propaedia of the 15th edition of the *Encyclopaedia Britannica*. One reference worth quoting is under Algebraic Topology: “the methods of killing homotopy groups”!

SELF-HELP GROUPS

Steve Davies

I cannot stress too strongly the support and encouragement to be gained from belonging to a self-help group if at all practicable. We (six of us, M100 at High Wycombe) met regularly on Monday nights in each others’ homes whenever there was no tutorial. This weekly contact was very useful in promoting continuity and relieving feelings of isolation. In a group your weak point may be another’s strong point (and vice versa); so mutual help results.

We composed our assignment answers and discussed any differences. On occasions we had to agree to differ.

At revision time we had past papers and plenty of unanswered questions from all through the course to practise on.

Our course results confirmed the value of ‘Self-Help’.

M 1 0 0

Unit 1 from John Hampton

Consider any finite set of $n \geq 1$ elements, $\mathcal{C} = \{x_1, x_2, ..., x_n\}$. How many different functions f: $\mathcal{C} \to \mathcal{C}$ are there? There are n possible images $x_1, x_2, ..., x_n$ for every element x_i (i =1, 2, ..., n) of $\mathcal{C}$ under f. Hence, since $\mathcal{C}$ contains n elements f may defined in n^n different ways.

How many of these have inverse functions? If f: $\mathcal{C} \to \mathcal{C}$ has an inverse function then there are n possible images for x_1 under f, but then there are only $n - 1$ possible images for x_2 under f (since $f(x_2)$ must be distinct from $f(x_1)$), and so on. Hence f may be defined in $n(n - 1)...2 = n!$ distinct ways.

That is there are $n!$ 1–1 functions f: $\mathcal{C} \to \mathcal{C}$ in all.

Unit 2 from Marion Stubbs

On January 2nd 1976, some five years after first struggling with it, I finally conquered M100 Unit 2. It is reduced to two postcards stuck on my cupboard door ready for the onslaught of M351. Peter Weir wrote "I look forward to your cupboard's postcards publication" so here they are, slightly elaborated. M100 students may find it useful sometime later in the year, but meanwhile are advised to press on regardless, morale boosted by knowledge that your predecessors were equally nonplussed, especially if they lacked any previous calculus. Hopefully M201 will find it useful, not to mention any other M351 who never saw the light.

1. e_x = APPROX – EXACT
2. $e_{f(x)} = f'(x)$(APPROX – EXACT) or $f'(x)e_x$
3. $r_x = \frac{e_x}{x}$ *or* $\frac{\text{APPROX}-\text{EXACT}}{\text{APPROX}}$ *or* $\frac{\text{APPROX}-\text{EXACT}}{\text{EXACT}}$
4. $r_{f(x)} = \frac{f'(x)e_x}{f(x)}$ or $\frac{f'(x)(\text{APPROX}-\text{EXACT})}{f(x)}$
5. $E_{f(x)} = |f'(x)|E_x$, where $E_{f(x)}$ is the *error bound* of the image
6. Scale Factor = $f'(x)$
7. Newton Raphson method (glibly tossed aside in six lines on page 34) comes up again in better detail in Unit 12 and can be skimmed hastily, but for the record the rearrangement of $f(x) = 0$ is:

$$x = \frac{xf'(x) - f(x)}{f'(x)} \quad or \quad x = x - \frac{f(x)}{f'(x)}.$$

M100 - please don't bother about all this until you know what $f'(x)$ means. I believe it can be translated as dy/dx which some M100 may know, but as I only read but cannot write Leibniz notation I am none too sure. I also fancy that e_x is some people's old friend δx but again shaky on translation. Hopefully Leibniz will teach me himself how to use his notation in AM289, if M351 leaves me any time.

SOME LETTERS

From Michael Gregory (to Peter Weir) - Congratulations on your election. I imagined you in the weeks before Christmas searching through piles of cheques and postal orders for your Christmas cards. I hope the job as membership secretary does not become too arduous.

The magazine has developed over the years; I think it is important to preserve a balance between the different aspects / functions, e.g. mathematics / the personal touch; OU / contact between staff and students.

From Michael Masters - In connection with the paragraph in number 26 entitled “Serial Numbers”, my number is E 0355200 and the other details as follows: Region - London; Occupation class - don't know (bank official); Sex - male; Courses - D100 (first choice, not allocated), M100.

(MS - *very* interesting! Male non-teacher in *London* OK at 35,000)

From Tom Bale - I seem to have busier since the exams than I was before (I think that’s because my wife reckons I should be available for lots more bridge!)

I suggested to Marion once that she should consider registering for VAT, and thus recover the VAT on materials and equipment. She wasn’t very keen as she thought it might mean a lot of extra work (it shouldn’t) - but now that she has some helpers it might be reconsidered. Perhaps it won’t make quite so much difference now though - the magazine as supplied by the printer should be zero-rated. But tax on the £700 of equipment mentioned in issue 28 amounts to £50!

I was intrigued by the solutions to my problem 27.3 (the sterling coins series). It reminds me of a science fiction story I read years ago where some top scientists were gathered to hear a tape recording left by an eminent colleague who had died. The quality was very poor, but it was just possible to infer that he had discovered anti-gravity (or force-fields or some such thing). The scientists felt that what he had done they could do, and indeed before long the rediscovery was made. It then transpired that the whole thing had been a hoax to spur them on. So perhaps if the proper question is put in the right way we’ll have some M500 reader inventing a new algebra or some such thing. (Wasn’t it Euler who left some marginal note that he had solved some problem which no-one since has solved? (No! Fermat. - Ed)

(MS - Yes, I am keen on VAT rebate IF someone would give it to us and IF someone with the knowhow will do all the applications and associated bookkeeping. Now that I have more helpers I also have more members and more work, not less! Parkinson's law I suppose. It is worth paying the VAT just to avoid the extra work, from my point of view. But why would anyone give us VAT rebate when the Charities Commission has already decreed that our aims are not charitable and that M500 is taxable? One of our own Inland Revenue M500 members opines that a tiny profit on Weekend '75 is taxable because it is “not M500”, so what point in applications to total strangers?

(Weekenders please note that I have given each of you $16.\dot{4}2857\dot{1}$p. refund and grateful thanks to each of you for returning it so rapidly as donations towards the ’76 deposit! Hope that solves the problem.)

From Mike Hodgson - Should anyone be interested my advice to new students is as follows: 1 - Try to get into a self-help group. It works wonders for the morale, when you realise that no-one else understands it either. And a collective moan is good for the spirit; 2 - If possible try to go to Summer School as late as possible. That way you can pick more brains on more subjects; 3 - Join M500. You may not understand some (or even any) of it but it is nice to see that maths has a fun side to it.

Information of Efron Dice would be appreciated. I can find no trace of it anywhere.

I enclose also a problem I was given years ago and finally completed when I learnt all about the Newton rule of approximation. However I believe there is a much easier way to arrive at a solution than involving an equation in powers of four. Most people look at it and promise an answer in ten minutes or so. I would like to see this done.

(Ed - Here follow a couple of variations on our “Ladders” problems from around M500 11 and 13. These were all solved using quartics, I believe, and iteration methods. Is there a simple solution to this type of problem - two ladders crossing in an alley, ladders leaning against things, etc?)

From Henry Jones - Living as I do where there is no other OU student of mathematics and science, I greatly relish the chatty bits in M500 as well as the actual mathematics. Indeed, I have come to feel a sustaining affinity with the correspondents, especially those who are not unkindly disposed to eccentrics like myself.

From Richard Ahrens - It seems to me that there is little point in publishing any problems based on Don Harper's “An Axiomatic Approach to Algorithms” (M500 29 1) as the article is completely incomprehensible. I will have to have the following questions answered before I can expect to think about a problem.

1) Why does θ - the output set - never get a mention after the first definition?
2) *S* is not given as an ordered set but subsequently seems to be ordered (at least partially) although the order relation is not given except possibly in a garbled form in example 2.
3) Is *F* a function or an M100 mapping?
4) The appearance of F^{-1} in the article suggests that *F* is one–one but that was never stated.
5) Why is “termination” defined and then never mentioned again?
6) The input set *I* is ignored as completely as θ, but I suppose x_0 is a member of *I* in the subsequent passages.
7) The word integer appears several times although we were not told that S consisted of

integers and to further confuse me integer seems to include vector in the examples.

Perhaps Don Harper made the mistake of writing as though he expected people to read and understand what he had written. Had he adopted the attitude of S Pantos, who obviously intends that no word he writes should be meaningful, he could have avoided criticism.

(Ed - There are no problems this month on the subject because Don Harper has written to say some of the article was missing. Here, then, is the emended example 3:

3) It is fairly easy to show that if $K, L, Q, R, \in \mathbb{Z}^+$ then the transformation

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} K & RL \\ L & K + QL \end{pmatrix}^n \begin{pmatrix} K \\ L \end{pmatrix}$$

satisfies the axioms; and consequently it can be used to generate all solutions to the equation $x^2 + Qxy - Ry^2 = 1$ from the initial solution (K, L) if it exists. i.e. $S = \{x, y\colon x^2 + Qxy - Ry^2 = 1; x, y \in \mathbb{Z}^+\}$, $x_0 = (K, L)$ e.g. for $x^2 + xy - y^2 = 1$, $x_0 = (K, L)$, $Q = R = 1$.

$$F\colon \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

n =	0	1	2	3
x_n =	1	2	5	15
y_n =	1	3	8	21

F satisfies the axioms therefore all positive integer solutions are in the above sequence.

Don Harper.)

From Jeremy Humphries - About the exams (M500 29 14). You can ask now. The marking is very lenient or I got somebody else's. (M201)

Prom Peter Weir - The error analysis on e was interesting (29). Like decorating, not half so bad once you get started.

π should keep the wires going for a year more. I wonder what Machin would have said about things like $\sqrt[i]{1} = e^{-\pi/i}$ etc.

From Tony Brooks - I am now living in Belgium and am therefore somewhat cut off from contact with other OU students. Hence if any M500 reader feels like corresponding directly with an exiled OU mathematician cum philosopher then please do so. I don't promise a quick reply but all letters would be welcome.

I shall not be doing any maths courses in 1976. I have been persuaded by Professor Godfrey Vesey that I need to do A402 before I can do a higher degree in philosophy with the OU, therefore I shall be faced with the problem of listening to the radio broadcasts (there is no TV for A402). If any kind reader would be prepared to record the A402 broadcasts on cassettes or open reel tape and send them to me I would be most grateful. I could arrange (by fair means or foul) to reimburse the cost of postage.

MONGE'S SHUFFLE

Marion Stubbs

(This piece is supposed to be intelligible to M100. Complicated expressions near the beginning are intended to scare you to death. They may be skipped without compunction. I have said all this because M100 often think that items in M500 are written for and by some elite, which is not so.)

In M500 26 Eddie Kent described Monge's shuffle. Take a pack of cards. Place the top card under the second, the third card under both of them, the fourth card on top, and so on. When there are 2p cards in the pack we can define a map which sends a card from its initial position x_0 to the position $f(x_Q) = x_1$ by:

$$f(x) \to \begin{cases} \tfrac{1}{2}(2p + x + 1), & x \text{ odd} \\ \tfrac{1}{2}(2p - x + 2), & x \text{ even} \end{cases}.$$

Shuffling the pack m times moves any card from its original position x_0 to the position $f^m(x_0) = x_m$ (i.e. repeating f m-times). Then, according to Monge, the following relation holds:

$$2^{m+1}x_m = (4p + l)(2^{m-1} + (-1)^{m-1}(2^{m-1} - 1)) + (-l)^{m-1}\, 2x_0 + 2^m + (-1)^{n-1}.$$

In M500 26 Eddie omitted a summation sign in the middle of the right-hand-side of this equation, but I have now expressed the sum concerned as $(2^{m-1} - 1)$ which is a lot simpler.

Using a pack of 6 cards, which I shall call a 6-pack, this relation is quickly shown to be useless; neither Eddie nor I can locate the error. I think but cannot prove that a simple single equation of this type cannot cope with all possible $2p$-packs, and as far as I can see Monge's equation does not hold for any $2p$-pack! This does not prevent us from exploring the shuffle itself in other ways. It is useful to start with a 6-pack because of its convenient size. Since $2p = 6$, $p = 3$.

Take the 6-pack and shuffle once. If the values of the cards initially matched their positions, so that a card with face-value 3 was in the 3rd position for example, we can make a table showing where each card moved to, viz.:

1	2	3	4	5	6
4	3	5	2	6	1.

Card 1 is now in position 4, card 2 in position 3, agreeing nicely with the defined function f, since $f(1)$ should be 4, $f(2)$ =3 and so on.

Wow, just for the hell of it, rewrite this permutation in the form (1 4 2 3 5 6), which means that 1 maps to 4, 4 to 2, 2 to 3, 3 to 5, 5 to 6 and 6 maps back to 1. For inscrutable mnemonic reasons I shall call this perm VALPOS_6. The inverse of this perm, or a shuffle which would restore the original order, I call POSVAL_6 = (6 5 3 2 4 1), namely VALPOS_6 written backwards. More correctly POSVAL_6 should be written (1 6 5 3 2 4) with the 1 shunted round to the front.

Then VALPOS_6 = (1 4 2 3 5 6) tells us that

	GIVEN = VAL		RESULT = POS
the card with value	1	is now in position	4
	4		2
	2		3
	5		5
	5		6
	6		1 ,

while POSVAL_6 = (1 6 5 3 24) tells us that:

	GIVEN = POS		RESULT = VAL
the card in position	1	has face-value	6
	6		5
	5		3
	3		2
	2		4
	4		1.

In each case the required Result is found one place to the right of the Given in VALPOS or POSVAL, and the mnemonics now seem less inscrutable, perhaps.

Now shuffle again = 2nd shuffle. Without looking at the faces of the cards you can Amaze Your Friends by telling them positions or values according to their commands, reading off from VALPOS or POSVAL (written on your cuff?) TWO places to the right. At this stage we may as well abandon POSVAL and obtain POSVAL Results as required by reading off from VALPOS backwards. Test yourself by writing out the table for GIVEN = POS and RESULT = VAL after the third shuffle, then do the shuffle and check the actual cards against the table. In this case the results will be 3 places to the right or left of the given, depending whether you want VALPOS or POSVAL.

In general call the state after the mth shuffle $(\text{VALPOS}_6)^n$ and read off the results m places to right or left of the given. Needless to say, $(\text{VALPOS}_6)^6$ restores a 6-pack to its original order.

Next explore a 12-pack in the same way. After one shuffle the table is

1	2	3	4	5	6	7	8	9	10	11	12
7	6	8	5	9	4	10	3	11	2	12	1.

Rewrite in convenient form VALP0S_{12} = (1 7 10 2 6 4 5 9 11 12) (3 8), which means that all except 3 and 8 map in the way previously described, while 3 and 8 have a simultaneous private mapping of their own. Test a few shuffles and see. After every even number of shuffles 3 and 8 are back in their original positions.

$(\text{VALP0S12})_{10}$ =10 shuffles restores a 12-pack to its original order. For results after the mth shuffle read off m places to the right (or left) within brackets as for the 6-pack.

Finally try other 2-packs. Is there a way of calculating VALP0S_{2p} without ever handling a pack? It's a straight question. Can you make Monge's equation work? I cannot.

T H E M 5 0 0 S O C I E T Y

It has been pointed out by an M500 member who has lots of experience in chartered accountancy that if the turnover of The M500 Society passes £5000 it is liable for VAT at 8%. Since we hope the weekend will attract 150 people @ £25 each and there are 400 odd subscribers to this paper we are obviously approaching the edge. It is therefore essential that the two aspects are separated. The simplest solution is that the Weekend remains a dictatorship run by Marion while M500 floats off on its own. To this end The M500 Society Constitution is enclosed. You will see that no part of it is binding for longer than it takes to vote it out. Please send in your reasoned dissention by return.

STATUS

Peter J Allender

Since I work in an engineering design department I am taking engineering topics in 1976. Nevertheless, this would probably be valid for applied mathematicians since “Mechanics of Continuous Media” is a London honours topic.

It might be worth mentioning that the Institution of Mathematics and its Applications has a firm council ruling that three full credits in “mathematical” subjects at second or higher level from the OU is acceptable for Licentiate of the Institution. This is, to my knowledge, the first positive statement by a professional institution on the OU. “Mathematical” should be interpreted as either a course containing an M or a course with high mathematical content.

SCIENCE MUSEUM COMPUTING GALLERY

Marion Stubbs

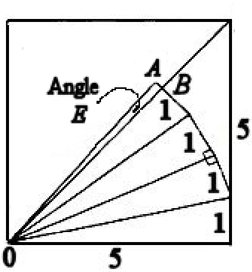
Students who entered the Science Museum competition in 1974 at the instigation of SCS, for programs suitable for the proposed computing gallery at the Science Museum will be interested to know that the gallery is now open. ‘The museum staff are aiming at “reasonably intelligent teenagers”, but with much to interest both younger children and elder computer cognoscenti,’ according to a late-1975 Times report (from which the date has been scurrilously torn by the non-bibliographically-minded reader who sent it to me). Amongst the delights of Babbage machines and Manchester University's differential analyser (1935-vintage) resides an armour-plated 1970’s computer, supposed to be small-boy-proof, which can be used by visitors. Sounds like worth a visit to South Kensington for individuals and school parties.

DEGREES FOR WOMEN: Fair, fairer, fairest.

(*Punch*. 5 Feb 1876) - from MS.

P I

First, *from Peter Weir*: a picture and a problem:



Tan $E = \frac{5/239}{5} = \frac{1}{239}$; since $AB = 5/239$. This "can be proved using only the geometry of similar triangles".
Can it indeed!
Any offers?

A formula leading to π which converges faster than Machin's is one due to Ramanujan and given in *Computer Bulletin* # 2. It is below:

$\frac{\pi}{4} = \frac{1123}{882} - \frac{22583}{882^3}\cdot\frac{1}{2}\cdot\frac{1\cdot 3}{4^2} + \frac{44043}{882^5}\cdot\frac{1\cdot 3}{2\cdot 4}\cdot\frac{1\cdot 3\cdot 5\cdot 7}{4^2 . 8^2} - \ldots$ where the numerator is $1123 + 21460n$.

From Bill Shannon - I think perhaps Marion has been led slightly astray by the sloppy use of the word "nearly" in M500 26. Machin's formula is neither an approximation nor an identity but an equation connecting three transcendental numbers. Machin was interested not in arctan but in finding convergent series for π. His method is ingenious since all he needed to know about arctan 1/5 was that it was slightly greater than π/16.

Don't underrate the old time mathematicians! As long ago as 1596 ten-figure tables of all 6 trig functions were available They were compiled by Rheticus with the aid of a team of number crunching human computers for angles at intervals of 10 seconds. It took 12 years and the final volume occupied 735 pages each 9 × 15 inches.

For those interested there was a calculator supplement in *New Scientist* 13 November 1975. For calculators without a π key 355/113 gives an easily remembered approximation with a relative error of less than one in ten million.

(See *Makers of Mathematics* by Hooper, and *Mathematics and the Imagination* by Kassner and Newman.)

Mathematics, however ingenious, is not a proper substitute for knowledge.

A. M. Lee, *Applied Queuing Theory* MacMillan 1966
(from Rosemary Bailey)

POCKET CALCULATORS

Peter Needham

I have just obtained a Sinclair programmable; you can program 24 operations on it and I wonder if anyone can think of an interesting program or game that can be done on it.

It's functions are

+, –, ×, ÷ , $1/x$, x^2, $-x$, sin, cos, arctan, $\sqrt{x}$, log, antilog, memory, store (STO), recall (RCL), exchange ($x \leftrightarrow m$) and VAR; this displays the result to be observed or number to be entered. Putting a number in the program requires 2 operations plus one for each number. e.g. if you wish 73 in the program then / ⊂ / 7 / 3 / ⊃ / thus requiring 4 operations.

Example of a program: – Subtraction game.

/ enter / $x - m$ / rcl / – / var / $x \leftrightarrow m$ / enter / ⊂ / 4 / ⊃ / – / rcl / + / sto / enter / var /

The operator selects a starting total and then subtracts 1, 2 or 3 from the total. The calculator does the same and play continues alternately.

The object is to leave your opponent with 1 in the display.

(Executions	/ Total sto enter /	n = 1, 2 or 3
	/ n exec /	operator plays
	/ exec /	calculator plays
	⋮	.)

CREATIVE COMPUTING:

Nuclear Research	Das Whizkidden grupe
Administration	Das Oudtgeschmardten grupe
Rocket Engine	Firenschpitter mid schmoken-und-schnorten
Management	Das Ulzerenbalden grupe
Nuclear Warhead	Das eargeschplitten laudenboomer
Hydrogen Device	Das eargeschplitten laudenboomer mit ein grosse hollengraund und alles kaput.

(reprinted with permission from *Creative Computing* vol 1 no. 4 May–June 1975, published by Ideametrics, PO Box 789-M, Morristown, NJ 07960, USA.) This issue also contains a full list of *Super Star Trek* with rules and notes and - double wow! - a computer-poster of Mr Spock. It also gives the inside information on algorithms for PONG, or TV tennis, after which you can win; and 76 pages crammed full of program lists, analyses, articles and contests. *CC* seems to be directed towards schools and under graduates. Don't miss it. It is all in BASIC or in English and is published 6 times a year. (MS)

ANOTHER SELECTION OF BOOKS - Michael Gregory

Recreations in the Theory of Numbers, A. H. Beiler, Dover 1964.

This is a book for anyone who solves or compiles numerical puzzles. It contains 103 tables including a factor (24 digits) of $2^{2^{73}} + 1$, and the values and equations of polygonal numbers. There are also lots of useless but readable bits of information, such as the division of numbers into male, female, virile and effeminate.

Applied Numerical Methods, B. Carnahan *et al.*, Wiley 1969.

In 600 large pages numerical methods are clearly explained with detailed examples; topics covered include interpolation, and the solution of systems of equations and of differential equations. The theory of each subject is followed by an explanation and algorithms for the use of selected techniques; listings of Fortran IT programmes with input and output data are also given. Although rather expensive the combination of theory and practical computing will be a worthwhile buy.

Cambridge Tracts In Mathematics And Mathematical Physics, No. 13, *The* 27 *Lines Upon the Cubic Surface*, A, Henderson, Hafner 1911.

If you wish to pursue the subject introduced by Richard Ahrens in M500 7, this gives a clear and exhaustive account. (Ed -This is a reference to Richard's statement, in an article on developable surfaces, that on a general cubic surface there are 27 straight lines, each intersecting 10 other lines.) You can find the "double six" in many books, but most dealing with the 27 lines assume a considerable knowledge of a complicated subject.

Diagrams, A. Lockwood, Studio Vista 1969.

This is not really maths, but illustrates the presentation of data in graphs (including perspective, logarithmic and distorted triangular) and informative diagrams. Many interesting examples are taken from magazines, newspapers and trade literature, with adequate explanation but not theory. You could borrow this from the library if you want to brighten up your assignments.

THE AXIOM OF ARCHIMEDES

Peter Weir

I may be going (gone) daft, but I see a hole in Datta's proof, M500 29 5. In II, where he tries to prove that $\sim p \Rightarrow \sim q$. It is true that $\sim p \Leftrightarrow \exists k \in \mathbb{N}$: $k \in \mathbb{N}$ but $k + 1 \notin \mathbb{N}$; but k need not be the largest natural number – consider 1 2 3   5 6.

The hole in his proof?

PRONUNCIATION

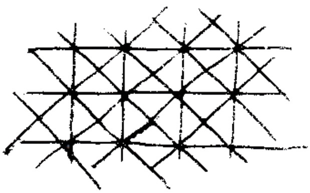
Jeremy Humphries

Said a man to his offspring, “Indeed, Ron
It is obvious that what you need, Ron,
Is a twelve-sided hat.”
(For the boy he begat
Had a head like a dodecahedron.)

PACKAGING

Margaret Corbett

In the November 1974 issue of the *British Journal of Dermatology*, 91, 507 is an article by Schellander and Headington:”The stratum corneum - Some structural and functional correlates.” They describe how they looked at strippings of the outer layer of the skin obtained by pressing a drop of Permabond adhesive on a glass microscope slide on to the skin. The Permabond polymerises at once and the outermost layer of skin is pulled off with the slide. It retains its normal shape and can be looked at through the microscope without further processing. This outer layer of skin is pliable but inelastic. It is therefore folded - in the obvious way, along skin creases, and with finer folds interposed. The basic pattern is either a rectangular lattice subdivided into triangles; or a more equilateral, triangles only, arrangement. The smallest triangles may show round or polygonal subunits each made up of a variable number of horny cells. The horny cells are geometric with either hexagonal or pentagonal outlines.



My question is this: is there any obvious relevant mathematical comment to be made? I suppose these hexagons are the best way of packing solids with a minimum boundary tightly together - a sort of minimum energy solution. But I know hardly anything about hexagons. If you are talking maths with anyone would you like to discuss this? I should be very interested in any ideas which come up.

THOUGHTS UPON RECEIVING MY EXAM RESULTS - from Sinbad

“It’s just not worth it,” said the Queen of Spain,
“Five minutes pleasure, for nine months pain,
Three months rest, then at it again.”

PS Did any of you M202ers see the blue knickers shown by Esther Rantzen in “That's Life”? They appeared to be an unusual example of a Möbius Band.

SOLUTIONS

28.5 *ROLL ME OVER*

Play to get the bottom faces of 8 dice in a square formation on top without sliding or leaving the square.

L = roll to the left, etc.
LURD RULD DRUU LLRD ULD, DRU LDRR UULD DRUL DLUR. 38 moves. This is symmetric and probably minimal. Based on a solution in *Jnl of Recreational Math* volume 7 number 3.

29.1 *NEW YEAR RESOLUTIONS*

Keep numbers 2, 4, 6, 8, 10. Michael Masters.

29.2 *NEWTON'S COWS*

a (a', a'') cows graze b (b', b'') fields bare in c (c', c'') days. What relation connects the nine magnitudes a to c''?

Peter Needham showed that $ac/b = a'c'/b' = a''c''/b''$ which leads to Jeremy Humphries's solution:

$$\begin{vmatrix} b & bc & ca \\ b' & b'c' & c'a' \\ b'' & b''c'' & c''a'' \end{vmatrix} = 0.$$

29.5 *DONUT SLICER*

From 3 simultaneous cuts through a doughnut one gets no more than 13 pieces (including 2 minute pyramids). The formula for n cuts: $(n^3 + 3n^2 + 8n)/6$. - Marion Stubbs and Jeremy Humphries.

(Marion also states that the three cuts will produce 15 pieces if the hole is not perfectly round and 18 pieces if one is permitted consecutive cuts with the pieces rearranged after each cut.)

PROBLEMS

30.1 *SUBSETS* - John Hampton

$A = \{a_1, a_2, ..., a_n\}$ is a finite set of $n \geq 2$ elements. The number of proper subsets of A (i.e. excluding A and $\varnothing$) containing one element only is ${}^nC_1$, containing two elements only is ${}^nC_2$, ,... containing $n - 1$ elements only is ${}^nC_{n-1}$. So the number of proper subsets of A is $\sum_{i=1}^{n-1} {}^nC_i = \sum_{i=1}^{n-1}(n!)/((n-1)!\,i!)$. However, by induction the number of proper subsets of A is $2n - 2$. Prove that $\sum_{i=1}^{n-1} {}^nC_i = 2n - 2$.

problems continued

30.2 *THE OU CAB* - Ken Wogan

If 600 = CAB what is 342,764,853,755,904?

30.5 *POINT CONSTRUCTION* - Bob Margolis

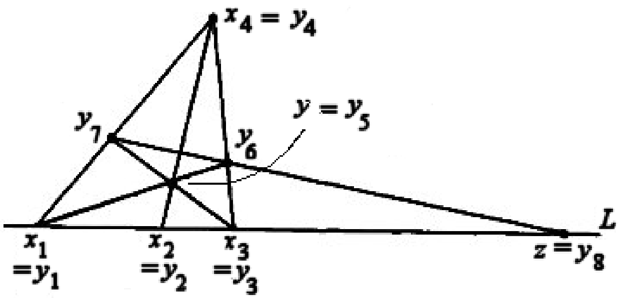
Suppose x_1, x_2, x_3 are three given points on a line L. x_4 is an arbitrary point not on L and y is any point on the line joining x_4 to x_2.

A *construction* is given for producing a point z on L. Prove that z is independent of the choice of x_4.

(In this context a *construction* is a finite sequence of points $y_1, y_2, ..., y_n$ and a set of rules such that $y_i = x_i$ $(i \leq 4)$, $y_5 = y$ and y_{k+1} is obtained by intersecting joins of points already available, i.e. $y_k...y_1$.

e.g.

$$\left.\begin{aligned} y_6 &= (y_3\, y_4) \cap (y_1\, y_5) \\ y_7 &= (y_1\, y_4) \cap (y_3\, y_5) \\ z = y_8 &= (y_6\, y_7) \cap L \end{aligned}\right\}$$ where $(y_i\, y_j)$ indicates the line joining y_i and y_j.



If you can do that, state and prove a more general theorem.)

30.4 *FIBONACCI COPRIMES* - Krysia Broda

Show that any two consecutive Fibonacci numbers are coprime.

30.5 *NEXT TERMS* - Tom Dale *and* Max Bramer!

O, T, T, F, F, S, S, E, N, ?

Editorial

Although there is still a lot of mathematics (of various kinds) on hand we are very short of letters. Nearly every bit of heavy maths that comes in is accompanied by a letter – usually explaining where the idea came from and what the author is doing – which would be ideal for M500; but marked "Personal" or "Not for Publication." Why? Are mathematicians unusually retiring people? M500 exists so that we can all get to know one another and what better way than contributing a piece of chat. Makes for easier reading too over the toast and marmalade.

Min Kent (now aged 14) has just come running in to tell me the solution to 28.3 on page 16 doesn't work. So we checked the one I copied it from in JRM and that doesn't work either. Now perhaps someone would like to try it and see where the mistake is. Sorry about that!

We've been having some votes come in. One person approves the title "The M500 Society": that's 100% in favour so far. And Ian Turner doesn't much care for MOUTHS as a name. I wonder how many people know what MOUTHS stands for. But that doesn't matter much so long as everyone knows what it is.

As a change from the above this is being typed in 1½ spacing. It is obviously better than the usual single spacing from an aesthetic point of view and is probably easier to read; but is it enough better to justify using it throughout the magazine and so getting less in? (Always assuming there is sufficient material to fill up.) I would really like to know before starting on number 31.

Eddie Kent.

Letter to Marion Stubbs: (12 January 1976)

Dear Mr and Mrs Subbs, I have been told about your game of a copy of *Dice Star Trek* by one of my teachers who had it and thinks it is very good. I am including a 50p. postal order and would be obliged if you would send me a copy of this game. Yours Sincerely, A J Wright. (Reading, Berks.)

MS - To said teacher. If you will please let me know who you are I can send extra copies of the STARFLEET COMMAND SIGNALS as and when they are published, for distribution around this school. (I thought they were coming from Shell-Mex!) Other teachers may like to note this; and also to note Prof. Pengelli's latest comment which is "As far as *Dice Star Trek* is concerned I was suggesting that at middle school level it should be used for teaching purposes; in my view mathematics should be fun and that material seems to fit that description nicely." 100 copies of the 2nd impression now exist, awaiting customers, @ 50p.