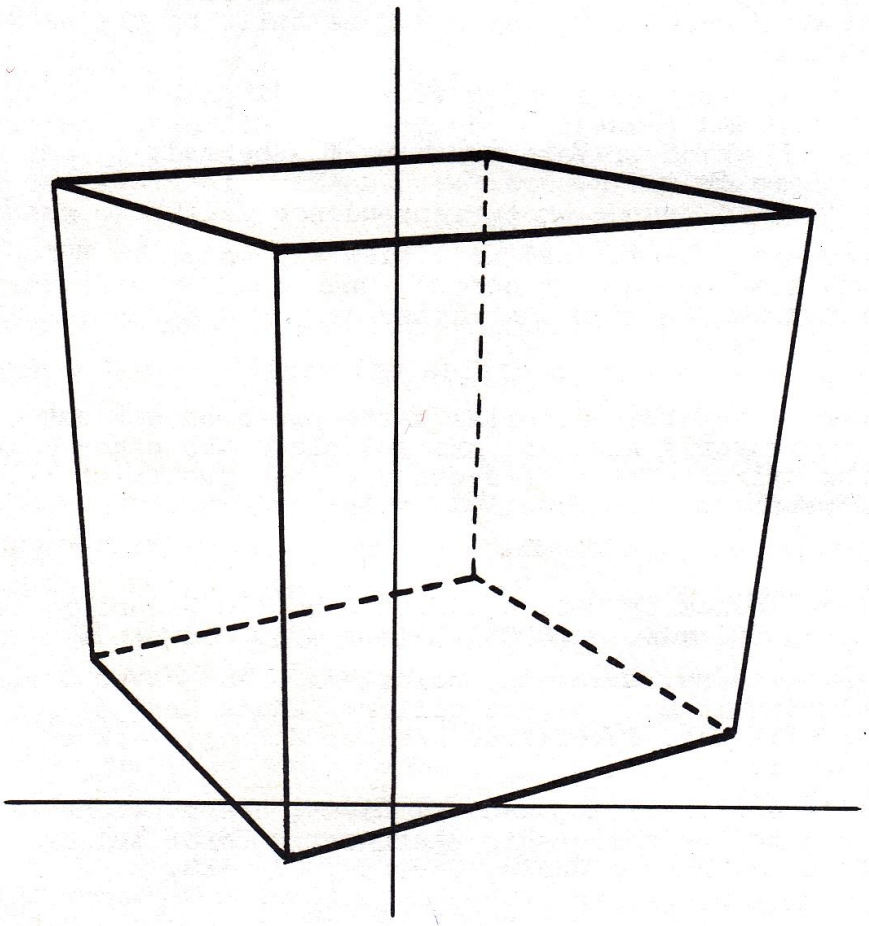


M500 31

M500 is a non-profit student-operated and student-owned magazine for Open University mathematics students and staff, and for any others who are interested. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct but invite criticism and comment. Articles submitted for publication should normally be less than 600 words in length, although subjects which require more space may be split by the authors into instalments.

MOUTHS is a list of names addresses and telephones, together with previous and present OU courses, of voluntary members; by means of which private contacts may be made by any who wish to share OU and general mathematical interests or who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned, and do not necessarily represent those of either the editor or the Open University.

The cover. Not the editorial waste paper basket but a cube. To see it properly shut one eye and place the other 4 inches above the origin (the intersection of the two fainter lines). From a transformation supplied by Lewis Johnson. See p 1.

M500 is published by Marion Stubbs.

Articles, letters, problems, solutions - and cover designs - for inclusion in M500 to the editor: Eddie Kent.

Membership applications, subscriptions, change of address, MOUTHS data to the membership secretary: Peter Weir.

The Treasurer is Austen F Jones.

Cheques and postal orders (which should continue to go to Peter Weir) should be made payable to THE M500 SOCIETY and crossed "a/c payee only, not negotiable" for safety in the post.

Anything sent to any of the above and not marked as personal will be considered for possible inclusion in M500.

M500 31 published March 1976.

Subscription: £3 for ten issues.

Membership: 380 (rising daily).

Printed by Tizengrove Ltd Queens Terrace Southampton.

CUBE

Lewis Johnson

The rows of the matrix D are the coördinates of a set of points $\{x_1, ..., x_8\}$ in $\mathbb{R}^3$. $D' = DM$ gives the coördinates of the set $\{x_1', ..., x_8'\}$ in $\mathbb{R}^2$ on the cover.

$$D = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 2 & 0 & 2 & 1 \\ 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 2 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 2 & 2 & 0 & 1 \end{pmatrix} \qquad M = \begin{pmatrix} 0.866 & 0.250 & 0 & 0.108 \\ 0 & 0.866 & 0 & -0.125 \\ 0.500 & -0.433 & 0 & -0.188 \\ -1.366 & 0.683 & 0 & 1.592 \end{pmatrix}$$

This is a true perspective representation of $\{x_1, ..., x_8\}$ as seen from a point $4''$ from the origin on the axis perpendicular to the paper.

Since, to provide all the required degrees of freedom, the transformation needs a 4×4 matrix, D has a fourth column of 1's to give compatibility of multiplication. Similarly the final matrix D' must have its fourth column reduced to unity (normalised) by suitably dividing through each row.

M is actually the product of five submatrices as follows:

T_1 - translation of the circumscribing cube so as to place its centre at the origin;

Ry - rotation by 30° about the *y*-axis;

Rx - rotation by 30° about the *x*-axis: these operations improve the view and give a three-vanishing-point perspective;

T_2 - translation of –1.5 along the *z*-axis: this ensures the solid is wholly behind the (*x*, *y*)-plane (so as not to get over-the-shoulder projection and need eyes in the back of one's head);

P - projection of each point onto $z = 0$ along a line through (0, 0, 4).

So, finally, D' = DT1RyRxT2P (normalised) and in general these submatrices do not commute.

Notes: (1) no originality is claimed; computer buffs will recognise it. (2)I should be glad to send more detail of the derivation of the matrices to anyone interested.

MEASUREMENT OF CHANGE

Willem van der Eyken

I wonder if any readers are ploughing their way through all these maths courses for any reason other than the appreciation of the beauty of abstract thought, the pleasure of learning a foreign language, etc. In particular is anyone interested in the application of mathematical ideas and techniques to the problems of social sciences?

Perhaps a personal concern might make my question clearer. I began studying maths because I am keen to apply mathematical ideas to questions in education and educational research. I do not mean the more obvious applications such as occur in analysing quantitative data (the sort of thing one gets in MDT241 for example) but rather in applying these to problems that are barely perceived.

Let me give an example. Assessment in schools is largely based on the "batch" principle that, given 30 pupils, there is some norm to which they all tend in performance -a group mean, if you like. Let us therefore set a test (call it an exam) and set a pass mark within, say, one standard deviation of this mean, and pass all those who reach this point. This sort of approach is of course disastrous to the child who comes to a subject with no understanding of it at all, and after considerable work gets to a point which falls just short of this mark. In school terms this child "fails" and yet in personal terms this same child might have made more progress (from zero to 36 per cent say) than another child who moves only from a base of about 36% to 45%.

What we therefore want to do is to make individual assessments of children, measuring not their relationship to a group ability but rather a measurement along some linear path. This implies that we test our pupils before they ever take a course and then again immediately after it, and hence obtain two bench marks - before and after. There are however great problems associated with such strategies, notably the fact that children change dramatically over the course of time - and do so at differing rates - and that we are therefore not dealing with a simple linear model at all.

This problem, described as the "measurement of change" is a very fascinating one that mathematicians like Rao and Coleman have given some attention to, but it is quite clear that to "learn" mathematics in order to apply it to problems of this kind involves a sophisticated knowledge and facility with mathematics which I am not likely to have for many years, if ever.

Does this suggest that we ought to try and get someone to design a course of "Mathematics for the Social Sciences"? Does it mean that one should not set out to learn mathematics for such purposes, because unless you devote yourself exclusively to the study you will never make much of a contribution anyway? Does it mean that "learning" mathematics has really very little practical value, and that the best one can hope to do after wading through all those M courses, is that one has a very general appreciation (not to be sneered at) of what modern maths is all about, and can perhaps read maths books without blanching? I wonder!

THE AXIOM OF ARCHIMEDES

John Reade

It must he clear that the Axiom of Archimedes cannot be equivalent to the Axiom of Infinity, since if this were so, then there would be no need for two different names for the same thing.

Of course, this is only a "meta-mathematical" argument. If we wish to demonstrate their non-equivalence beyond all reasonable doubt we must exhibit a number system with an infinite number of integers in which the Archimedean Axiom is false. Such a number system is the so-called "Non-Standard Real Numbers". This is the number system in which Non-Standard Analysts do their Non-Standard Analysis. The construction is quite complicated, but essentially amounts to extending the "standard" real number system by introducing infinitely large numbers and infinitesimally small numbers. For the details see *Non-Standard Analysis* by Abraham Robinson.

The mistake in Datta Gumaste's argument (M500 29 5) purporting to prove the Axiom of Infinity implies Archimedes' Axiom is in line 3 ("consider the integral part of $x \in \mathbb{R}^+$"). To prove a real number has an integer part you need Archimedes' Axiom, so the argument assumes the result it is trying to prove.

Archimedes' Axiom is true in $\mathbb{Q}$ (the rational numbers) since given any positive rational $m/n \in \mathbb{Q}$, we have immediately $m/n \leq m$. Other ordered fields in which Archimedes' Axiom holds are, e.g.:

$$F = (r + s\sqrt{2} : r, s \in \mathbb{Q}),$$

$$G = (r + s\sqrt{3} : r, s \in \mathbb{Q}),$$

etc. (Staff)

COINCIDENTAL BIRTHDAYS - Nicholas Fraser

Having just joined MOUTHS and read M500 I felt I'd like to contribute a problem. It's not mine, I pinched it from Arthur C Clarke; but it amused me when I read it and if any readers are like me then they might like to see through the mathematical logic.

22 people are trapped in ship on the moon, to pass the time a man postulates how many people you needed before you had a 50-50 chance that two of them had the same birthday.

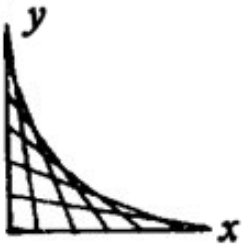
He stated that if there were 24 or more the odds are better than even that two of them have the same birthday and also if there were more than 40, nine times out of ten two will have the same birthday. The question to you high-brow lot is why?

The book is *A Fall of Moondust*, Pan Books; chapter 9.

Ed - Can you generalise this? If there are *m* people what is the probability *n* of them share a birthday?

CONSTRUCTIONS I I

Max Bramer



I can take Roger Claxton's analysis (of the curve produced by intersecting lines as in the figure) a little further. First, perhaps I should point out that the equation for e_0 is actually $\frac{x}{10} + y = 1$. The equation given, $x + (9/2)y = 9$ is for e_1.

Denoting the intersection of the lines e_n and e_{n+1} by (x_n, y_n) and rewriting the formula for x_n slightly, we have

$$x_n = \frac{(q-n)(q-n-1)}{(q+1)}; \quad y_n = \frac{(n+1)(n+2)}{(q+1)}; \quad n = 0, 1, ..., q-2 \qquad (1)$$

(q was 10 in the original article). In passing it is easy to verify by substitution that $y_k = x_{q-2-k}$; $x_k = y_{q-2-k}$; $k = 0, 1, ..., q-2$; hence (with $q = 10$) $x_0 = y_8$, $y_0 = x_8$, etc.

To find the equation of the curve we need to eliminate n from (l) thus giving a relationship between x_n and y_n in terms of q. Replacing $q + 1$ by Q in (l) gives

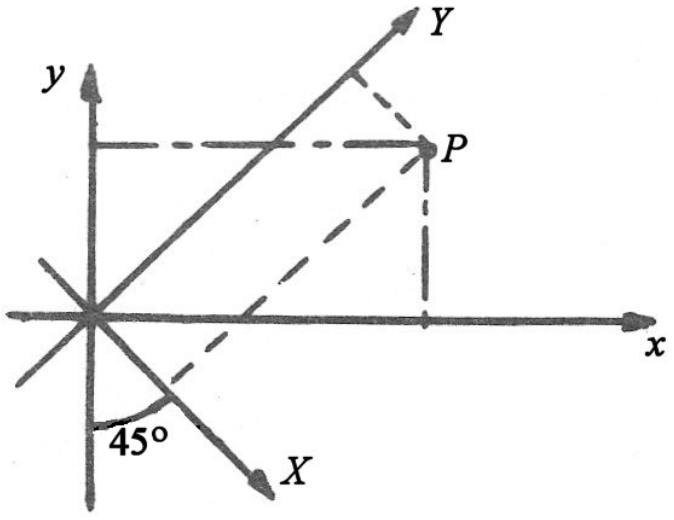
$$xn = \frac{(Q-(n+1))(Q-(n-2))}{Q}; \quad yn = \frac{(n+1)(n+2)}{Q} \qquad (2)$$

so $x_n - y_n = Q - 1 - 2(n + 1)$, i.e. $2(n + 1) = Q - 1 - (x_n - y_n)$. (3)

Substituting for n+1 and n+2 in the formula for y_n ((2)) gives

$$x_n^2 - 2x_n y_n + y_n^2 - 2Q(x_n + y_n) - Q^2 - 1 = 0. \qquad (4)$$

Dropping the subscript n gives the equation of the enveloped curve. This is in fact a parabola, which can be demonstrated as follows: take new axes at 45^o clockwise from the old. If the coördinates of a given point are (x, y) relative to the old axes and (X, Y) relative to the new, then (x, y) and (X, Y) are linked by the relations



$$X = x \cos 45^o - y \sin 45^o$$

$$Y = x \sin 45^o - y \cos 45^o.$$

Substituting $X = \frac{1}{\sqrt{2}}(x - y)$;

$$Y = \frac{1}{\sqrt{2}}(x + y) \text{ in (4)}$$

- without the subscript again - we have $Y = \frac{X^2}{Q\sqrt{2}} + \frac{Q^2-1}{2\sqrt{2}\,Q}$. Thus, relative to the new axes, the curve is a parabola of the form $Y = aX^2 + b$.

The next step would be to investigate the case of a curve generated from lines joining points on the two axes as before, but with different spacing for each of the two axes. Would it still be a parabola?

(Staff)

Ed - from Steve Murphy there came a particular solution to the original problem, which can be derived from Max Bramer's. But he ends thus:

As the step length tends to 0, increasing n so that $nq = 1$, the curve tends to an "Astroid" (mentioned in M100), the equation of which is

$$x^{2/3} + y^{2/3} = 1.$$

This curve can be generated as the locus of a point P on the circumference of a circle of radius 1/4 which rolls without slipping on the inside of a fixed circle of radius 1.

"FUNCTIONAL ANALYSIS"

Percy Sillitto

I recently bought the above book by W Rudin (McGraw-Hill 1973) on the strength of its being recommended further reading in Unit 13 (Hilbert Spaces) of M331. Anyone else considering its purchase should be warned that in at least a dozen places it invokes in an essential way, advanced theorems and results from the same author's other book *Real and Complex Analysis*, McGraw-Hill, 2nd edition 1974. Therefore unless one is familiar with, or possesses another book dealing with, such things as the Riesz Representation Theorem, the Rodon–Nikodym Theorem, Mergelyan's Theorem, etc, Rudin's *Functional Analysis* is *not* really recommendable as a self-contained book on the subject matter of its title.

ST VALENTINE'S DAY

A reader received this and passed it on to M500

Of CMAs and TMAs your life it is so full.
Of "Axioms" and "Morphisms" and other kinds of Bull.
There's Numerical Solutions of Algebraic Equations
Not to mention all the problems of Techniques of Integration,
There's Computation, Transformation, and even Exponentiation,
Orthogonal, Pythagoral (?) and if this don't make you blue
You can always spend your time on Differential Equations II.
There's Radio and TV and Units by the score
And still the good old postman brings more and more and more.
But even though the old OU takes up all your precious time
I hope you'll think of me today and be my Valentine!

"THANK YOU, MARION"

Marion Stubbs

Lytton Jarman, while twiddling with radio knobs, suddenly observed that the person talking about colonels was yours truly. Since this masterpiece has been going out on the air for three or four years I would like an opportunity to apologise.

"What masterpiece?" I hope some of you are saying. If you have not heard it, DON'T! It was a radio programme about Use of Libraries. I think I was dragged in on it because I had just produced a small pamphlet for Solent students on library facilities in the Hampshire area. The programme sounds as though several students are being interviewed. In fact each of us was totally alone, except for a producer, in a soundproof studio somewhere, probably in the bowels of Ally Pally.

My producer was almost totally silent. At intervals he said I was to talk about this, that or the other. I recorded two sides of tape, during which the batteries ran down and the sound was like a dull monotone-decreasing function. My thesis, which I stuck to continuously for five hours, was that Public Libraries alone were insufficient for a University education, and that some survey such as I had produced for Solent should be done for every area, so that students who wanted to use specialist materials would know which special libraries were willing to help. (Most of them, except for the local University, were very willing, as a matter of interest.)

Out of all this They edited four sentences and used them. I use the word 'edited' advisedly since they totally reversed my meaning by clever deletions, viz.

P: Talk about any time you used Public Libraries during M100.

MS: ... I went to the College of Technology library when I needed extra help ref kernels ...

(the general sense of the statement was that the PL was U/S, at least on this topic.)

They deleted "College of Technology" so that on the programme I apparently say that "I went to the library" - and you cannot even hear a change in voice level!

The other students also recorded a full tape each; and they were lucky to have one sentence broadcast.

Since then I have never listened to Open Forum nor to any programme where students are apparently "interviewed" without wondering just where the lies are. When I subsequently founded M500 the policy from the start was to print exactly whatever the author wrote. I cannot think of anything which would induce me to go on any OU radio programme again!

Mr Paul Getty ... is quoted as having said that a billion dollars is not worth what it used to be.

Nubar Gulbenkian.

MONGE'S SHUFFLE

David Asche

Concerning this shuffle of a pack of cards (which involves placing the top card under the second, the third under both of them, the fourth on top and so on), can you please say where you found that formula?

The following little formula is a conjecture of mine:

$$(4p+1)k \pm p = 2m - 2 .$$

Here 2p is the number of cards and *m* is the number of shuffles needed before the original order is again arrived at. The number *k* is some suitable non-negative integer and the sign attached to *p* is sometimes positive and sometimes negative. It should be possible to determine *k* and the sign for *p*, but so far it eludes me.

p	1	2	3	4	5	6	7	26
k	0	0	1	0	1	10	141	10
sign	+	+	+	+	-	+	+	-
m	2	3	6	4	6	10	14	12

Ed - The original formula mentioned by David, from which mine in M500 26 and Marion's in M500 30 were both taken came from *Mathematical Recreations* by Maurice Kraitchik (Allen & Unwin 1949) p322.

> If the pack be shuffled *m* times ... we find the following relation connecting the final position x_m and the initial position x_0 of any card:
>
> $$2^{m+1}x_m = (4p + 1)(2^{m-1} + (-1)^{m-1}(2^{m-2} + ... + 2 + 1)) + (-1)^{m-1}2x_0 + 2m + (-1)^{m-1}.$$
>
> A pack of cards will regain its original order after a certain number *m* of such shufflings. This number may be found by substituting x_0 for x_m in the preceding formula and solving for *m*. ...

CHESS - Roger Claxton

The M500 Chess Group is now five strong and we are setting up a small friendly league to play games by post and by telephone. The league will probably run until the end of the current academic year (correspondence chess is a leisurely affair!). This has the advantage (like the OU) that players can play when they want and at their own pace.

Five is not felt to be the optimal membership by any means so if you would like to join in please contact me. All experience levels are welcome although I admit that the novices (like myself) probably gain more from exposure to better players than the good players do from the beginners.

LETTERS

From David Weller - Dear Miss/Mrs/Ms/Marion Stubbs (delete in accordance with current fashion); I now find the above address form necessary. Since taking note of your mini-lecture and adopting the form of address suggested by you (MS - I said correct form is Ms if one does not know, and that all OU and Midland Bank call me "M Stubbs" whether I like it or not, and always have) I have received several complaints. Perhaps my female friends are not yet liberated in letter writing.

Many thanks for everything else I have received from you. I am constantly amazed at the way you manage to make contact through such a large organisation appear so personal.

I am still battling on with DST, although I haven't yet discovered whether it is a full or a half-credit course. I am now trying to introduce more random events, in keeping with the cheaper tv imitation of the game.

Any situation will suit me for the MWI, as long as it is after July.

From Cyril Whitehead - If anyone is interested I have now changed sides!! I am the new (1976) M100 Tut/Couns for the Rugby study centre, having been a student since year one (A005...). (Staff)

From David Hayman - Thank you for the back number of M500. You will note from my subscription form that I am rather a meagre applicant to your merry band. I don't mind being approached by other members, although I am not likely to be of much use to them in the foreseeable future. Having recently changed jobs I haven't given a daytime telephone for two reasons: firstly I would probably feel unable to stop talking to anyone who 'phoned, and I honestly can't remember the number. Any help that I could give at the moment would be limited to a mutual joint bailing out between myself and another.

If you are still looking for high student numbers in 1975 I attempted a post experience course as a filler until I could join more fully and was given the number E9043302.

From Tony Brooks - I must say how much I enjoy M500; it's come a long way from the original *Solent* M202 *Newsletter*. I still have one copy, number 6 I think.

From Ray Tiver - I have really wanted to do maths all of my life. Many is the time I have actually started. Get yourself a school book, pen and paper, a nice quiet corner, start to work, and you strike a rock before you turn the first page. The BBBB book has no answers. Just to stop you from cheating. It also stops you from working back from the answer when you are stuck. Just like going to an OU tutor for advice. I have not found them helpful. I know they are not supposed to give you the answer to a TMA but they could give you an example. They only drive you to another student who gives you the answer anyway. In my S100 group there were

five teachers who taught one another how to get A's. Personally I prefer women teachers, much more patience, not so kranky as men, pay more attention to detail, may not be as mathematical but those I have met will do me. If you can get one to take you on, worth their weight.

From Colin Davies - I want a Dice Star Trek but I can't find any details of price or anything else in the last two issues of M500. Can you send me one on trust?

Ed - Don't you keep old copies of M500? I think that's sad. I wonder how the editor of *The Times* feels when he sees someone eating fish and chips out of it. Anyway DST costs 50p from Marion who will probably supply details in exchange for an sae. The original description was on the back of M500 27.

From Arthur F Stanway - Enclosed is £5 deposit for the Weekend Workin and also 50p for a copy of Dice Star Trek. If, as advertised, it grips me to the extent of interfering with my studies, can I claim this as a mitigating factor when I get poor exam results in November?

I hope Marion is resigned to being re-elected Publisher for the next *P* years (*P*: the largest known prime).

From Ken Hegerty - I am a middle-aged widower, living alone in a bed-sitter in North London and tolerably contented and happy. Slightly disabled and enjoying keeping inflation down by keeping the unemployment figures up, if only by one! Interests include philosophy and psychology (and even para-psychology!), music, films, Esperanto, and travel. Pet dislikes: smoking, dogs and children. (The comma after "smoking" is important.)

From John Carter - DST has proved such a success that I'd like another copy please. This second copy is for my own use primarily but will also be lent out to the many people who are interested in playing DST. The first copy is with a friend's son and he enjoys playing the game so much that I'm letting him have it on permanent loan. I'm not including a deposit for the maths weekend, despite your plea which I fully sympathise with. The reason is that I have decided it's unfair to leave my wife for the weekend. However there's a chance that we can both come but it's early days yet and the probability is low (babysitter for four children needed!)

From Geoff Strutt - The magazine is delightful and despite losing the personal touch of hand-winding the new format is both neat and good value for money.

THIRD LEVEL COURSES

year	course	finally regist'd	exam'd	course results				
				distinc'h		pass 2	pass 3	pass 4	fail
1974	M321	443	196	25	12.7%	36	64	54	17
1975	M321	331	158	16	10.1	42	50	39	11
1975	M331	228	141	22	15.6	32	38	22	26
1975	M332	341	250	38	15.2	76	87	35	11
1974	SM351	243	177	19	10.7	58	36	34	30
1975	SM351	228	142	24	16.9	41	32	26	19

This table is taken from one sent by John Baker. The original had much more explicit material, percentage figures and the like, but nothing that cannot be deduced from the above. Since it had been reduced to A4 size the further reduction to our A5 would have been ludicrous. I will tell you though that the highest percent pass rate of examinees was in M332 - 94.4% got grade 4 or better. And that course had the highest pass rate of students finally registered as well. (For those astute enough to notice the above figures do not balance, some 1975 results are still pending.)

JOBS IN COMPUTERLAND

Peter Weir

Unlike most sectors of employment, there are many vacancies in computing. A glance at any of the computer rags will confirm this - they are bursting with vacancies.

So, if you are unemployed or your job is in danger, why not consider a job in computing? By computing I mean commercial computing. No OU course gives much idea of commercial computing. TM221 gives heavy emphasis to the minicomputer, but commerce is not (yet?) so biased. Most vacancies are for programmers on large computers.

If you already have relevant commercial computing experience you should have little difficulty in getting a job. As for those without this experience, sell yourself! Ask for interviews - take along some OU material that you have mastered, show off a good maths TMA, it will impress them. Your maturity plus your ability to learn will give you advantages over more people than you might think. You might even have the good fortune, as I did. to find out that your prospective employer is an OU student (shades of freemasonry)!

One good source of job adverts is the newspaper ‘Computer Weekly’, available at larger newsagents and public libraries.

THE M500 SOCIETY

From Austen F Jones - I am willing (or foolish enough!?) to take on the job of M500 Treasurer. I qualified as a Chartered Accountant in 1973.

MS - Many thanks! "Credentials" checked and Austen is now our Treasurer. He is currently considering financial systems - and good luck to him - on top of M100 + T100. This offer was received, checked and approved while M500 30 was still at the printers, hence M500 was still a capitalist dictatorship and so I appointed him without an election since the instant constitution had not then been published. This relieved me of bed-at-2am working on M500 finances ever since early December, and I am truly grateful. Any other new members who are CAs and who think of applying could well help out. It is a hefty job and might well be sharable among several suitably qualified members. Apply to Austen Jones if willing to assist him in any way.

Austen also sent the following to the editor of M500, to whom it seems rather a heavy load; especially for someone who is both in gainful employment and doing two full credits.

TERMS OF REFERENCE: TREASURER

The Treasurer will be responsible for

a maintaining proper books of account;

b receiving cash (from the other officers and/or members of The Society) and recording all transactions;

c maintaining and controlling cash balances in appropriate secure places whilst ensuring good interest rates;

d preparation of the annual accounts;

e preparation of quarterly financial statements;

f ensuring that The Society obtains maximum benefit from tax avoidance measures (and any other statutory requirements).

PRONUNCIATION

Norman

As regards the matter of pronunciation raised by Eddie Kent, there are three names of mathematicians which I wish our teachers at Walton Hall would get right, especially when they're on the box. The first two are the Norwegians Abel and Sylow, who are pronounced (as a member of a university department of Scandinavian Studies assures me)

Ah-bell (not Able or Ah-bl);

Seal-off (not Sile-(Sill-)ow(oh,ov)).

The third is a Frenchman, namely Rolle, but they will persist in pronouncing him as if he were a German, thus Rol-uh, instead of the correct: Rol.

THE KRONEKER DELTA

Philip Newton

Last year an answer to a TMA question was marked “incorrect - the delta must appear under a summation sign”. The expression I had used was of the form .

$$\sum_{1}^{N}(\ )\cdot\delta_{ij}\sum_{1}^{N}(\ ).$$

Intrigued by this I checked various books and found common agreement with the definition that “δ_{ij} is to be taken as n^2 scalars each of which takes the value 0 ($i \neq j$) or 1 ($i = j$). Furthermore no summation sign is necessary since the indices do not represent the components of some geometrical identity”. Surprisingly, neither Kreider *et al*., nor Nering (set books, M20l) do more than use the delta in connection with bases. In Eisberg (SM351) there is some delicious wriggling since the poor chap has not heard of the delta nor of the Dirac delta function, (see pages 183 to 191). He juggles with both discrete and continuous sets of eigenfunctions and ends up with

$$\int_{-\infty}^{\infty}\psi_L^*(x)\psi_n\,dx = \begin{matrix}1, & L=n\\ 0, & L\neq n\end{matrix}; \qquad (7.54).$$

“These properties often allow a considerable simplification of quantum mechanical calculations” ;

The other set book, Gillespie (SM351) has only heard of the delta, which is used with abandon until we trip on 2-39b on page 22.

All these matters are correctly treated in Schiff, *Quantum Mechanics*, a book which is recommended to 3M351 students. As for the tutor - he was too busy to go into the matter ...

(Remember that we sometimes demand definitions for the sake not of the content, but of their form. Our requirement is an architectural one: the definition is a kind of ornamental coping that supports nothing.)

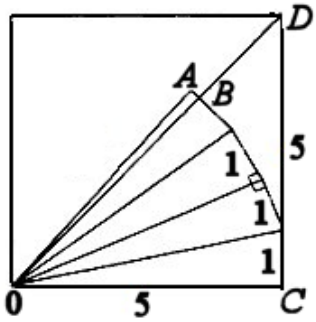
“Philosophical Investigations”,

Ludwig Wittgenstein.

PI

or, Machin resolved; by Harold Moulson

A solution to Peter Weir's problem in M500 30: In the figure



$\angle A0B = \angle A0C - \angle D0C$

$= \arctan \frac{1}{5} - \pi/4$

$= \arctan \frac{120}{119} - \arctan 1.$

These results follow from the identities

$\arctan A \pm \arctan B = \frac{A \pm B}{1 \mp AB}$.

Therefore

$\arctan 1 = \arctan \frac{120}{119}\ \angle A0B$

$= 4 \arctan \frac{1}{5} - \arctan \frac{1}{239}$;

and $\pi = 4 \arctan 1$

$16 \arctan \frac{1}{5} - 4 \arctan \frac{1}{239}$.

This also solves Marion Stubbs's problem in M500 27 because nowhere is a circular measure used, just anti-tangents and the fact that a quarter of a semi-circle has angle $\pi/4$ or 45° at the internal angle. (Marion was worried about using arctan to calculate π when π is defined as 4 arctan 1.)

ESSAYS IN MATHS COURSES - Dave Diprose

I did not see the original letter on this subject, but I think most mathematicians prefer not to write (does M500 receive many articles for publishing?), and I am firmly against compulsory essay questions in exams (M500 26). If the choice was made wide then the choice for other questions would have to be narrowed and that would discriminate against the mathematician who is poor with words. I think there is a place for essays, but I would wish to have some practice in TMAs first, and a compromise in exams with part-essay questions.

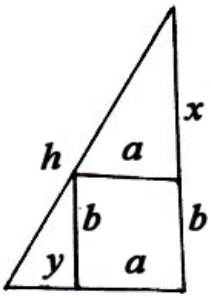
POCKET CALCULATORS - Michael Gregory

Re Michael Masters' query about calculators, there is a *Which?* report on the subject but rather out of date. The January 1976 edition of "Electronics Today International" did a comparison of about 30 models listing the functions etc on each, no assessment of reliability was made. It is certainly worth consulting before making a purchase.

LADDERS

Richard Tombs

The ladder against wall problem can be easily solved, by sleight of hand and quadratics only. From diagram



$(x + b)^2 + (y + a)^2 = h^2$.

Put $xy = ab$ giving quadratic

$(x + y)^2 + 2b(x + y) = 2ab - a^2 - b^2 + h^2$.

Solve the quadratic for $(x + y)$ and hence

(using $xy = ab$ again) for x, y.

WHAT IS MATHEMATICS? Datta Gumaste I liked Professor P's comment that "mathematics should be fun." (M500 30 18). Mathematics that I have met so far is fun. Is there a branch of mathematics that is not fun?

And yet, according to A W Lee (M500 30 12), mathematics is not a proper substitute for knowledge. Just think of the state of our knowledge without natural numbers - let alone other areas of mathematics. However, in one sense Lee is absolutely right. Mathematics is well beyond knowledge. *It is wisdom*.

In my view, there is only one thing worth knowing and that is how hopelessly ignorant one is; and the surest way to experience one's ignorance is to do mathematics; It's only when you are ignorant, you have some chance to know anything at all! Maybe that is what wisdom is all about!!

M500 is getting better and better. I hope to send my mathematical proof that "Mathematics is an art" soon.

M500

A few readers have written in reply to the various questions that have been asked, explicitly or otherwise, in this journal. For instance I keep getting brief notes about what people would like to see. *Brian Woodgate* listed a few "thoughts on the contents":

1 Serious mathematics; by staff or outside mathematicians;
2 Fun mathematics and problems;
3 Faculty news - courses, policy etc;
4 The M500 Society news;
5 Facts about the "world of mathematics". What outside courses are available; what are our BAs worth etc;
6 Letters to the editor;

And let us stay independent and out of the clutches of the OUSA, Sesame and Open Forum

clan.

Michael Gregory also had a numbered list (is this symptomatic of the subject?):

1 Creating a sense of unity among maths students and staff;

2 arousing interest in new aspects of mathematics;

3 transfer of information;

4 overcoming isolation.

He goes on "possibly under the influence of TMA we seem to overstress the need for completeness of solutions etc. Often the publication of incomplete solutions, or just suggestions for a new approach (see my "Constructions" in 28) would provide the stimulus to complete the problem."

Mike Coombs likes the new style of printing because it causes much less eyestrain than the old. This is quite a typical reaction. (I can't lay my hands on any more letters at the moment but I'm sure I've seen them.) Actually I'm a little surprised because I personally preferred the old, larger printing; but that's my problem. This month I have used a darker ribbon almost throughout, and hated it. It's very messy and difficult to correct mistakes; but it should give a darker image.

No-one has dissented from the constitution that came with 30, so we'll take that as carried. All agree that 1½ line spacing though attractive is nevertheless too wasteful; and *Mervyn Savage* has taken me severely to task for my notation.

Marion was wondering recently why the OUGS has more members than us. I'm sure there are many reasons but perhaps one of them has something to do with the fact that you get a free membership of OUSA thrown in when you join. What can we offer as our free gift?

Keep the stuff rolling in please. There isn't quite enough material on hand for next issue; partly because three items that are packing out the file are not usable in their present state; I am waiting for replies from their authors. Also I have masses on Machin's π which needs editing, and on Hoops. Both of these blew up in my face and I need a quiet time to organise them. But they are not ready for publication yet, so come on you people who keep promising things. Now is the time.

Eddie Kent

A letter that just came (green ink on mauve paper; brightened my day I can tell you):

From Anne Dennett - As a new M500er I must say I like the magazine. The 1½ spacing looks far better from the aesthetic point of view, but far, far better I think is to stick to single and get more in. I even approve of "The M500 Society" too, though it does conjure up visions of secret passwords before being allowed into the study centre! I can't help but feel Ian Turner's "It" is counting itself among the staff, hence 1003 instead of 1002.

OBITUARY: WERNER HEISENBERG

Professor Werner Heisenberg died on February 1 at the age of 74. He was born in Wurzburg on December 5 1901. His father was professor of Greek at the University of Munich where he followed as a student under Sommerfeld. His main interest was in reconciling the problems of quantum mechanics, necessary to describe atomic processes, with Newtonian mechanics and Maxwell's electrodynamics.

Max Planck had first introduced the "quantum" into physics. Einstein and Bohr took the idea further to develop a theory of atoms and their interactions with radiation, but there still remained the classical theory to which empirical rules were joined as necessary. The rules did what was required but contradicted some postulates of the old theory and had no consistent theory of their own. By 1924 Niels Bohr had developed his correspondence principle which specified a limit within which classical results must apply.

Sommerfeld introduced these ideas to Heisenberg who moved to Göttingen in 1924 to become Privatdozent and assistant to Max Born. In the same year he visited Bohr's institute in Copenhagen. He became convinced that the key to understanding lay in banishing concepts which could not be verified empirically. Thus in the spectrum of atoms the frequencies and amplitudes associated with line intensities were described but the electron orbits were unobservable and therefore meaningless ideas to be excluded from the theory.

In 1925 Heisenberg applied these concepts to the simpler pendulum problem and was able to formulate a new system in which the quantum rules arose naturally and Bohr's correspondence principle applied. Together with Max Born and Pascual Jordan, Heisenberg, at the age of 24, developed the Quantum Mechanics (or matrix mechanics) which is the theoretical basis for all physics. He received the Nobel Prize in 1932.

(Erwin Schrödinger's wave mechanics which also gave a consistent description of quantum phenomena was published shortly after but was soon shown to be a different formulation of the same theory.)

In 1927 Heisenberg was Professor of Theoretical Physics at Leipzig and in 1941 Professor of Physics and Director of the Kaiser Wilhelm Institute, Berlin. After the war he became eventually Director Emeritus of the Max Planck Institute for Physics.

His most, famous and far reaching contribution to quantum mechanics was the uncertainty principle, which has had a great impact in fields outside his own, through no fault of Heisenberg's. He also proposed the idea of isotopic spin to facilitate the notion of the proton and the neutron as different states of the same particle, and introduced the S-matrix theory into the study of the interactions of elementary particles.

He stayed in Germany through the war and led the "Urainverien" working on applications of nuclear fission. For this he was interned at Godmanchester by the Americans, but then returned to help with the reestablishment of scientific research in Germany. In 1953 he became president of the Alexander von Humboldt Foundation. He was an FRS, an accomplished musician and a keen hiker. He spent the last 20 years in an attempt to construct a unified field theory of elementary particles.

He married Elisabeth Schumacher in 1937. They had three sons and four daughters.

SOLUTIONS

28.3 ROLL ME OVER - The problem was to get the bottom faces of 8 dice in a square formation on top without going outside the square. The solution printed last month was wrong. Jeremy Humphries has picked it up: Moves 15 and 16 are transposed. If LLRD (left, left, right, down) is changed to LLDR the solution works; but it is not minimal since LLRD = LD.

That makes the solution now LURD RULD DRUU LLDR ULD, DRU LDRR UULD DRUL DLUR. Marion Stubbs says this problem is discussed in *Chez Angelique* so let us possess ourselves in patience.

29.2 NEWTON'S COWS - a (a′, a″) cows graze b (b′, b″) fields bare in c (c′, c″) days; what relation connects the nine magnitudes a to c″? The determinant solution last month was correct but Jeremy Humphries states the relation ac/b = a′c′/b′ = a″c″/b″ is only true if the grass doesn't grow:

If M is the initial amount of grass per field;
m is the growth per day per field;
Q is the consumption per cow per day;

then after c days (for a, b, c) the amount of grass remaining is $bM + cbm - caQ = 0$. Whence $ac/b = (M + cm)/Q$; and so on.

30.1 SUBSETS - Prove $\sum_{i=1}^{n-1} {}^{n}C_1 = 2^n - 2$.

By the binomial theorem (which can itself be proved by induction) $(1 + x)^n = \sum_{i=0}^{n} {}^{n}C_i x^i$. Substitute $x = 1$ in this and the result follows. - Mervyn Savage, Max Bramer, A J S Winter. And a straightforward induction proof from Peter Weir.

30.4 FIBONACCI COPRIMES - Show that any two consecutive Fibonacci numbers are coprime.

$F(n + 1) = F(n) + F(n - 1)$. Hence if k divides $F(n)$ and $F(n - 1)$ it also divides $F(n + 1)$; etc. - Mervyn Savage, Roger Claxton, Peter Weir, A J S Winter.

30.5 NEXT TERMS - O, T, T, F, F, S, S, E, N, ?

T, E; initials of the English number words - Mervyn Savage, Roger Claxton, Jeremy Humphries, A J S Winter.

or: W, W, A, S. - Initials of "Onwards to the fantastic frenetic Summer Schools. Educational, notwithstanding wine women and song." - Peter Weir.

P ROBLEMS

31.1 EULER'S POLYGON DIVISION Jeremy Humphries.

In how many different ways can a plane convex polygon of n sides be divided into triangles by diagonals?

problems continued

31.2 MRS READ'S KNITTING MACHINE Richard Ahrens

Graham Read's wife has a knitting machine that uses thread wound on special spools. Some knitting patterns call for the simultaneous use of several threads (k say) which means having k spools with thread wound on them. Suppose we have n spools ($n \geq k$): s_1, s_2, ..., s_n, containing various quantities of thread t_1, t_2, ..., t_n respectively. What condition must the numbers t_1, ..., t_n satisfy so that it is possible to knit all the wool without ever having to wind wool from one spool onto an empty spool? It is possible to change spools as often as you like (i.e. before they are empty.) The pattern uses all k threads at the same rate. (Hint: try $k = 2$, $n = 3$ first.

(ii) Suppose we have the situation described above and the numbers t_1,..., t_n are such that it is possible to knit all the wool without rewinding a spool. Show that it is possible to knit all the wool with no more than $n - 1$ stops to change spools, but that in some cases $n - 1$ stops will be necessary.

(iii) (Unsolved and looks difficult) Find an algorithm for dividing the most efficient way of using all the wool (i.e. fewest stops to change spools) in those cases where the job can be done in fewer than $n - 1$ stops.

31.3 COMBINATIONS Max Bramer

Prove that = $\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!}$ is an integer; $n \geq r \geq 0$.

31.4 HOW OLD Marion Stubbs ("This is a problem taken from *Creative Computing* - details regarding sub rates and address from M Stubbs - but has little to do with computers. *CC* is not really sure about its source; it might have been in *American Mathematical Monthly* several years ago. It's not going to help you get started at anything; actually it might finish age problems, altogether"):

Ten years from now Tim will be twice as old as Jane was when Mary was nine times as old as Tim. Eight years ago, Mary was half as old as Jane will be when Jane is one year older than Tim will be at the time when Mary will be five times as old as Tim will be two years from now. When Tim was one year old, Mary was three years older than Tim will be when Jane is three times as old as Mary was six years before the time when Jane was half as old as Tim will be when Mary will be ten years older than Mary was when Jane was one-third as old as Tim will be when Mary will be three times as old as she was when Jane was born. How old are they now?

31.5 CUT WIRE Bill Shannon

Two cuts are made at random in a piece of wire. What is the probability that the three pieces of wire will form a triangle?