M500 32

M500 is a non-profit student-operated and student-owned magazine for Open University mathematics students and staff, and for any others who are interested. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct but invite criticism and comment. Articles submitted for publication should normally be less than 600 words in length, although subjects which require more space may be split by the authors, into instalments.

MOUTHS is a list of names addresses and telephones, together with previous and present OU courses, of voluntary members; by means of which private contacts may be made by any who wish to share OU and general mathematical interests or who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned, and do not necessarily represent either those of the editor or the University.

The cover design is taken, as authorised, from “Geometrical design and ornament”, by Edmund V. Gillon Jr., published in the U.K. by Constable, 1969 @ £1. This is one of the volumes in the Dover Pictorial Archives Series.

M500 is published by Marion Stubbs

Articles, letters, problems, solutions, cover designs for inclusion in M500 to the editor: Eddie Kent

Membership applications, subscriptions, change of address, MOUTHS data to the Membership Secretary, Peter Weir

The Treasurer is Austen Jones

Cheques and postal orders, which should continue to go to Peter Weir, should be made payable to THE M500 SOCIETY and crossed “a/c payee only, not negotiable” for safety in the post.

Anything sent to any of the above and not marked as personal will be considered for possible inclusion in M500.

M500 32 Published April 1976.

Subscription rate: £3 for 10 issues.

Membership: 365

Printed by TIZENGROVE LTD Queens Terrace Southampton.

TWO COROLLARIES

Daniel Dubin

(1)

One	*Two*
a	any universal covering space of a

connected, locally path-connected

space has a simply connected covering space iff it is semilocally 1-connected.	semilocally 1-connected space is simply connected.

(2) The student will easily verify that

$$
\begin{tikzcd}
 & H^n(A \times I \cup X \times \mathring{I}, X \times 0) & \\
H^n(B', b_0') \arrow[ur] \arrow[r] & H^n(A \times I \cup X \times \mathring{I}, X \times \mathring{I}) \arrow[u] \arrow[r] \arrow[d] & H^{n+1}((X, A) \times (I \times \mathring{I})) \arrow[ul] \\
H^n(PB', \Omega B') \arrow[r] & H^n(A \times I, A \times \mathring{I}) & H^n(X, A) \arrow[u] \\
H^{n-1}(\Omega B', \omega_0') \arrow[u] \arrow[r] & H^{n-1}(A \times \mathring{I}), X \times 0) \arrow[u] \arrow[r] & H^{n-1}(A) \arrow[ul]
\end{tikzcd}
$$

(by way of an idea of Roger Duke)

RULES, AND OTHER THINGS
Tony Brooks

I think enough has now been said on the topic of sequences by Hugh Mclntyre and I (see M500 19, 23 and 28). However he does raise two very general points to which I would like to see some opinions from other readers of M500.

Hugh asks if I will admit there is a point beyond which arbitrary rules have zero value. But where is this point? How do I fix it? What is the criterion for determining whether a rule is arbitrary? I would say that the value of a rule depends entirely on the context in which it is used. There is no internal measure of a rule’s value independent of the situation or mathematical structure in which it is used. A rule which is arbitrary and of no value for me may in fact be of great value to someone else in another situation. The rule for integration by parts has sometimes been useful to me in my work. However for most of the time it is of zero value to me, and to someone without knowledge of calculus it would probably seem to be arbitrary.

Hugh also asks what have such arbitrary rules to do with mathematics? This prompts me to ask, what is mathematics? Is it a structure in which only certain non-arbitrary rules (whatever they might be) are allowed? In fact what does making a rule consist of? Perhaps my arbitrary rules are not part of maths because they are not rules at all.

P I
John Reade

The simplest series for π is as suggested by Marion Stubbs in M500 27: $\pi/4 = \arctan 1 = 1 - 1/3 + 1/5 - 1/7 + \ldots$. To get an accuracy for π of two decimal places with this series you would have to take 200 terms. Arctan is calculated from its Taylor series, $\arctan x = x - x^3/3 + x^5/5 - x^7/7 + \ldots$ valid for $-1 \le x \le 1$. So Machin’s formula

$$\left(\pi = 16 \arctan \frac{1}{5} - 4 \arctan \frac{1}{239}\right)$$

gives the series $\pi/4 = 4\left(\frac{1}{5} - \frac{1}{3.5^3} + \frac{1}{5.5^5} - \ldots\right) - \left(\frac{1}{239} - \frac{1}{3.239^3} + \ldots\right)$ which converges much more rapidly, though still not as fast as one would like. It is nevertheless the best there is. There are other identities but they are either much more complicated or give series which converge more slowly. The recent computer calculation of π to a million places of decimals used Machin’s formula.

ERRATA

(1) On the back of MOUTHS list 2, line 6, it is implied that Birmingham has moved. This is not so - it is still "just off the **M6**".

(2) Both Steve Murphy and Peter Weir pointed out that the formula given in M500 30 12 converges to $4/\pi$ (not $\pi/4$). Here it is again, simplified:

$$\frac{4}{\pi} = \frac{1123}{882} + \sum_{n=1}^{\infty} \frac{1123 + 21460n}{(-1)^n 882^{2n+1}} \cdot \frac{4(n-1)!}{2n(3n+13)/2(n-1)!} .$$

I won't admit to how long that took, but I couldn't have managed without Bob Margolis's help.

(3) Max Bramer writes: I am very suspicious about Steve Murphy's remarks about an "astroid" (M500 31 5). However, as I cannot really understand the meaning of "as the step length tends to 0, increasing n so that $nq = 1$" I cannot be sure he is wrong.

Note that there are two misprints on page 4. Formula 4 should have $+Q^2$, not $-Q^2$ and the formula for y is really

$$y = x \sin 45° + y \cos 45°.$$

(4) I seem to have left some manuscript additions out from page M500 31 18; although it's fairly obvious that, in problem 31.2, $n \geq k$ and in 31.3 $n \geq r \geq 0$. EK

<u>RIEMANN IN TROUBLE</u> In the spring of 1854 a young German mathematician named Bernhard Riemann was greatly worried about his future and about a test he faced immediately. He was already 28, and still not earning - he was living meagrely on a few thalers sent each month by his father, a Protestant minister in a small Hanover town. He wrote modestly to his father and brothers that the most famous university professors, in Berlin and in Gottingen, had unaccountably been extraordinarily kind to him. He had his doctor's degree; now, to obtain an appointment as a lecturer (without stipend), he had to give a satisfactory lecture before the whole Faculty of Philosophy at Gottingen. He had offered three subjects. "The two first ones I had well prepared," Bernard wrote to his brother, "but Gauss chose the third one, and now I'm in trouble..."

For the thoughts which Riemann did make public on that June day of 1854, and what Gauss thought of him, see "The Curvature of Space", by P Le Corbeiller in *Mathematics in the Modern World: Readings From Scientific American*, Folkestone: Freeman, 1968; from which the above is quoted.

MS

"THE BULLNOSE AND FLATNOSE MORRIS"

A book by Lytton Jarman and R I Barraclough published by David & Charles.

A new edition (March) of this chronicle of "the car which moulded motoring history". From before the Great War to the early 30's the documentation covers the 6-cylinder models, Leon Bollees, Morris Commercials, Empire Oxfords and MGs; with another chapter for Bean Cars.

The book was originally published in 1965 and the new edition takes the story of the Flatnose up to 1934 with sixteen new line drawings and eight photographs.

Send £6.62 to Lytton Jarman, 27 Oakfield Road Rugby for your signed copy.

AMATEUR COMPUTER CLUB

Richard Shreeve

The ACC is open to anyone interested in the design, construction or programming of computers, as a hobby.

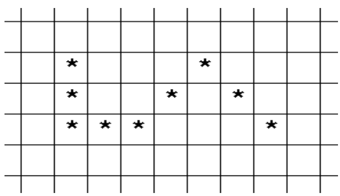
Membership of the ACC for the year 1 April 1976 to 31 March 1977 costs £1 (50p for UK members aged 16 or under on April 1st 1976), and includes the subscription to number 4 of the *ACC Newsletter* (6 issues, starting from April 1976.) Copies of volumes 1, 2 and 3 of the *ACC Newsletter* are still available at 75p for volume 1 and £1 each for volumes 2 and 3.

The ACC organises seminars and visits to computer users and manufacturers, supports local amateur computing groups, designs computer equipment such as the 'Weeny-Bitter' £50 computer (Volume 3) and a cheap keyboard/VDU (Volume 4), and communicates with the international membership through the Newsletter.

The *ACC Newsletter* carries articles on soft and hardware computer techniques, data on interesting new devices, general news and members' letters.

Enquiries to M. Lord, 7 Dordells, Basildon, Essex, SSI5 5BZ.

For all *Life* addicts, what happens to this:



I have followed it for 135+ generations with no signs of stabilising.

FIVE SUNDAYS

Lewis Johnson

The other day my wife walked in and said ‘Did you realise that there are five Sundays in February this year, you are a numerical analyst; when did this last occur and when will it occur again?’ Actually I dislike numerical analysis but can any of you readers pick holes in the following?

Normally the day for a given date moves forward one each year, except leap years when it moves forward two days. Since every fourth year is leap (normally) any date moves forward five days in four years. So In $4n$ years It will move forward $5n$ days, $n \in \mathbb{N}$. So for the day-date to repeat (i.e. February 1st again to be a Sunday), and the year leap, $5n$ must be the least multiple of 7; therefore n=7 and “the years required $4 \times 7 = 28$. So the situation last obtained in 1948.

Will it recur in 2004? No; since the year 2000 is not a leap year in the Julian Calendar (sic) *. So In 2005 we shall be a day short, i.e. February 1st 2005 is a Saturday.

In a subsequent span of $4n$ years from 2000 the day will move forward $5n$ places and this must equal $7m + 1$ $n, m \in \mathbb{N}$ to reach a Sunday. We thus have the Diophantan equation $5n = 7m + l$, and the least solution is $m = 2$, $n = 3$. So $4n$ equals 12 bringing us to the year 2016. By which time all you eager young M100s should be graduates.

*Ed - Some of us have been using the Gregorian Calendar since 1582.

PRONUNCIATION

John A Wills

If a German tries to English his pronunciation of Manchester he comes up with Menchester, revealing that he does not know much English. If he sticks to a straightforward German pronunciation no-one knows how much English he knows.

Mutatis mutandis the same for the speaker of English. Someone without French may get by with ‘Paree’ but we think he’s a bit silly. During the year OU-1 I heard an S100 lecturer trying to German ‘Einstein’. Someone had told him that ‘stein’ is pronounced ‘shtein’ and extended the principle to Einstein. Goofed again: st is sht only at the beginning of words.

No-one can learn all the pronunciation schemes (some of which approach the lunacy of English) of languages mathematicians have spoken. By pretending we demonstrate our ignorance and in no way improve communication.

Anytime things appear to be going better you have overlooked something.

Chisolm’s Second Law of Human Reaction.

INAUGURATION OF AN M100ER Nicholas Fraser

FOREWORD - Having started the M100 course and met some of my fellow students I wonder if I am typical.

ACT 1 Scene I: Home. August 1975.

My God! I've been offered a place; I'd forgotten all about it. Point is though do I want to do it?

Of course you do that's why you applied isn't it?

I suppose so, but will I be able to do the course; it's totally different to anything I've done before!

Scene II: Home. September.

Refresher booklets - crikey I don't know anything, what am I going to do when the course starts proper.

Scene III: Preparatory course. October.

5 of us start the course but by the end of the 5th week only 2 of us are left! Is the course proper going to have this effect?

Tutor doesn't deal with the RB material but elements of the course. This stimulates but I am still worried about those refresher booklets.

Scene IV: New student meeting. November.

20 bemused students talked at by student who's experienced it all. Trouble is, Ml00 for him is 3-4 years previous; no one to tell me what it's like.

Scene V: Course material. December.

5 parcels arrive packed with material; heck do I have to read all this stuff now. CMA, TMA, jargon galore. How to study, Handbooks for everything except for going to the toilet in the study centre!

Scene VI: Course material. January.

I start to study, find it hard going but it is enjoyable. This notation is going to be difficult to remember. Apart from that is the more serious problem: I can't even draw the curly brackets!

Scene VII: M500

What on earth is that all about? All this high falutin talk on abstruse maths problems. Ah! at least there is some English written. Wait a minute, I can do that problem. GREAT.

ACT 2 (Takes place at the same time as Act 1.) Scene I: Work.

Well boss, I am about to do an OU course. (Double take) Are you, Oh! (Pause) Let's talk it over at the pub.

Scene II: Pub.

I think it's a good idea. We may be able to help you. Ask Personnel.

Scene III: Personnel.

Business not good; but we are prepared to sponsor you (Great start.)

Scene IV: Friends.

I am going to take an OU course. (Double take) Are you Oh! (Pause) What in?

Well you don't take it in anything but I am doing maths first. Are you good at figures then?

Well it's not really about figures; it's New Maths. (Bemused look. End of conversation.) (This scene may be repeated 20 times.)

Scene V: Pub; evening.

We don't see you much now Nick; Well I'm studying with the OU. Are you clever then?

where've you been getting to? (Double take)

Scene VI: Pub; lunchtime. What's new maths then?

Well I've only just started the course so I can't really tell you properly.

What are you studying there?

It's about Operations and Morphisms. (Blank look from the both of you because you are trying to work it out, so how on earth can you be expected to explain it!)

Scene VII: Friends.

Why are you doing it then?

How does it work?

How could I get on it? (Rare)

Well, it's going to be hard. (I know!)

I wouldn't do it.

I couldn't do it.

<u>ACT 3</u> ?

M100 AND MST281: COURSE RESULTS 1971-1975

year course	finally registered	examined	distinction		pass	fail	pending
1971 M100	4539	3423	390	11.4%	2389	644	
1972 M100	3818	2629	368	14.0	1772	489	
1973 M100	2656	1876	354	18.9	1286	236	
1973 MST281	1105	777	167	21.5	484	126	
1974 M100	1959	1350	274	20.3	967	109	
1974 MST281	695	540	122	22.6	375	43	
1975 M100	2507	1772	293	16.5	1227	246	6
1975 MST281	646	467	71	15.2	343	30	23

E F R 0 N   D I C E   Bob Curling

Following the publication in M500 30 of Mike Hodgson's request for information concerning the Efron Dice, I remembered being intrigued myself by a mention of them at Reading last summer. A hazy recollection of "sure winner" prompted me to look through my notes made at Summer School until I finally found a reference. I have only details of dice numbering but assume the game is played as fellows.

Two players decide to play a game wherein each chooses one die from the four Efron Dice after which they throw their respective dice simultaneously. The one with the higher score wins. All very simple and straightforward.

But the owner of the dice (and the villain of the piece) can ensure that he always has a greater probability of winning than his opponent. This he does by appearing to be very generous and allowing the opponent to choose his dice first.

If my probability theory is correct, I reckon that if B has first choice of die then A can always choose one of the remaining dice so as to give the following probabilities:
P(A wins) = 2/3; P(B wins) = 1/3.

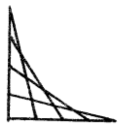
The dice are numbered as follows:

1)	2	3	3	9	0	11
2)	0	1	7	8	8	8
3)	5	5	6	6	6	6
4)	4	4	4	4	12	12.

So in future beware of the owner of 4 dice offering a "fair" game. Confound him by insisting he chooses his die first!

CONSTRUTCTIONS II   Richard Ahrens

On Roger Claxton's constructions (M500 30); Max Bramer astonished me by actually fitting a curve to those points of intersection, because I had thought that Roger had asked the wrong question. If instead of looking for the curve through the point3 of intersection you find the curve which has all the threads as tangents, then problems about step length disappear. Putting in more threads just gives a better approximation to the same curve,



Problem: Suppose we have a family of straight lines whose equations are given by an expression of the form

$$a(t)\cdot x + b(t)\cdot y = c(t)$$

a, b and *c* are differentiable functions. Each value of *t* gives the equation of one line in our family. (In Roger's original problem all his threads belonged to the family $\frac{x}{10-t} + \frac{y}{1+t} = 1$, $t \in \mathbb{R}$.) Prove that the curve that has all these lines as tangents is given by eliminating *t* from

$$\begin{cases} a(t).x + b(t).y & = ct \\ a'(t).x + b'(t).y & = c'(t). \end{cases}$$

I don't see how Steve Murphy could have obtained an "astroid" as the limiting curve since it is not true. The limiting curve is still a parabola. You can get an astroid by using stitches of equal lengths - not equal spacings at the ends as Roger prescribed.

CHESS John Parker

Roger Claxton writes to say his chess set is flourishing, and it is well known that we have some excellent programmers at our disposal. So why aren't we writing a chess program?

I would suggest that the main program would be outlined in M100 (any offers?), tossed around for a couple of months, agreed to by a committee, and then individuals delegated to actually code it into BASIC, type it up and store it; each individual to control a section of program. The committee to decide (a) main outline, (b) variables permitted.

If the general consensus of opinion is that this task is too bold , then a preliminary draughts program may well be in order. A big difference, but no mean task!

For the chess program I would outline as follows:

1 - Start
2 - Read a move
3 - Consider all possible moves, writing each on a file
4 - Consider all replies for every possible move
5 - Determine and play best move
6 - Print board on request
7 - Return to step 2.

Obviously it would be necessary to include in the loop routines to determine the winner, routines to detect illegal moves etc; but OK let's start somewhere. Steps 3 and 4, carefully controlled, should be repeated as often as backing store permits.

I am well aware that I have over simplified the problem, so please don't tell me so!

My old cosen, parson Whitney (James Whitney, c1680), told me that in the visitation of Oxon in Edward VI's time, they burned mathematical bookes for conjuring bookes, and if the Greeke professor had not accidentally come along, the Greeke testament had been thrown into the fire for a conjuring booke too. - Aubrey: *Brief Lives*

(from Jeremy Humphries)

M500 Max Bramer

After reading back issues of M500 right down to number one I am inclined to think that Roger Bridgman and John Hampton *both* have a point (M500 29).

The items published in M500 range from the sophisticated (Hoops, Bob Margolis's Point Construction problem), through the well-polished (the Cubic Hypercube solution in M500 28), the obscure, to the speculative/dubious (private list supplied on request!) and simple errors of fact. Unfortunately it may take a great deal of experience to be able to tell which is which. No wonder some readers think the material is trivial while others think it is far too difficult for them! Marion Stubbs's summary of M100 Unit 2 was a good idea, except for the two definitions of r_x (which one is right?) and the mention of the illicit Leibniz notation; ($f'(x)$ *is* like dy/dx, roughly speaking - but those two words need a lot of explaining).

Surely it's time to think about exercising a little more editorial control over what is published - or at least more editorial comment. It really does not help anyone to print articles they cannot understand, particularly if they are wrong! What about setting up an (unofficial and unpaid) editorial panel of volunteers to help 'vet' material and pick up any errors? That is the normal practice elsewhere. The final decision would of course remain with the editor. For a small bribe I would comment on computing topics, M100 and interesting looking problems myself. Any other volunteers?

Ed - Roger Bridgman felt that there are too many ideas packed into too small a space so that it is difficult to concentrate both on eating one's kipper and filling in the gaps in an argument at the same time. John Hampton would like to see fewer issues with the mathematics more editorially selected.

When an article comes in for publication in M500 I read through it; if I can understand it and it seems to make sense, in it goes. If I can't follow it but it's from a member of staff I print (after all one must believe in something); if not I either put it in anyway if it looks pretty enough or I hold onto it and write to the author for further explanation - and *rarely get a reply* (I hope you are squirming out there).

Anyway if someone writes lousy mathematics you can use that as a warning not to ring him for help with your own problem. Or perhaps you can ring him and point out his mistake and so start a collaboration.

When a thing has been said and well said, have no scruple; take it and copy it. Give references? Why should you? Either your readers know where you have taken the passage and the precaution is needless, or they do not know, and you humiliate them. -

Anatole France.

NOTATION Coral Bytheway

In one mindless moment my spouse declared ‘since sec is just cos to the minus one, why don’t we forget about secs’. Are people suffering from secs-phobia turned on by cos’s? (Cossie: Northern dialect for bathing costume.)

But seriously why does conventional mathematics (particularly in M251) neglect the minus power notation; i.e. why write a^{-1} as $\frac{1}{a}$ (thereby sabotaging a simple linear arrangement of symbols) and why write $\cos^{-1} x$ as $\sec x$? Indeed why not save hours of agony and teach differentiation of functions of functions with a liberal use of brackets; i.e. why not write $\cos^{-1} 3x$ as $(\cos(3x))^{-1}$ and

$$\frac{1}{\sqrt{\left(1 - \cos^3\left(\frac{x-1}{x+1}\right)\right)}}$$

as $(1 - (\cos((x-1)(x+1)^{-1}))^3)^{-½}$?

In this way students who are methodical and take easily to computer programming will acquire some increased mathematical insight and those full of the mathematical art might learn to be a little more error free. Incidentally it could also be typed on an ordinary typewriter with no mathematical symbols!

CONCEPTS OF MODERN MATHEMATICS - I Stewart, Pelican, 80p.

Brian Woodgate

Having entered M100 from an engineering background, I was confused by the abstract contents of the course, when I wanted to do what I called ‘real’ mathematics.

The above paperback would have helped as it includes Abstract algebra; Linear algebra; Analysis; Groups and Topology, etc. It is readable and concise; e.g. fields and rings are simply explained in a few lines; and it has lighter points such as a treatment of the game of solitaire.

For a bonus there is Gődel’s theorem which shows that we have no firm foundation in mathematics, not even ‘set theory’.

<u>From Jeremy Humphries</u> - Put me with Ken Hegerty (M500 31 9) on the anti-smoke list. Exposure to the foul stuff quickly makes me ill. How much longer will this method of assaulting other people be condoned?

I asked the OU why they didn’t ban smoking at Summer Schools.

‘Oh - we couldn’t do that - some people can’t work without smoking.’

Well, hard luck, say I. It’s their fault and why should I suffer for it? Suppose I couldn’t work without playing the bagpipes. Suppose I couldn’t work without punching people on the nose. What’s the difference?

SUPERFIELDS

Datta Gumaste

I. Consider a field, say F.

Then (i) $(F, G, +)$ is an abelian group. Call it G_1. + is the binary operation of 'addition' and 0 is the identity.

(ii) $(F - \{0\}, 1, \bullet)$ is an abelian group; call it G_2. • is the binary operation of 'multiplication' and 1 is the identity.

(iii) • is distributive over + (on both sides).

II. Ask the following question: is it possible to define another group, G_3, as follows.

(a) $\{F - \{0, 1\}, e, \times)$ where $\{F - \{0, 1\} = G_3$ is a group, $\times$ is the binary operation and e is the identity;

(b) $\times$ is distributive over • on both sides.

III. This is an attempt to provide a partial answer to this question. We demonstrate that there do exist some finite fields for which G_3 can be so defined.

Consider first $F = \{0, 1, 2, 3\}$; $o(F) = 4 = 2^2$. See tables.

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

G_1

•	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

G_2

×	0	1	2	3
0	0	0	0	0
1	0	1	1	1
2	0	1	2	3
3	0	1	3	2

G_3

G_1 represents additive group, identity 0.

Non-zero entries in the second table represent G_2, identity 1, non-zero and non-one entries in the third table represent G_3, identity 2.

• is distributive over +, and $\times$ is distributive over •. Also note that $a \times 0 = 0$, $\forall a \in F$.

IV. Consider a more general case. Let F be some finite field such that $o(F) = 2^k$, $k \in \mathbb{Z}^+$ and $2^k - 1 = p$, p prime. We know that $F = G_1$ is a group. $F - \{0\}$ is a group, G_2 say, and $o(G_2) = p = 2^k - 1$. So G_2 is in fact a cyclic group because its order is prime. We claim that G_2 can be turned into a field, and hence we get another group $G_3 = F - [0, 1\}$ which satisfies conditions (a) and (b) above.

V. $G_2 = F - \{0\}$ can be made into a field.

(1) First note that there is a theorem which says that if C_n is a cyclic group of order n then it is isomorphic to $(\mathbb{J}_n, +)$, the cyclic group of integers modulo n.

(2) Apply this result to the group G_2 :-

G_2 is cyclic $\Rightarrow G_2 \cong (\mathbb{J}_p, +)$ where $p = 2^k - 1$. Now because p is a prime we know that the integers modulo p form a field $(\mathbb{J}_p ,+, \bullet)$ (see Herstein, p.91), so if f is an isomorphism from $(\mathbb{J}_p ,+, \bullet)$ to G_2 we can use f to induce an operation '$\times$' on the elements of G_2 corresponding to the operation '$\bullet$' in $(\mathbb{J}_p ,+, \bullet)$. All we need to do is define $\times$ by: $f(a) \times f(b) = f(a \bullet b)$, $a, b \in \mathbb{J}_p$. Then $(G_2, \bullet, \times)$ is a field isomorphic to $(\mathbb{J}_p, +, \bullet)$.

(3) Since G_2 is a field its multiplicative group is cyclic (see Herstein p.317). Call it $G_3 = G_2 - \{e\} = F - \{0, e\}$. Further, the binary operation $\times$ is distributive over $\bullet$ in G_2. (Problem: We have not defined $\times$ for the zero of our original field. Prove that $\times$ remains distributive over $\bullet$ if and only if $a \times 0 = 0, \forall a \in F$.)

VI. To summarise, we have shown that if F is a finite field such that $o(F) = 2^k$ and $2^k - 1 = p$, p prime then (1) $(F, 0, +) = G_1$ is a group; (2) $(F - \{0\}, 1, \bullet) = G_2$ is a cyclic group; (3) $(F - \{0, 1\}, e, \times) = G_3$ is a cyclic group; (4) $\bullet$ is distributive over $+$ on both sides; (5) $\times$ is distributive over $\bullet$ on both sides.

VII. Call such a field a superfield. The primes which satisfy $2^k - 1 = p$, $k \in \mathbb{Z}^+$, are called Mersenne primes. The argument above shows that with each Mersenne prime we can associate a superfield.

I am grateful to Richard Ahrens who gave me the essential clue to solve the problems that I have posed in II above.

MONGE'S SHUFFLE

From Steve Murphy - One thought someone might find more useful than I have is the function

$$f\colon x \longmapsto \begin{cases} ½(2p + x + 1), & x \text{ odd} \\ ½(2p - x + 2), & x \text{ even} \end{cases}$$

(M500 30 9) could be written as

$$f\colon x \longmapsto p + ¾ + ¼((2x + 1)(-1)^{x+1}, \quad x \in \mathbb{Z}.$$

This suggests (no more) that 'x_0' should appear as an exponent on the RHS of the relation:

$$2^{m+1}x_m = (4p + 1)(2^{m-1}+(-1)^{m-1}(2^{m-1}-1)) + (-1)^{m-1}2x_0 + 2^m + (-1)^{m-1}.$$

Is it possible that there is a misprint?.

From Krysia Broda - If one lets

$$g(x_m) = ½(2p -(-1)^{x_m} \,.\, x_m + \frac{(-1)^{x_m} + 3}{2}$$

then the result (provable by induction) becomes

$$2^{m+1}x_m = (4p + 1)(2^{m-1} - \sum_{i=0}^{m-2}\left(-1\right)^{i+1}2^i P_{i+1}) + 2^m + 2x_0P_0(-1)^m + P_0(-1)^{m-1}$$

where $x^m = f^m(x_0)$ and $P_i = (-1)^{x_i}(-1)^{x_{i+1}} \ldots (-1)^{m-1}$.

This is not much use for calculation. So I looked at the largest cycle for each p. For those I looked at there seemed to be no pattern except that, for those p for which there was only one cycle number, $4p + 1$ was prime; and that for all the others it was not.

A LETTER (the only one this month) *from Richard Ahrens*

I was puzzled by Philip Newton's impassioned note on behalf of the Society for the Prevention of Discrimination Against Deltas - Kronecker's, Dirac's or the Nile's (M500 31 12). I must confess I was also a little suspicious - Why did he not tell us what index he was using on the second Σ ? Could it have been *i* or *j*? Was his tutor right after all?

I had a feeling once about mathematics - that I saw it all, Depth beyond Depth was revealed to me - the Byss and the Abyss. I saw - as one might see the transit of Venus or even the Lord Mayor's show - a quantity passing through infinity and changing its sign from plus to minus. I saw exactly how it happened and why the tergiversation was inevitable - but it was after dinner and I let it go. -

Winston Churchill (from RA)

SOLUTIONS

30.2 *THE OU CAB.* If 600 = CAB what is 342,764,853,755,904?

$600 = 2^3 \times 3^1 \times 5^2$ - i.e. successive primes, each raised to the power of the corresponding letter's position in the alphabet.

Hence $342764853755904 = 2^{15} \times 3^{30}$ = OU. - Ken Wogan.

30.3 *POINT CONSTRUCTION*. There is a solution for this, by Steve Murphy; but at the moment it is somewhere between him and Bob Margolis. We hope not lost.

31.1 *EULER'S POLYGON DIVISION*. Not a solution. We have been told that the formulation of the problem was ambiguous. It might have been better as:

> In how many different ways can a plane convex polygon of n sides be divided completely into triangles by non-intersecting diagonals?

Two congruent shapes are different unless one is a translation from the other, in which case they are same but the universe is different. So even a square can be divided in two different ways . Euler gave the problem to Goldbach in 1751.

He said 'The process of induction I employed was quite laborious.'

31.2 *MRS READ'S KNITTING MACHINE*. (The problem (M500 31 18) specified the simultaneous use of k spools with thread wound on them. If $s_1, s_2, \ldots, s_n$ are n ($\geq k$) spools containing quantities of thread $t_1, t_2, \ldots, t_n$ what condition must the numbers t_i satisfy to be able to avoid winding from one spool on to an empty one. Suppose the t_i are such that it is possible to knit all the wool without rewinding a spool, show that no more than $n - 1$ stops to change spools are needed; find an algorithm to minimise those stops.)

A necessary condition on $t_j^{(i)}$ is that $\forall j$, $j = 1, n$, $t_j^{(i)} \leq \frac{t(i)}{k}$, where $t_j^{(i)}$ is the amount of yarn on s_j before step i and $t^{(i)}$ is the total amount of yarn before step i, $i = 1, 2, \ldots$. We would like i to go from 1 to n giving at most $n - 1$ stops.

Algorithm At each step i,
(1) Order the spools so that $t_1^{(i)} \geq t_2^{(i)} \geq \ldots \geq t_k^{(i)} \geq t_n^{(i)}$.
(2) Choose $d^{(i)} = \min\{t_k^{(i)}, t^{(i)}/k - t_{k+1}^{(i)}\}$.

The first restriction is obvious, and the second comes from the fact that at each step (1) must be satisfied by all t_j. It is easy to show that if (1) is satisfied at step i then after removing $d^{(i)}$

from $t_1, ..., t_k$ (1) is still satisfied by all t_j

(3) Remove $d^{(i)}$ amount from $t_1, t_2, ..., t_k$. If any yarn left go to i again.

It is easy to see that because of (1) if at any stage there are only k spools left with yarn on them these spools must have t^i/k on each - and hence the algorithm stops. Also it is impossible for the algorithm to leave more than $n - k$ spools empty at any time, except at the end.

Proof. At each stage either one or more of the t_i is reduced to zero and/or one or more t_i becomes maximal. Thus if we take the minimum that can happen at each stage (to maximise the number of steps) we find the following: the very last step removes wool from k spools, so that the step before reduces one or more spools to zero *and* makes one spool maximal; so leaving these two steps out we have: number of steps = $(k - 1) + (n - k - 1) = n - 2$. (✳) With the last two steps we get the maximum number, n. The two terms in (✳) come from realising that once a spool becomes maximal it stays so, and hence $k - 1$ spools can become maximal in the steps considered, and at most $n - k - 1$ can go to zero. (We must have $k + 1$ non-empty spools at the beginning of the penultimate step, else all spools would have equal amounts of wool and the step would not be penultimate.) - Krysia Broda.

31.4 *HOW OLD* are Tim, Jane and Mary?

(3, 8, 15) - Krysia Broda;

(25/3, 46/3, 77/3) - Mervyn Savage.

31.5 *CUT WIRE* - Two cuts are made at random in a piece of wire. The probability that the three pieces will form a triangle is: 1/4. - Krysia Broda, Bill Shannon.

(Two footnotes. A shower of mail pointing out that the Fibonacci coprimes proof was no such thing. I know. Just interchange $F(n + 1)$ and $F(n - 1)$ after hence and use some imagination. Surely no-one wants full TMA-answers to that kind of thing? And a heap of calculations from Krysia on problem 31.1 - Euler. It ends with the formula for an $(n - 2)$gon of $2n!/n!(n + 1)!$ It's different from Euler's but gives the same results up to the decagon so is probably correct. I'll give Euler's next month.)

PROBLEMS

32.1 *SUPERFIELDS* - Datta Gumaste. (See page 12)

i) Are the Mersenne superfields the only finite superfields?

ii) Are there any infinite superfields?

iii) If F is a superfield, is $\times$ distributive over + ?

iv) Let S be some non-empty set. Consider the set of all functions on S into F where P is a superfield. Call such a set T; i.e. $T = \{f : f \text{ is a function on } S \text{ to } F\}$. Can we transform T into a superfield?

32.2 *NOUGHTS AND ONES* - Richard Ahrens

If

$$\begin{aligned} K = {} & (x_1 - x_2)^2 + (x_1 - x_3)^2 + (x_1 - x_4)^2 + \dots + (1 - x_{2m})^2 \\ & + (x_2 - x_3)^2 + (x_2 - x_4)^2 + \dots + (x_2 - x_{2m})^2 \\ & + (x_3 - x_4)^2 + \dots + (x_3 - x_{2m})^2 \\ & + \dots \\ & + (x_{2m-1} - x_{2m})^2, \end{aligned}$$

$$= \sum_{\substack{i,j=1 \\ i<j}}^{2m} (x_i - x_j)^2$$

find the maximum possible value of K if it is known that each x_i ($i = 1, 2, ..., 2_m$) has the value 0 or 1.

32.3 *FIND THE NEXT TERM* - Eddie Kent

43, 50, 55, 65, 76, 89, ... ,

taking into account the following information: the sixth number minus the first is equal to the sum of the digits in the first five numbers.

32.4 *THE FALLING STONE* - Bill Shannon

A stone falls the last half the height of a wall in half a second. Find the height of the wall. (A two line solution is possible.)

32.5 *RATIONAL TERMINATION* - Max Bramer

What rational numbers a/b have a finite (i.e. non-recurring) N-cimal (e.g. decimal, bicimal) expansions in base N. Prove your result.

EDITORIAL

I hope someone can make something out of John Parker's Chess piece on page nine, and that I haven't made too many mistakes in copying it. That's the trouble when you have to transcribe something written both in lousy handwriting and a foreign language. There was also a computer printout but I really didn't understand that at all. Perhaps John will write and say if there is something he wants me to do with it. I am quite happy to print anything you send along but you must bear in mind that I know nothing about computing having undertaken the minimum at M100 (just enough to 'Get Snoopy' and play blackjack). I lost interest when it became clear that the instruction 'wipe' (or whatever it was) did not clean out the entire WH library. I find computing the most footling activity known to man. It should be left to trained apes. Why, the machine even made a mistake on that 'Get Snoopy' printout.

Max Bramer (page ten) having read M500 back as far as issue one must have seen the Ketley saga. I trust what he has to say doesn't raise that spectre again. But at a slight tangent to Max's views I would like to ask for a little help. Every so often an article in M500 gives birth to a stream of others on the same subject. Hoops did it; and Pi. Now I have bundles on Monge, Coincidental Birthdays and Constructions. Each of these starts with some common basic ideas then goes on to develop them in individual ways. So there is something of value in each contribution and much that would be repetitive to continuing readers (though not to new subscribers).

What is needed therefore is someone to go through a particular subject and edit an article out of the mass of information and ideas, giving credit where due. Perhaps the simplest way of doing this would be for intending contributors to contact the original author first, and he could perhaps organise some sort of forum to construct a follow-up article. Of course in the last resort I am always prepared to do the job myself. And many people just might not like the idea and will insist on complete publication or nothing. But it's worth thinking about.

So we finally ended up with two letters in this edition. That has tilted the bias rather sharply in the direction of straight mathematics and away from the old duplicated newsletter image. Well if that's what you want. I suppose everyone is now getting down to serious OU work and has no time for fripperies; never mind. But don't forget that there are eighteen pages to fill every month.

And now a story to bring tears to your eyes. It's about this poor old typewriter - a staggeringly old Olympic portable. I took the top off because I had to keep changing the ribbon, and lost it; then some screws came out and it feels very unstable and one of the sides gets mixed up with the spacer bar occasionally. The shift feels a bit spongy and won't go all the way down (if you look hard you can see the capitals are slightly high); three of the keys almost always stick and three more are loose and don't always land in the right place (thank heaven for Tippex) so please support your equipment fund or we might not make it through the year.

Eddie Kent