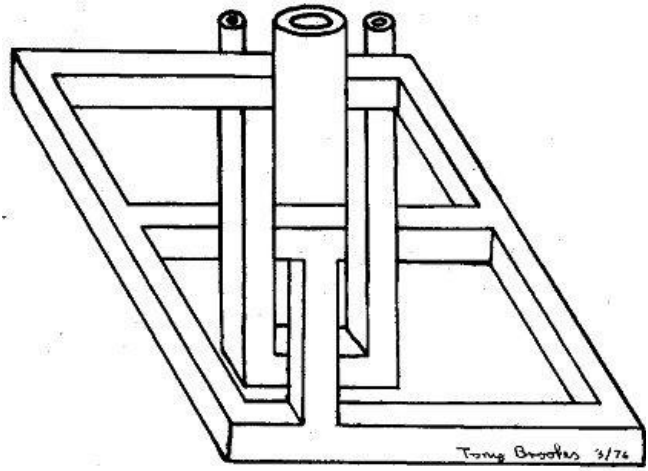

M500 33

M500 is a non-profit student-operated and student-owned magazine for Open University mathematics students and staff, and for any others who are interested. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct but invite criticism and comment. Articles submitted for publication should normally be less than 600 words in length, although subjects which require more space may be split by the authors into instalments.

MOUTHS is a list of names addresses and telephones, together with previous and present OU courses, of voluntary members - by means of which private contacts may be made by any who wish to share OU and general mathematical interests or who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned, and do not necessarily represent those of either the editor or the Open University.

The cover design is by Tony Brooks, who writes: In the impossible picture I have tried to combine two impossible shapes into one, so as to create as much visual confusion as possible.

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Articles, letters, problems, solutions, *coverdesigns* for inclusion in M500 to the editor: Eddie Kent.

Membership applications, subscriptions, change of address, MOUTHS data to the membership secretary, Peter Weir.

The Treasurer is Austen F Jones.

Cheques and postal orders, which should continue to go to Peter Weir, should be made payable to *THE M500 SOCIETY* and crossed "a/c payee only not negotiable" for safety in the post.

Anything sent to any of the above and not marked as personal will be considered for possible inclusion in M500.

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COULD YOU HAVE WON A PUBLIC SCHOOL EDUCATION?

Richard Ahrens

The following question appeared in a recent Common Entrance Exam – taken by thirteen-year-old applicants for places at Public Schools.

> Write the numbers 1 to 20 in a column. In a second column, alongside the first, write the first twenty terms in the sequence 1, 3, 4, 7, 11, ..., where further terms are obtained by adding the two previous terms. In a third column write the remainder when each term in column 2 is divided by the corresponding term in column 1. Thus the sixth term in column 3 is 0 because 18 when divided by 6 leaves a remainder of zero.
>
> There is a theorem relating numbers in column 1 to numbers in column 3. What do you think it is? Test your guess by extending the table a few more rows.

If you want to answer the question at the top of this article you had better stop reading and have a try at the problem because I am about to say what I think the expected answer is.

Let us call the sequence in column 3, $\{v_n\}$, then my guess at the theorem is:- $\forall n$, n prime $\Rightarrow v_n = 1$.

Of course the question set did not ask for a proof but M500 readers won't be satisfied until they have proved it.

Now I am quite sure that this one theorem does not exhaust the interesting properties of the sequence $\{v_n\}$ and I would like to suggest that M500 should form a collection of results about this and related sequences.

A few questions that suggest themselves to me are :-

1) Is the converse of the above theorem true? i.e. does $v_n = 1$ imply n is a prime?

2) Is $v_{2^k} = 2^k - 1$ a theorem?

3) For which n is $v_n = 0$?

It would be useful if someone who enjoys computing would calculate rather more terms of $\{v_n\}$, so that conjectures can be tested more thoroughly. I suspect that we will find that the subject should be broadened to include the Fibonacci type sequences in the arithmetic of remainders modulo n.

Ed - I shall begin by suggesting an answer to question 3. $v_n = 0$ for $n = 6k$, k odd. Construct a fourth column, $\{w_n\}$, of elements in column 2 mod 18. This column is cyclic with period 24. Also it appears that $w_{6k} = 0$; $w_{6k+j} + w_{6k-j} = 18$, j even, or 0, j odd; $w_n + w_{n+12} = 18$.

FIVE SUNDAYS

Eddie Kent

In an article under this heading last month Lewis Johnson asked if any reader could pick holes in his analysis of the five-Sunday-in-February phenomenon. As editor I feel in a somewhat ambivalent position about contributing (you know the e e cummings poem about “mr u will not be missed who as an anthologist sold the many on the few not excluding mr u”) and though I frequently write replies I usually tear them up later.

However on this occasion I feel a somewhat proprietary interest having raised the subject of calendars myself back in M500 24 (“Monday’s Child”). Among other items of wisdom I gave the definition and derivation of Gregorian Calendar. Briefly it was a device to shorten time by discarding some of Julius Caesar’s leap years. (JC had believed there were 365.25 days in each year whereas there are nearer 365.242, an error of about 0.8 days per century or 0.002%.)

In the Julian Calendar every fourth year was a leap year. But after Pope Gregory XIII in 1582, of the years ending in 00 only those whose century number is divisible by 4 are leap years. Referring to the Gauss formula given in my article (and making the necessary adjustment for February) it can be seen that February 1st 1976 and February 1st 2004 are both Sundays. February 1st 2016 leads to $(4 + 1 + 0 + (6 - 2)) \bmod 7$ which is 2, or Monday. February 1st 2005 $\equiv (4 + 1 + 0 + (6 - 1)) \bmod 7 = 3$, Tuesday.

Of course Gregory’s adjustment wasn’t enough, but by less than 0.001% (2 days a millenium, actually) and there might have been alterations to the calendar since Gauss’s time. If there have been and Lewis knows of them could he perhaps give references?

From Ron Aitken - Your last paragraph in M500 32 has touched me - as intended. Small donation enclosed. If all 365 members gave about 50p you’d be OK for a year or so. How about it, fellow MOUTHS?

Nicholas Fraser’s chronicle is similar to my experiences to date. I have found MOUTHS of immeasurable benefit in providing a sounding board at the instant I need it off which I can bounce my interpretations of the underlying ideas. Although I talk to myself - and listen - it’s a great help to involve someone who has met and surmounted my immediate problem. Thanks again to the benefactors of clods like me who began and persisted with M500/M0UTHS.

Chess problems are the hymn tunes of mathematics.

G H Hardy.

SMOKING AT SUMMER SCHOOLS

John A Wills

Ken Hegerty (M500 31 9) and Jeremy Humphries (32 11) are right to protest about smoking at Summer Schools. All OU students are over the age of 18 and have no cause to smoke except for personal enjoyment. None of us needs to demonstrate how big he is by walking around with a dummy in his mouth. It did not occur in lectures and tutorials in the year OU 1. It should never have started, particularly as most of the host universities forbid their own students the affectation in lecture halls and we show great discourtesy by ignoring our hosts' not unreasonable preferences. If there really is anyone who cannot work without smoking he should be in hospital. As nicotine is a depressant its users can in general study less well than its abstainers: those who give it up for the Summer School week will be better off than those who persist. And staff should obviously not distract or annoy students by smoking on duty.

Lytton Jarman

It is the first time I have ever seen anyone speak out for those of us who loathe tobacco smoke in all its aspects. Surely there must somewhere be a chemist who could produce a deodorant that would be pleasant or odourless to non smokers but make smokers feel as ill as we do when they contaminate the air and even our clothes with their stench. If they want to smoke they should erect little airtight conveniences where they can set fire to whatever they wish in total privacy. May I join Jeremy Humphries and Ken Hegerty as founder members of the Smokeless Summer School Society.

COMPUTING

John Parker

You state the criteria for publishing an article on page 10 of M500 32. Since my article was not pretty, you could not understand it, and I am not a member of staff, why on earth did you print it? As to the printout, I *did* include instructions as to what to do with it. But maybe you couldn't read them. Your attack upon my handwriting was fully justified, I bow humbly before the truth.

Your attack upon computing, however, was not justified, and clearly stems from your (admitted) lack of education in this field. Computing the most footling (footle: to bungle; to be incompetent) activity known to man? Hardly, when we rely on computers to control the throttles, flight direction, take-off and landing, and stability in flight of our airliners; guide spacecraft to the moon and back; produce otherwise impossible X-ray pictures of hospital patients; calculate with precision the correct point those patients require radiation treatment etc, etc.

Machines can solve differential equations faster than we can write them down and calculate the tedious without fault.

But these things are trivial when we consider the enormous potential in these machines. If I may draw a very crude analogy, today's computer is like the string kites of World War I. Tomorrow's computer will be (society permitting) like Concorde.

When we solve problems we first devise a method of solution and then apply that method. Mathematicians, more than most, are people who enjoy devising methods of solution for problems. It is this ability that we call intelligence, leaving machines and trained apes to carry out the methods and find the particular solution.

The human brain is a machine. The mind however is not material, but is inextricably linked to the brain, unable to function without it. This is similar to the computer and its program, each being useless without the other. And the program is not material either.

It follows that if we can define the method by which we devise methods then we can write it down. If we can write it down then a machine can carry it out. If we are to discover the true nature of man and intelligence then we will need the model that the computer provides.

SECOND LEVEL COURSE RESULTS: 1973–1975

year	course	finally registered	examined	distinction		pass	fail	
1973	M201	1077	834	101	12.1%	619	114	
1974	M201	1083	889	83	9.4	741	65	
1975	M201	847	633	69	10.9	511	37	(15)
1973	M202	881	675	101	15.0	502	72	
1974	M202	402	289	30	10.4	233	26	
1975	M202	245	173	26	15.0	133	8	(4)
1974	M231	769	548	70	12.8	434	42	
1975	M231	1024	636	49	7.7	547	34	(6)
1973	M251	1110	948	94	10.0	682	77	
1974	M251	1211	820	117	14.3	666	37	
1975	M251	1033	577	77	13.3	471	25	(4)
1974	MDT241	2204	1735	153	8.8	1515	67	
1975	MDT241	1920	1379	81	5.9	1215	73	(10)
1973	MST282	1077	732	63	8.6	540	129	
1974	MST282	911	606	21	3.5	485	100	
1975	MST282	712	468	47	10.0	374	47	
1975	TM221	949	691	49	7.1	607	34	(1)

COINCIDENTAL BIRTHDAYS

In M500 31 Nicholas Fraser asked for a justification that the probability of two or more people in a random selection of 24 having the same birthday is more than 1/2 and for 40 people there is a better than 9/10 chance.

All those who replied made the initial assumption that birthdays are equally spaced throughout the year - which seems unlikely. Perhaps there has been some work done on this. Also all made the additional simplification of ignoring leap years.

Margaret Corbett quotes from *Lady Luck: the Theory of Probability* by Warren Weaver (Heinemann, 1963):

> The event that at least two persons share a birthday is complementary to the event that they all have distinct birthdays. We start with one person. Whatever day it may happen to be he has a birthday. The probability that person number two has a different birthday is clearly 364/365 since it will be different if he was born on any one of the 364 days remaining after we cross off, so to speak, the birthday of the first person. When we advance to the third person there are 363 permissible days left, so the probability that the third person's birthday differs from that of the first and the second is 363/365. These are independent events, so the compound probability that number two differs from number one and that number three differs from both number one and two is 365/365 × 364/365 × 363/365. ...
>
> It is now easy to generalise to the formula for the probability of all distinct birthdays for *n* persons. It obviously is
>
> $$\frac{365 \times 364 \times 363 \times 362 \times 361 \times \ldots}{365 \times 365 \times 365 \times 365 \times 365 \times \ldots}$$
>
> with *n* factors in both the numerator and denominator of the fraction. Therefore the probability of the complementary event, namely that at least two persons share the same birthday, is
>
> $$1 - \frac{365 \times 364 \times 363 \times \ldots \times (365 - n + 1)}{365^n}.$$

Krysia Broda calls this probability $1 - Q_n$ and says - we want $Q_n < \frac{1}{2}$; how can we find n? We can write $365 - k = 365\left(1 - \frac{k}{365}\right)$, and Q_n becomes $\left(1 - \frac{1}{365}\right)\left(1 - \frac{2}{365}\right)\ldots\left(1 - \frac{n-1}{365}\right)$. Using the approximation $\frac{1}{e^x} \approx 1 - x + x^2/2! \approx 1 - x$ for small x we require $\exp -\left(\frac{1}{365} + \frac{1}{365} + \cdots + \frac{m-1}{365}\right) < \frac{1}{2}$ or $\exp -\frac{m(m-1)}{2 \times 365} < \frac{1}{2}$ or $m(m-1) < \ln 2 \times 730 = 730 \times 0.693$ and this quadratic has solution $m \approx \frac{1 \pm 45}{2}$; i.e. 23.

Sidney Silverstone calculated $1 - Q_3 = 0.0082$; $1 - Q_{23} = 0.5073$ or slightly greater than evens; and $1 - Q_{41} = O.9032$, or approximately 9/10.

CHESS

P C Hoad

It seems to me that John Parker (M500 32 9) has not so much simplified the problem as ignored it altogether. This arises in his step 5 (determine and play best move). In fact a practical method of determining the best move is not yet known, although quite a lot of people, including an ex-world champion, have been working on this for at least ten years. (John Parker's suggested time scale is, to put it mildly, over optimistic.)

The present position on computer chess is that several programs have been developed which will play a moderate game of chess (the best being about lower-board county standard) and that tournaments for computers have been held (in America, of course). Before diving in at the deep end I would suggest that anyone interested should look at the present state of the art. Some references are below.

I am not a draughts expert and do not know of any computer programs for this game, but I expect that some will have been developed by now.

Scott, J J: "A chess playing program", *Machine Intelligence* IV; American Elsevier, NT, 1969.
Botvinnik, M M: *Computers Chess and Long-Range Planning*, Springer-Verlag, NT, 1969.
Greenblatt chess program, *Proc AFIPS* 1967 *FJCC* Vol 31, AFIPS Press, Montvale NJ, pp 801-810.
Good, I J: "A five year plan for automatic chess" in *Machine Intelligence* 2, Oliver & Boyd, Edinburgh, 1967
Bell, A G: "How to program a computer to play legal chess". *The Computer J* 13,2, May 1970.
Levy, D N L: "Computer chess - a case study on the CDC 6600". *Machine Intelligence* VI; American Elsevier, NT, 1971.

The description of the Cooper-Koz program (*Communications of the ACM*, July 1973, volume 16 number 7, pages 411-426) contains a large number of other references (including, incidently, some on draughts).

From Ian McCook - I enclose a cheque for 50p for *Dice Star Trek* - I've read so much about it I must have one! Several comments follow which you may ignore.

I am no brilliant mathematician but after having completed about half of M334 I am thinking "Can the rest be so straightforward?"

Has anyone ever got a first class honours whose best four credits are not all distinctions?

Why do books invariably define 0! =1 when $k! = (k + 1)!/(k + 1)$ which implies 0! =1!/1 = 1?

What about NOUS - non-member of OUSA?

EFRON DICE

Chris Pile

I had not heard of Efron Dice until mentioned in M500, but I would think that there must be more to the game than as described by Bob Curling (M500 32 8). The range of numbers from 0 to 12 appears unnecessary. I think that the dice could be numbered:

A) 1 1 1 5 5 5
B) 0 0 4 4 4 4
C) 3 3 3 3 3 3
D) 2 22 2 6 6.

The dice could then be spotted as an ordinary die except for the blank faces of B. This also has the advantage that whoever chooses die C can save himself the energy of throwing it!

Ron Aitken

I support Bob Curling's assertion that whatever die you choose I can choose one from the three remaining dice to give me a probability of beating you of 2/3. However, probabilities like these are based on a long succession of coups. In practice unless you are the casino owner they don't help much in an individual coup.

I have analysed the probabilities and state that there is a marginal advantage if forced to choose first in choosing die number 4. (1- 2 3 3 9 10 11; 2- 0 1 7 8 8 8; 3 - 5 5 6 6 6 6; 4 - 4 4 4 4 12 12.) Here your long run probability of winning is 0.52 compared with 0.50 for dice 1 and 3 and 0.48 for die 2. The casino owner should always throw die 4 under these circumstances and, provided punters chose their dice without reading M500 (i.e. an even distribution between 1, 2, and 3), would thus secure a house take of about 2% - small but steady over a long term.

John Parker

The dice can be tidied up by noticing the equivalence classes ↘

6
6
4 6 8
3 4 6 8 11
1 3 4 5 8 10 12
0 2 4 5 7 9 12

1 - 2 2 2 6 6 6; 2 - 1 1 5 5 5 5;
3 - 4 4 4 4 4 4; 4 - 3 3 3 3 7 7.

Four sided dice improve the probability if the first and last digits of the rows are lopped off:
1 - 2 2 6 6; 2 - 1 5 5 5; 3 - 4 4 4 4; 4 - 3 3 3 7.

May I give a large container to Marion (holds pies)?

3 1 4 1 5 9 2 6 5 4

Geoff Bennett

MONGE'S SHUFFLE

Max Bramer

The formula given for x_m is certainly wrong (M500 30). I have another formula (easily proved by induction), which unfortunately is quite useless for calculating numerical values! I have instead been looking at "cycle lengths" for individual cards. That is, for a given pack of 2*p* cards and a card in a particular starting position x_0, if the card first returns to x_0 after *k* shuffles the cycle length is *k* and $x_k = x_0 = x_{2k}$, etc.

When a card returns to a position it has already occupied, the definition of function *f* (✳) ensures that it will repeat its previous behaviour, e.g. for $2p = 20$, card number 5 moves through the sequence 5, 13, 17, 19, 20, 1, 11, 16, 3, 12, 5, then again 13, 17, 19, 20, 1, etc. *k* is 10.

Eventually (after at most 2*p* shuffles) *some* repetition must occur (since there are not $2p + 1$ different positions in a pack of 2*p* cards!). However it is not entirely obvious that any card will eventually return to its starting point - there might be cycles such as 9, 14, 6, 5, 6, 5, ... Even if every card eventually returns to its starting point, they may not do so simultaneously. Thus for some pack card 1 might return to its position every 20 shuffles, but card 2 might return every 19 - so 380 shuffles would be needed before they were both together in positions 1 and 2 at the same time. A complete pack of 2*p* cards might require $(2p)!$ shuffles.

So far by analysis only; but a simple `BASIC` program soon reveals some surprising results. These are all true for values of *p* from 2 to 50 inclusive, i.e. packs of from 4 to 100 cards:

i) each card eventually returns to its starting point;

ii) the longest cycle length for any card is *N*, $1 \le N \le 2p$;

iii) in some (degenerate?) cases, cards cycle in less than *N* shuffles but in each case the cycle length is an exact divisor of *N*.

iv) Combining these results gives the unexpected result that the entire pack will be in its original order after some number of shuffles *N*, *not greater than* 2*p*.

For all *p* there is an integer *N*, $1 \le N \le 2p$ such that $x_0 = x_N$, ($x_0 = 1, 2, ..., 2p$). There is no obvious relationship between *N* and *p*. As an example, for a 10-pack ($p = 5$) the cycle length for positions 1, 3, 6, 7, 9, 10 is six; for position 4 it is 1 (i.e. invariant), for positions 2, 5, 8 it is 3. (Note that 1 and 3 are exact divisors of six.) The entire pack is back to its original order after 6 shuffles.

As far as I can tell David Asche's formula for the smallest *N* (✳✳) is correct, although it is difficult enough to verify, without attempting a proof! The attached computer printout shows, for values of *p* from 2 to 50, *p* tabulated against $4p + 1$ and each of the cycle lengths for the individual cards (the largest of these being *N*). Inspecting the table shows that the following seem to hold:

i) there is a cycle of length 1 (i.e. at least one invariant card) for every third value of *p*, i.e. $p = 3h + 2$ or $4p + 1 = 12h + 9$ - which can be verified from the original definition of function *f* ✳ with $x_0 = 2h + 2$ - and no other values of *p*;

ii) There is only one cycle length in the table if and only if $4p + 1$ is prime.

These include all the instances where $N = 2p$, p or $p/2$ and whenever this occurs p is odd for the $N = 2p$ case and even for for the other two! Naturally any or all of these results might be pure coincidence, but I doubt it. Can anyone else fit all of these pieces together?

p	$4p+1$	cycle lengths	p	$4p+1$	cycle lengths
2	9	3 1	26	105	12 6 4 3 2 1
3	13	6	27	109	18
4	17	4	28	113	14
5	21	6 3 1	29	117	12 6 3 1
6	25	10 2	30	121	55 5
7	29	14	31	125	50 10 2
8	33	5 1	32	129	7 1
9	37	18	33	133	18 9 3
10	41	10	34	137	34
11	45	12 4 3 2 1	35	141	46 23 1
12	49	21 3	36	145	14 2
13	53	26	37	149	74
14	57	9 1	38	153	24 8 4 3 1
15	61	30	39	157	26
16	65	6 2	40	161	33 11 3
17	69	22 11 1	41	165	20 5 4 2 1
18	73	9	42	169	78 6
19	77	30 5 3	43	173	86
20	81	27 9 3 1	44	177	29 1
21	85	8 4 2	45	181	90
22	89	11	46	185	18 2
23	93	10 5 1	47	189	18 6 9 3 1
24	97	24	48	193	48
25	101	50	49	197	98
			50	201	33 1.

Ed - the two formulas mentioned (✳ and ✳✳) are

$$(\ast)\ f: x \to \begin{cases} ½(2p + x + 1, & x \text{ odd} \\ ½(2p - x + 2, & x \text{ even} \end{cases};$$

and David Asche's conjecture in M500 31 7:

$$(\ast\ast)\ (4p + 1)k \pm p = 2^{N-2},$$

with p and N as above and k a suitable nonnegative integer. For instance, with $p = 50$ we have $k = 10\,683\,998$ and the sign attached to p is positive.

Imagine the perplexity of a man outside time and space, who has lost his watch, his measuring rod and his tuning fork. -

Exploits and Opinions of Doctor Faustrall 'Pataphysician, A Jarry.

M500 - REPLIES TO MAX BRAMER

Michael Gregory

I find Max Bramer's offer (M500 32) to supply a list of "speculative/dubious" items very distasteful; I am confident that this is not normal practice, and trust that it will be treated as it deserves.

M500 is a mathematics student/staff magazine. Errors are to be expected - they occur in textbooks, course units and even computer journals - and yet he offers to "pick up any errors." (Could Max Bramer be too exalted to be in the MOUTHS lists?)

There are other reasons why it would be a mistake to vet articles before publication. If these corrections are made by members of staff, writing anything for M500 would be like doing a TMA, and the magazine would probably die. Who would vet the staff articles? (Need I ask?) If we are prepared for a script to go to a referee surely we would send it to a journal. I think M500 offers a chance to be creative; which may be as important as attempting perfection.

Henry Jones Ian Ketley was no spectre, nor is Max Bramer scary, so why funk hitting back again?

Ian named those whose mathematics disturbed him. Max on the other hand expects us to go to the trouble of writing to him privately, which for me is not on.

The OU's mathematical faculty itself may not be above criticism. In M100 it enthusiastically preaches the gospel of unification and generalisation. Why then is this principle rejected in the domain of the calculus with all its ramifications? The OU should answer me and any other student who aspires to be creative. (See M500 14 15.)

In writing thus, I hope it doesn't give the impression of opposition to some possibly constructive suggestions by Max. But don't change too much. In spite of my own grumpiness I like the humour and the 'faults'.

Marion Stubbs

Is it "dubious", as an activity, to speculate? If so how does personal knowledge expand? And, more particularly, why do I pay taxes and rates so that adolescent students may learn in an environment where vocal speculation is encouraged? They might as well read their books diligently at home and keep quiet about their ideas, as Max seems to think OU students should, since we have rare occasions when we can vocalise anything and must rely on written speculation.

At the risk of resurrecting the entire Ketley saga (M500 15 *ff*, 1974), I opine that any OU mathematics students who want a "perfect" magazine for undergrates should subscribe to

Mathematical Spectrum (from University of Sheffield - really essential for all undergraduates) and *Manifold* from University of Warwick. Both of these are solid mathematics and nothing else and both of them need new subscribers. The new undergraduate magazine for computer-types is *Creative Computing*, which similarly wants new subscribers and throws way-out ideas in all directions, mostly in BASIC and English. M500 can scarcely compete with these three and should not wish to; one very good reason being that they are run by professional mathematicians, whereas M500 is a student-operated magazine.

Perish the thought that any editor of M500 should be ham-strung by some Editorial Committee - he deserves his bit of Power! He knows well enough he can call on experts whenever he *wants* help; and it does not take long for editors to find out who are the experts who can be relied upon to reply quickly - which is the prime editorial requirement. Eddie is now the second of two editors of M500 who have been told by well-intentioned staff that they are fairly incompetent.

I say that staff have their moments of Power - in assessment situations - and should stop trying to hold all the Power. Let them join in the debate after some “dubious” speculation has reached print instead of attempting to suppress speculation at birth; and let them write some “perfect” original pieces which will enliven our cornflakes and widen our horizons and ease the Editorial burden by providing those elusive “Page 1” abstruse mathematical pieces which are now a tradition in M500. Some of us might even feel inclined to argue with staff perfection.

LADDERS

Dorothy Sharpe

Thank you for saving my pride. One of my sixth form boys brought me a ladder problem he had obtained from a university student he had met on holiday. The lad had spent ten hours trying to solve it and now my reputation was at stake. Within the hour it was solved using the method suggested in M500 31 4.

VOLUNTEERS are NEEDED to man THE M500 SOCIETY DISPLAY (in hourly shifts) at the OPEN DAY at WALTON HALL on June 5th. Please, contact MARION STUBBS if willing!

⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕

"UNDERSTANDING SPACE AND TIME"

From the Faculties has come news of a new course; with the above title. But it won't be ready before 1978 at the earliest. The man to write to if you are interested is Alan Cooper of the Science Faculty.

It will be designated S354 and have the usual 16 units. No M in the designation means that any mathematics will be used for clarity of formulation of key arguments only. So there will be no reliance on technical facility. There will be some vectors and some tensors, if you know what they are.

"The major part of the course will be concerned with explaining the concepts of space and time as used in modern science. In the process many different observations in astronomy and experiments in nuclear and fundamental particle physics will be discussed. The emphasis will be on the way that the ideas of space and time are tested by these experiments. The course will end by using the concepts developed in the course to attack the questions of the origin of the universe and its future evolution.

"The nature of space and time are so fundamental that the course should be of interest not only in its own right but also to anyone wishing to have a firm basis for the exact sciences, especially modern physics."

The provisional unit titles are:

1- Newtonian Mechanics in absolute space-time; 2 - Laws of Classical physics; 3 - Convariance; 4 - Special relativity; 5,6- Laws of local phenomena; 7 - Principle of Equivalence; 8 - General relativity; 9 - Application to spherically symmetric systems; 10 - Models of the Universe; 11- An overview of physical laws; 12,13,14 - Parity, Time reversal and charge conjugation; 15- Models of the Creation; 16 - The Evolution of the Universe.

Alan says that a good introduction to the course is *The Character of Physical Law* by Richard Feynman (MIT press, paperback edition).

POCKET CALCULATORS

Tony Brooks

Calculation of e^x and e^{-x} with four function calculators.

I work in the field of reliability analysis and as a result I frequently need to calculate exp(-x). Therefore I have found it useful to develop an easy but accurate routine for finding exp(-*x*) with a basic four function calculator. Now that scientific calculators have become so cheap the publication of this routine is of less use than it would have been two years ago. However there must still be many thousands of simple calculators still in use and to those who own them this routine could be useful.

Several articles have appeared on scientific function calculation on four function calculators (see *Wireless World Annual* 1975, pp. 79-82. This article unfortunately contains many printing errors). However I have found the routine I give here particularly simple and accurate. It is based on the following equation:

$$e^{-x} \approx \left(\frac{x-2n}{x+2n}\right)^n \quad \text{for large } n.$$

To make calculations simple the value of *n* needs careful choice. *n* should be a power of 2 so that repeated squaring will suffice. With $n = 256 = 2^8$ the error is only 0.13% for $x = 10$ and less for smaller *x*.

The following tested routine works on a machine without a memory but with a K (constant) button which can hold the last entry (such as the Sinclair Cambridge). C is the clear button which clears the constant and the calculator display. However the number is still retained by the calculator chip until overwritten. A is the value for which exp(– x) is required. ×, + , – , ÷, = are the usual functions.

$$\texttt{A + 512 = K 1024 - ÷} \underbrace{\text{C K} \times}_{\text{repeat 8 times}}.$$

If e^x is wanted the above routine can be followed by C K – = which gives 1/exp(−*x*) = exp(*x*), or instead exp(x) is given directly by

$$\texttt{A - 512 = K 1024 + ÷} \underbrace{\text{C K} \times}_{\text{repeat 8 times}}.$$

For those with machines with a memory and the capability of squaring simply by repeated pressing of the button, the following routine for exp(–*x*) (i) and exp(*x*) (ii) is suggested: Let ST mean store in memory, RCL mean recall from memory

A + 512 – ST 1024 ÷ RCL × = (i)

↑

repeat 8 times

↓

A – 512 + ST 1024 ÷ RCL × = . (ii)

CONSTRUCTIONS II

Steve Murphy

Please accept my apologies - I boobed (M500 31).

If we assume that the lines are drawn from the family $\frac{x}{c-t} + \frac{y}{t} = 1; t - (0, c)$ where *c* is a positive real number then for positive values of *x* and *y* the equation of the limiting curve is

$$x^{1/2} + y^{1/2} = c^{1/2}.$$

As Richard Ahrens and Max Bramer point out this equation does not represent an astroid but represents a portion of a parabola.

PI

John Hampton

Two interesting formulæ are:-

$$\arctan x + \arctan \frac{1}{x} = \begin{cases} \frac{-\pi}{2}, x < 0 \\ \frac{\pi}{2}, x > 0 \end{cases}$$

and

$$\arctan x + \arctan \frac{1-x}{1+x} = \begin{cases} \frac{-3\pi}{4}, x < -1 \\ \frac{\pi}{4}, x > -1 \end{cases}.$$

(I M Ryzhik and I S Gradshtein, *Tablitsy Intergralov, Summ, Ryadov I Proizvedenii*, Gostekhizdat, Moscow; 1951.)

Arctan $x = \sum_{k=0}^{n-1}(-1)^k \frac{x^{2k+1}}{2k+1} + E_{2n}(x)$ where $|E_{2n}(x)| \leq \frac{x^{2n+1}}{2n+1}$, $x \in [0, 1]$ as has already been pointed out in these pages; so if we wish to estimate arctan *x* correct to *m* places we must choose *n* to satisfy

$$\frac{x^{2n++1}}{2n+1} < \frac{1}{2} \cdot 10^{-m}.$$

Suppose that $m = 7$ then if $x = 1$ we require $n \approx 10^7$ but if $x = 1/5$ then $n = 5$ and if $x = 1/239$ then $n = 1$.

To approximate = 4 arctan 1 using this method is clearly impracticable but π = 16 arctan 1/5 – 4 arctan 1/239 may be trivially achieved for relatively low precision. Using multiple precision arithmetic procedures developed by I D Hill ("Procedures for the basic arithmetical operations in multiple-length working", *The Computer Journal* vol II p. 232*ff*, 1968) I wrote an Algol 60 program to compute π to 99 decimal places using the Taylor series above, on the ICL 1905F at Lancaster University. The 1950F has a cycle time of 650 nanoseconds and evaluation was achieved in an execution time of 538 seconds. The result obtained:

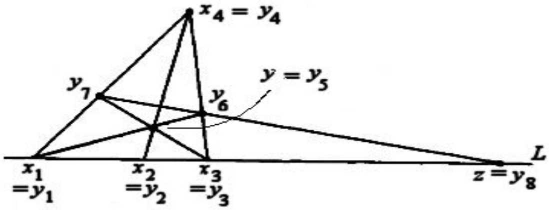
π = 3.141 592 653 589 793 238 462 643 383 279 502 884 197 169 399 375 105 820 974 944 592 307 816 406 286 208 998 628 034 825 342 117 068.

72 iterations were required to compute arctan 1/5 and 21 for arctan 1/239.

Apart from power series expansions, techniques for the evaluation of arctan include Chebyshev polynomial expansions, continued fraction expansions, polynomial approximations and rational approximations. All these methods are reviewed in *Handbook for Computing Elementary Functions* by L A Lyusternik, O A Chervonenkis and A R Yanpol'skii, Pergamon 1965; translated by G J Tee. Only polynomial and rational approximations are particularly suitable for hardware implementation within any computing device. In all such cases the accuracy achieved will be inherent in the method employed, but this can usually be stated with some precision.

SOLUTIONS

30.3 *POINT CONSTRUCTION*: x_1, x_2, x_3, three given points on a line. x_4 is not on the line and y is on the line (x_4, x_2). Prove that a construction given to produce z is independent of x_4.



$y_i = x_i$, $i \leq 4$; $y_5 = y$; and y_{k+1} is obtained by intersecting joins of points already available

From Steve Murphy

Let A_r represent the position vector of the point y_r then since y_1, y_2, y_3 are collinear there is a number p such that $A_2 = pA_1 + (1 – p)A_3$ (i);

similarly there is a q such that $A_2 = qA_4 + (1 – q)A_5$ (ii).

Hence $pA_1 + (1 – p)A_3 = qA_4 + (1 – q)A_5$; $pA_1 – (1 – q)A_5 = qA_4 – (1 – p)A_3$ and $(pA_1 – (1 – q)A_5)/(p + q – 1) = (qA_4 – (1 – p)A_3)/(p + q – 1) = A_6$ (iii).

The last deduction follows from the fact that the LHS of iii represents a vector dependent upon A_1 and A_5 while the terms involving A_4 and A_3 show it also to be linearly dependent upon these. It must therefore represent the point of intersection of the lines y_1y_5 and y_4y_3. In a similar way $(pA_1 – qA_4)/p – q = (1 – q)A_5 – (1 – p)A_3/(p – q) = A_7$ (iv).

From iii $pA_1(1 – q)A_5 = (p + q – 1)A6$ (v).

From iv $(l – q)A_5 – (l – p)A_3= (p – q)A_7$ (vi).

Adding v and vi: $pA_1 – (1 – p)A_3 = (p + q – 1)A6 + (p – q)A_7$ (vii).

Hence $(pA_1) – (l – p)A_3)(2p – l) = ((p + q – l)A_6 + (p – q)A_7)(2p – 1)$ and noting that $(p + q – l) + (pq) = 2p – l$ we deduce that $A_8 = (pA_1 – (l – p)A_3)/(2p – l)$; and since p is defined by the vectors A_1, A_2, A_3 which relate to given points we deduce that $z = y_8$ is independent of the choice of $y_4 = x_4$.

31.1 *EULER'S POLYGON DIVISION*: In how many different ways can a plane convex polygon of n sides be divided completely into triangles by nonintersecting diagonals?

Answer:

$$E_n = \frac{6 \cdot 6 \cdot 10 \cdot \ldots \cdot\ 4n-10)}{(n-1)!}.$$

(JH)

solutions 2

32.2 *NOUGHTS AND ONES*; Find the maximum possible value of $K = \sum_{\substack{i,j=1 \\ i<j}}^{2m} (x_i - x_j)^2$ if it is kown that each x_i has value 0 or 1.

$K_{max} = m^2$; EK, Ron Aitken and Gordon Thompson who justifies it as follows: Terms of K are equivalent to the set of integer pairs (i, j) with $1 \le i, j \le 2m$, $i \neq j$. If n of the x_i are 1 then $2m - n$ are 0.

The number of $(x_i - x_j)^2$ terms with $x_i = 0$, $x_j = 1$ or $x_i = 1$, $x_j = 0$ is $n(2m - n)$. Each equals 1. All other terms equal 0. Hence $K = n(2m - n)$ and will be maximum when $n = 2m - n$ or $n = m$.

32.3 *FIND THE NEXT TERM*: 43, 50, 55, 65, 76, 89,

106 (= 89 + 8 + 9). Gordon Thompson, Ron Aitken. Let n_q be the qth number in the list, S_p the sum of the digits in the number p, then we have

$$n_i = \sum_{j=1}^{i-1} S_{n_j} + n_1,$$

from which one can deduce the given information: the nth number minus the first is equal to the sum of the digits in the first $n - 1$ numbers. The sequence is known as Kaprekar's Digitaddition. One amusing problem is to devise a method to show the number of different ways any given number can be produced. For instance both 91 and 100 produce 101 as the next term in some sequence.

32.4 *THE FALLING STONE*: A stone falls half the height of a wall in half a second; find the height of the wall.

Bill Shannon: $s = 16t^2 \Rightarrow \sqrt{s} = 4t$ so $\sqrt{x} - \sqrt{(x/2)} = 4 \cdot ½ = 2$. So $\sqrt{x} = 2/(1 - \sqrt{½}) = 6.83$; so $x = 46.6$ ft.

PROBLEMS

33.1 *VECTOR SUBSPACES*: Rosemary Bailey (Staff).

If we have two vector subspaces, A and B, of a vector space V, we can also form the vector subspaces

$$A + B = \{a + b : a \in A, b \in B\}$$

$$A \cap B = \{v : v \in A, v \in B\}$$

of V, thus obtaining four possibly distinct subspaces of V.

Now suppose we start with *three* subspaces, A, B, C, of V. How many possibly distinct subspaces of V can we obtain by repeated use of + and $\cap$? (For example $A \cap (B + C)$ and $(A \cap B) + (A \cap C)$ are distinct in general.)

33.2 THE KING’S MOVE: Jerremy Humphries.

1 a) Put the king on the bottom left-hand corner square and allow him to move only north and east. By how many different routes can he reach the top right-hand corner square?

1 b) How many routes would he have across an $8 \times 8 \times 8$ cube between diagonally opposite corners? Again allow no detours.

1 c) General problem. The king is at point $(x_1, ..., x_n)$ in n-dimensional space and he wants to reach point $(y_1, ... , y_n)$. He can move a distance 1 along, one coordinate at a time and is allowed no detours. How many routes from $(x_1, ..., x_n)$ to $(y_1, ... , y_n)$?

2 a) Do 1(a) but allow the king to move diagonally also.

2 b) Find an expression for the number of routes from $(x_1, ..., x_n)$ to $(y_1, ... , y_n)$ on the standard chessboard. The rule for moving is the same as in 2(a).

33.3 *FIND THE NEXT TERMS*: N J A Sloane; *Jnl Rec Math*.

(1) 1,4,9,16,25,36,49, ...

(2) 1,3,6,10,15,21,28, ...

(3) 1,1,2,3,5,8,13,21,34, ...

(4) 1,2,5,12,29,70,169,408, ...

(5) 1,1.3,1,5,3,7,1,9,5,11,3,13,7,15,1, ...

(6) 1,21,21000,101,121,1101,1121,21121, ...

33.4 *THE PROFESSOR*: Peter Needham *and* Richard Tombs.

A professor told his assistant that he had given a party for his wife and two nieces. “The sum of my wife and nieces ages,” he said, “is twice your age and the product of their ages is 2450 years. How old are my nieces?” The assistant complained he had insufficient information so the professor said. “I was the oldest person present.” Then the assistant gave the answer. How old was the professor?

33.5 *MAXNIM*: Max Bramer.

Two players take part in the following game: Each removes any number of coins from a single pile, which begins with $N > 1$ coins. Players move alternately and the player who takes the last coin wins. There are two restrictions:
i) all N coins must not be taken on the first move,
ii) after the first move no player may take more than *three* times the number just taken by his opponent.
Are the following starting positions won or lost for the first player? N = 5, 8, 21, 25, 55, 125.

Suggest a general strategy for playing the game.

EDITORIAL Max Bramer has written to ask if we can publish a deadline in advance. In general we don't need one because I see my job as trying to construct as well balanced a magazine as possible out of what is available. But there are particular items which lose their validity after a certain date. Details of anything due to happen on or after say the 13th ought to be here before the end of the previous month. But I am in MOUTHS so you can always telephone me to see if the manuscript has gone yet.

Did anyone notice the report that Maria Reiche, the German mathematician, has measured the length of the standard unit used in the prehistoric Nazca ground drawings in Peru? It is 235 cm: the length of a string or sisal cord held between outstretched arms. She claims to have found five figures which fit this measurement exactly - some so large as to be properly visible only from the air. But it also fits the British remains at Woodhenge. An exhibition of Miss Reiche's photographs of these drawings was shown at the ICA; but it started on Thursday 8th April so to be of use to us the notice would have had to be here before the end of March (or before the end of February for the preview or to actually meet Miss Reiche.)

Talking of "did you notice", I found the report in *Nature* 260 417 (1976) about the inverse square law to be at least amusing. Dr Daniel Long of Eastern Washington State College describes an experiment in which there was a discrepancy of a fraction of 1% between the attraction exerted by 50 kg at almost 30 cm and that by 1 kg at 4 cm, which should have been the same. Dr Long proposes a small modification to the inverse square law. He is not a popular man.

And so back to Max Bramer. Many people wrote here to ask if they are on his "black list"! I can assure them that they were not. There wasn't one. Anyone who thinks he has been guilty could write direct to Max only he's not on the MOUTHS list. What about it Max?

As to computing, I feel that John Parker has argued my case for me so I will say no more, except thank you those that have written to help. For next month's M500 there is on hand an article by Steve Murphy on Coincidental Birthdays; more on Monge and Pi and another of Datta's extravaganzas. So no more on these subjects for a while, unless someone would like to prove Max's Monge Theorem. Concerning the Public School problem of Richard Ahrens's, I tried finding the remainder on dividing by 18 of the standard Fibonacci terms and got results very similar to those shown on page 1; so I would suggest this is a sterile approach.

Before I hit the bottom of the page there are one or two requests I'd like to make (again). Could manuscripts have the author's name and MOUTHS list number on each page please. Also it would help if people could avoid writing on both sides of the paper (unless with a continuation of the same item), as it messes up the filing system. And if you are going to use a name or technical term that I may not have heard of could you print it please?

Eddie Kent.