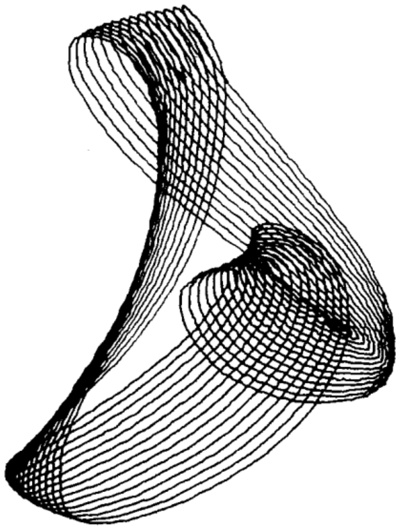


M500 36

M500 is a student operated and owned magazine for Open University mathematics students and staff, and for anyone else who is interested. It is designed to alleviate student academic isolation by providing a forum for public discussion of individuals' mathematical interests.

Articles and solutions are not necessarily correct but invite criticism and comment. Articles submitted for publication should normally be less than 600 words. Anything longer ought really to be split into instalments.

MOUTHS is a list of names, addresses, telephone numbers and previous and past courses of voluntary members, by means of which private contacts may be made by any who wish to share OU and general mathematical interests or who wish to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors concerned and do not necessarily represent those of either the editor or the Open University.

The cover design is by John Parker, who has sent a set of similar covers. They are produced by driving a pen horizontally and vertically in combinations of

$$A \sin w_1 t + B \cos w_1 t \qquad \text{and} \qquad C \sin w_2 t + D \cos w_2 t$$

where w_2 is an integer multiple of w_1.

Publisher: Marion Stubbs.

Anything for inclusion in M500 to the editor: Eddie Kent.

Membership applications, subscriptions, change of address, MOUTHS data, to the membership secretary: Peter Weir.

The treasurer is Austen F Jones.

Cheques and postal orders (which should go to Peter Weir) should be made payable to THE M500 SOCIETY and crossed "a/e payee only, not negotiable" for safety in the post.

Anything sent to any of the above and not marked as personal will be considered for possible inclusion in M500.

M500 36 published September 1976.

Subscription £3 for ten Issues.

Membership about 400.

Printed by Tizengrove Ltd Queens Terrace Southampton.

NUMBER REPRESENTATIONS

Krysia Broda

The representation of positive integers that we usually use is of the following type: $N = \sum_{i=0}^{m} a_i\, x^i$. Usually x, called the base, is the number 10 although computers often use 2. Each of the a_i can be any number $\in \{0, 1, ..., x-1\}$. There are three similar systems which might provide some enjoyment if you like playing with numbers.

(i) Fibonacci systems;

(ii) Factorial systems;

(iii) Ternary systems.

(i) In Fibonacci systems we have a variable base. We use the sequence formed from a recurrence relation of the type $u_k = u_{k-1} + u_{k-r}$, $r \geq 1$, given that $u_0 = u_1 = ... = u_{r-1} = 1$. e.g. $r = 2$ gives the Fibonacci sequence 1,1,2,3,5,8,... . The multiple 1s at the beginning are dropped so we consider the sequence $\hat{v} = u_{r-1}, u_r, u_{r+1}, ...$. Any positive integer can be written as the sum of terms from $\hat{v}$, and the representation is unique if we constrain the suffices of each pair of the terms used to differ by at least r. i.e.

$$N = \sum_{i=r-1}^{r} a_i\, u_i \qquad (1)$$

where $a_i = 0$ or 1, and for any set of r consecutive terms the sum of all products of coefficients of pairs from the set = 0. (The coefficients are the a_i.) If $r = 1$ this is the normal binary representation - no restriction on the suffices. If $r = 4$, 49 = 36 + 10 + 3, all terms in the sequence 1, 2, 3, 4, 5, 7, 10, 14, 19, 26, 36.

For the Fibonacci sequence this theorem is known as Zeckendorf 's theorem. It can be proved in several ways. One way is to demonstrate by construction that every integer can be represented by (1), and then to prove by contradiction that this representation is unique. It is easiest to start with the Fibonacci numbers as a particular case and then to generalise. I leave this as problem 1. For fun you could try addition in these systems; can you find any easy rules?

(ii) Now consider representing $N = \sum_{i=1}^{n} a_i\, i!$ where each $a_i \in \{0, 1, ..., i\}$. (2)

We can use induction on i to show that the representation is unique, and that such a representation exists, $\forall N \in \mathbb{Z}^+$: Assume a unique representation $\forall N < k!$ This is certainly true for $k = 1,2$. Consider any number N from the set $\{k!, k! + 1, ..., (k + 1)! - 1\}$. We show that a unique representation of the form (2) exists. Certainly $a_i = 0$ for $i > k$ as $i! \geq k + 1 > N$ by definition. We must also choose $a_k = a$ where $a \times k! \leq N < (a + 1)k!$ $a \in \{1,2,...,k+1\}$. i.e. a_k has a unique value. For obviously $a_k < a + 1$ else $a_k \cdot k! > N$, and if $a_k \leq a - 1$ the maximum possible sum is

$$\sum_{i=1}^{k-1} i \times i! + a_k \cdot\ k! = k! - 1 + a_k \cdot k! = k!\,(a_k + 1) - 1 < \ k!a\ \leq N.$$

We are now left with representing $N - a_k \times k!$ uniquely as $\sum_{i=1}^{k-1} a_i\, i!$. Since $N - a_k k! < k!$ this is possible using the induction hypothesis. Therefore our result is true for $k + 1$ if it is true for k, and since it is true for $k = 1, 2$ it is true for all $k \in \mathbb{Z}^+$ by mathematical induction. Finally then, for a given N choose k such that $k! > N$ and we know N is uniquely representable in the form given by (2). Problem 2 Try and find rules for multiplication and addition in this system.

With our usual base-10 number system we can also represent *some* rational numbers as finite sums of negative powers of 10. These rationals (<1) for which this is possible must be of the form p/q (in lowest terms) with q a product of powers of divisors of 10; i.e. 2, 5.

Using fractional representation we can represent any rational <1 uniquely as a finite sum of the form

$$\sum_{i=2}^{m} \frac{a_i}{i!} \quad \text{where } a_i \in \{0, 1, ..., i-1\}.$$

(This does not mean that there is no infinite sun of this form equal to the given rational; cf 9/100 + 9/1000 + ... = 0.1 = 1/10.) Given any rational p/q, $0 < p/q < 1$, let N be chosen such that $N!$ is the smallest factorial divisible by q ($N \le q$), then $p/q = r/N!$ for some $r \in \{1,2,...,N! - 1\}$. For example if $p/q = 3/8$ then $N = 4$ and $r/N! = 9/24 = 1/3 + 1/24 = 2/3! + 1/4!$. The number of *different* fractions represented as $\sum_{i=2}^{N} \frac{a_1}{i!}$, $a_i \in \{0, 1, ..., i-1\}$ is $N!$. Problem 3 - Prove this. These are $0/N!, 1/N!, ..., (N! - 1)/N!$. Hence our given rational is one of these and our representation is in the required form. p/q is also uniquely represented for any other representation must include $a_k/k!$ where $k > N$ (by problem 3). But then we would have at least two representations of p/q as for some a_i, one including $\frac{a_k}{k!}$ and one not; but, again by problem 3, those fractions representable by $\sum_{i=2}^{k} \frac{a_i}{i!}$ are all different. Hence we have a contradiction and our representation is unique.

There are other proofs of these results. Problem 4: find some of them. Problem 5: try using the system.

Ed - (iii) will appear in M500 37.

AM289

Miek Warden

Are there computers which will compile notes from eleven units on “one of the threads” running through the *History of Proofs* and the like? Surely this must be one certain maths course for which compass and ruler need to be taken out of mothballs? Anyone expecting stories about Archimedes in the bath or Kepler and his prime casks to brighten up the long hard slog towards a degree should think again.

Why does everybody keep so quiet about this course?

DELTA

Richard Ahrens

In recent issues of M500 I thought I had detected a healthy trend in that more readers than usual seemed prepared to comment critically on articles. However when Phliip Newton saw fit to criticise me in M500 35 I recognised this trend for the pernicious and unwholesome influence that it is and I expect - now that I have pointed this out - that the editor will rigidly suppress any further contributions of this nature. On second thoughts the censorship should start in the next issue so that I can get in a last word.

The Kronecker delta, δ_{ij}, is just a convenient piece of notation. As Philip said in his first contribution

$$\delta ij = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}.$$

It is not an $i \times j$ matrix, but I suppose you could view δij as an entry in an $m \times m$ matrix if $1 \le i \le m$ and $1 \le j \le m$. The point I was trying to make was that an expression like $\sum_{i=1}^{m} \delta_{ij}$ makes sense and is equal to a_j.

Philip is quite right when he guesses that I am oblivious to the concepts expressed in Don Harper’s article on algorithms (M500 29 1). Try as I might I couldn’t understand it.

SIZEABLE PROBLEMS

Peter Weir

M351ers, currently suffering from linear programming, may be interested by some figures lifted from a 1970 Univac (a computer manufacturer) Users Association conference report. The solution method used is basically that described in Unit 6 of M351, the product form of the simplex method, with the addition of some unexplained method of introducing the two most profitable variables into the basis at each iteration. The source of the problems was not given but was probably operations research for Sheel France.

problem number	rows	columns	coefficients	time (min)	cost today's rates
1	1854	3708	16010	59	£590
2	2210	5500	22609	38	380
3	1277	2911	14628	42	420
4	2315	4591	18887	58	580
5	3135	2738	31727	118	1180

Imagine having to rerun the last one because one of the 31727 coefficients was wrong!

POINT CONSTRUCTION (problem 30.2) Bob Margolis

In MS00 35 is a neat solution to the first part of the problem (to show that a construction to produce a fourth point on a line, given three points, is independent of the choice of a point not on the line). The solution itself is straightforward and uses nothing more sophisticated than the idea that the set

$$\{\lambda x_1 + \mu x_2 : \lambda + \mu = 1;\ \lambda,\mu \in \mathbb{R}\}$$

gives the vectors for points lying on the line joining $x_1$and x_2. It *is* remarkable for the fact that it uses three dimensions for an essentially two-dimensional problem. I would love to know how the authors arrived at this idea. Unfortunately, nice though the solution is, it won't generalise in any manageable way to cope with the second part of the problem. The key to a solution is contained in the 3-dimensional view of the problem.

If your mind was particularly devious, you might consider the following. Think of an origin (eye) not in the plane of the problem. Rather than considering points, consider the lines joining these points to the origin; rather than lines, the planes formed by those lines and the origin.

Now if (a,b,c) is a point any $(\lambda a, \lambda b, \lambda c)$ lies on the same line through the origin as (a,b,c).

The *plane* containing x_1 and x_2 (and the origin) will be

$$\{\lambda x_1 + \mu x_2 : \lambda,\mu \in \mathbb{R}\}$$

with no restriction now on λ, μ.

The original problem can now be phrased in terms of lines and planes rather than points and lines with the advantage that when we choose coordinates we can be free about multiples of coordinates as they don't affect lines through the origin &c.

From now on x_1, x_2 etc. will denote the *lines* through the origin rather than the points.

We can certainly choose $(x_1, x_3, x_5\}$ as a basis of $\mathbb{R}^3$ i.e. we can give coördinates

$$\left.\begin{array}{l} x_1 : (1,0,0) \\ x_3 : (0,1,0) \\ x_5 : (0,0,1) \end{array}\right\}$$ i.e. x_1 is the *line* from (0, 0, 0) through (1, 0, 0), etc.

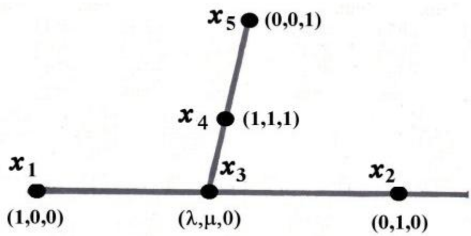
In this coördinate system x_4 will have coördinates (α, β, γ) say. Linear transformations of $\mathbb{R}^3$ that fix x_1, x_3, x_5 aren't going to affect x_8 which is on x_1x_2 so apply

$$T: (x, y, z) \longmapsto (x/\alpha, y/\beta, z/\gamma).$$

Now x_4 has the convenient coördinates (1,1,1). (Strictly x_4 is the line ... etc.)

NB $T(x_1) = x_1$ because $(1,0,0) \longmapsto (1/\alpha,0,0)$ which is just multiplication by a constant. Similarly with x_3, x_5.

Using an obvious shorthand we are in the following situation:



Any computation which now follows can only depend on λ and μ. Thus x_8 is independent of x_4, x_5 !!

WHO NEEDS ZERO? Harold Moulson

Way back in M500 12 Eddie Kent posed the problem: find two integers neither of which contains any zeros, whose product is 1 000 000 000. I met the problem a long time after this and was intrigued to know if this was unique representation. This year, having access to a programmable desk-top calculator with a 16-digit read out I have been able to investigate the problem empirically.

The first task was to construct a table of the powers of 2. At present the highest power is the 237th and as will be seen later there seems little point in going any higher. There are 35 powers of 2 in this range that contain no zeros in their expansion. These are: 1, 2, 3, 4, 5, 6, 7, 8, 9, l3, 14, 15, 16, 19, 19, 24, 25, 27, 28, 31, 32, 33, 34, 35, 36, 37, 39, 49, 51, 67, 72, 76, 77, 81, 86. Checking these values against the corresponding powers of 5 for expansions that contain no zeros, the list is reduced to ten:

$10^1 = 2 \times 5$
$10^2 = 4 \times 25$
$10^3 = 8 \times 125$
$10^4 = 16 \times 625$
$10^5 = 32 \times 3125$
$10^6 = 64 \times 15625$
$10^7 = 128 \times 78125$
$10^9 = 512 \times 1953125$
$10^{18} = 262144 \times 3814697265625$
$10^{33} = 8589934592 \times 116415321826934814453125$

The 237th power of 2 contains 72 digits and their distribution shows a fair degree of randomness, so I very much doubt there being any other solutions.

Every problem solved by exact science raises a hundred problems previously unforeseen; and so, in the measure that the exact sciences develop, our ignorance only increases!

Stefan Themerson: *Professor MMAA's Lecture*.

CHESS Roger Claxton

M500 Chess is flourishing and welcomes new members; please contact me if you are interested in telephone or postal chess.

Some figures for wonderment and possible dispute:
(a) theoretical maximum number of chess positions - 2×10^{50}.
(b) theoretical maximum number of possible chess games - 10^{l5790}.

Source: *British Chess Magazine*, January 1976. Any volunteers for the formulæ of the above?

PRIMES Brian Woodgate

Why do some textbooks consider 2 to be the first prime? If the definition is a natural number only divisible by itself and one then surely 1 qualifies. It may be trivial, but it meets the requirements.

There seem to be two camps, as for instance Hall & Knight define 1 as prime but Birkoff & Mclane do not. The problem arises when a book “considers the first *N* primes”, where do they start?

If 1 is not prime then why not?

Ed - If a prime is a number only divisible by itself and one and these are distinct then the only prime number is –l. If we drop the distinctness criterion we have two primes, 1 and –1. Insisting that only natural numbers can be prime means that either there are no primes at all or only the number 1. 2, for instance, has four divisors: 2, –2, 1 and –1; and these are all distinct.

Perhaps this is how prime should be defined: a positive integer with exactly four divisors.

Or, with Linderholm, “A number is prime if it has exactly two factors (where α and β are regarded as the same factor if $\alpha = u\beta$ with u a unit).

“The number 1 is not prime since any factor of 1 is *ipso facto* a unit, since (–1, 1) is the group of units in the ring of integers.”

(Are there composite ribs of beef?)

ERRATA

Max Bramer

In a rare moment of error our editor has managed to sabotage the third paragraph of my letter in M500 35 7. This should begin “Of course I *did* know ...”. Somehow the word “not” has crept in!

Among other misprints, the word “power” in paragraph six should be “purpose”.

SMOKING AT SUMMER SCHOOL

Bill Shannon

As a smoker for more than thirty years may I comment? It would not worry me in the least if smoking were banned in lecture rooms at Summer Schools. If M500 non-smokers were to put such a request in a reasonable way I would go along with them. A *Which?* survey (February 1975) showed that a surprising number of smokers would support a ban or restrictions in public spaces. However, some writers have chosen to adopt a singularly nauseating ‘holier than thou’ attitude, and as a fellow car-loather, I am glad to see Marion springing to the counter-attack. A number of unsubstantiated assertions have been made; can we stick to facts? For example in the UK in recent years road casualties have been about 360 000 a year, of which 100 000 were deaths and serious injuries (15 000 of them children). Source: Government statistics. In addition the infernal (*sic*) combustion machines belch out annually (UK): six million tonnes of carbon monoxide (a poison); six thousand tonnes of lead (another poison); four hundred and eighty thousand tonnes of unburnt hydrocarbons; two hundred and sixty thousand tonnes of aldehydes. Source: T100 Units 26, 27. I agree with Fred Holroyd that two blacks do not make a white, but in view of these appalling figures I would suggest that any car owner who adopts an air of moral superiority about smoking is guilty of the most sickening hypocrisy. Neither are non-car-owners in any position to sit in judgement: how many of them pollute the atmosphere by burning coal to heat their homes, or light bonfires in their gardens? Quite a few, I think.

---

and the final word, from

Datta Gumaste

Whatever else smoking may be it is an experience. I am neither a habitual smoker nor a habitual nonsmoker.

Most smokers, I suspect, know that with each finite set of puffs there is associated a ‘moment of truth’. To experience it you only need to carefully follow the movement of the smoke from the time you deeply inhale it until it reaches the bottom of your lungs and finally when it hits the centre of your brain! At this moment the smoker is the smoke.

On the other hand you don’t need to be an expert in yoga to feel the rhythm of the ‘rock and roll’ dance of the cells in your body when your lungs are filled with early morning fresher fresh air after deep and vigorous breathing.

In my experience the joy of nonsmoking is enhanced if you have experienced the ‘moment of truth’. In general it seems that the longer the interval between smoking and nonsmoking the greater the joy of both. (Of course the interval cannot be infinitely long.)

May I, therefore, submit that smoking at Summer Schools should be banned if and only if nonsmoking is banned!

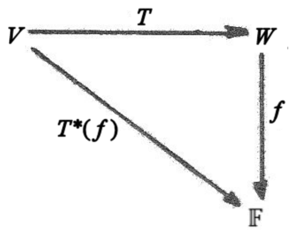
ALGEBRA IN THE FIELD Rosemary Bailey

(In M500 35 Rosemary described some basic statistical techniques used in her work of designing experiments in agriculture. She ended with the question: "How does abstract algebra come in?")

If you cast your mind back to the dreaded Unit 12 of M201 you will recall that if V is a vector space over a field $\mathbb{F}$ (ho ho - not the agricultural sort this time!) then there is associated with it a dual space V^* which consists of all linear transformations from V into $\mathbb{F}$. If you persevered to the end of the unit you may also remember that if Y is a subspace of V then there is a subspace of V^*, called the Annihilator of Y, consisting of all those functions f in V^* such that $f(Y) = 0$. Moreover (and I don't think this came in M201) if T is a linear transformation from V to another vector space W then we can define a linear transformation T^* (variously called the dual or adjoint of T) from W^* to V^* as follows: if f is in W^* then f is a function $W \to \mathbb{F}$; $T^*(f)$ is in V^* and so should be a function $V \to \mathbb{F}$; what could be more natural than to define $T^*(f)$ by

$$T^*(f): v \to f(t(v)) \quad \text{for all } v \text{ in } V?$$

That is, $T^*(f) = f \circ T$:



You are probably aghast at all of this. Certainly every student I have taught has always boggled at it when they met it for the first time. Indeed, I did myself: when I was an undergraduate I thought dual spaces were the most subtle form of torture ever invented. However, in spite of the apparent abstraction of all the above concepts I have had to use them in my work in design.

There again, what sort of fields are involved? Not, as you might think, the "natural" fields $\mathbb{Q}$ and $\mathbb{R}$ (though it has always been beyond me why a number system that needs equivalence classes of Cauchy sequences (or, worse, Dedekind cuts) to define it should be thought "natural") but the finite fields $GF(q)$ for prime powers q (vide end of M202). Personally I am rather finded-minded and am much happier with these fields than with $\mathbb{R}$, but I gather that the feeling is not universal: when I go along to the computing-machine boffins with a problem over $GF(9)$ they glare at me angrily and explain that their machine works "naturally" over $\mathbb{R}$ and it would require lots of programming to get it to cope with $GF(9)$.

Finite fields come with prime power order only, and not all experiments can be forced into this mould. Thus sometimes we require to take $\mathbb{F}$ to be a ring rather than a field (vide Unit 17 of M202): how much of the abstract theory still applies?

Eigenvalues and eigenspaces seem to have a crucial importance in the work on design. The day I discovered that "stratum", a word statisticians bandy about daily without ever defining, meant "eigenspace" a great light dawned on me. (Incidentally, if you really want to annoy a statistician with a question about an unexplained often-used word, ask him/her what a "degree of freedom" is. I think I have a partial answer: "n degrees of freedom" means n is the dimension of a certain subspace of a vector space which is orthogonal to certain other subspaces (the ones with different degrees of freedom) under some unspecified inner product.)

I'm primarily not a vector space theorist at all but a group theorist. Most of the group theory I've had to use falls outside the OU maths courses so I'll have to be content with mentioning topics. I've had to use Sylov theory, double transitivity and the lack of it, primitivity and the lack of it, fidelity a.t.e.l.o.i. (all this is getting to sound like one of the Sunday papers), and also a branch of category theory called character theory (which is not about giving references but about the interrelation between group theory and vector space theory), which I had hoped could safely be put in a museum show-case labelled "Pure Mathematics" and have the lid sealed for ever, but I understand that Physicists and Chemists use it too so perhaps I should not be too surprised at its appearance in statistics.

In conclusion I shall leave you with a problem which is of great relevance to design, which I can state in terms which I hope most M500 readers can understand, but whose solution needs, I suspect, much high-powered Pure Maths. (The flood of correct solutions will, of course, be gratefully received.)

G is a group of permutations of a set S. We define the ordered pairs (a, b) and (c, d) of elements of S to be equivalent if there is a group element g such that g carries a to c and b to d. (Check that this is an equivalence relation.) Suppose this equivalence relation decomposes $S \times S$ into n equivalence classes. Let $t_i, \ldots, t_n$ be n "independent variables" (I could be more precise here but I think most people will know what I mean). Let M be the matrix whose rows and columns are labelled by elements of S and whose (a, b) entry is t_i if (a, b) is in the ith equivalence class. M may already be symmetric; if it is not make it symmetric by adding it to its transpose.
Question: how many distinct eigenvalues has this symmetrised form of M?

Two items lifted (with permission) from recent copies of *The Times,* Diary of PHS.

First, 11 June 76: 'More metrical nonsense. The newly metricated edition of *Teach Yourself Mechanical Engineering*, by A E Peatfield, speaks of the use of a box spanner in a deep recess (vol 1 p. 13): "It might be found that the turn obtained was only 25.4 mm or 50.8 mm." The English from which this has been translated was, of course, "an inch or two".'

Then this, 22 June 76: 'A research document published by the Institute of Personnel Management reports that the white collar section of the General and Municipal Workers' union has 26 full-time officials, of whom 98% are male and 2% female. You try to work it out.'

UNENDED QUEST: AN INTELLECTUAL AUTOBIOGRAPHY, by Karl Popper

This book is available for the first time in paperback, Fontana £1. It was originally part of *The Philosophy of Karl Popper*, edited by P A Schilpp and published as volumes 14/I and 14/II of *The Library of Living Philosophers*, Open Court, Illinois, 1974, and hence largely inaccessible.

Now that Fontana have taken the plunge and brought the work out in a popular edition they ought to be encouraged: this is possibly one of the philosophical works which will be seen to characterise our century. Comparisons with John Stuart Mill's autobiography have been made. Like that work there is no great concentration on the facts of the author's life but rather an account of an intellectual journey.

Popper will of course be remembered best for his work on epistemology and his criterion of falsifiability as the distinguishing feature between science and pseudoscience. This is probably the basic concept in *Unended Quest* but there are so many and such long philosophical digressions that one nearly loses sight of the central theme at times.

Popper has always been an entertaining writer whose gifts have seemed almost too varied for one man. This book, looping all the strands of his cerebral life together, certainly explains how it was done. It is at last clear how the man who wrote *The Logic of Scientific Discovery* as a young man could later come out with *The Open Society and its Enemies* (a work which became known in certain circles as "The open society by one of its enemies!).

From Brian Woodgate - Congratulations on this year's Special Issue. I liked the layout and the results figures. It's advantage is that we can say what we feel about courses so that one gets the consumer view rather than the official view. Our contributions may be criticised in M500 later but surely there is nothing wrong with that as long as discussion both 'for' and 'against' is allowed.

May I suggest that to assist new members, including myself, all contributors who are not in the MOUTHS lists identify themselves. For example I have read through the above lists without finding a reference to s certain M Bramer who appears regularly.

From A H Clark - Many thanks for your fine magazine. I can only gasp at the erudition and expertise of your many contributors. It often takes me days to follow a simple proof but then I only joined to find out what "modern maths" was all about and I haven't managed to find my way back to the Arts world yet.

From Dave Diprose - I enjoy the present variety of M500. Some items look {and are) well above me, but there always seems to be something which starts me thinking; the joy of Independent thought and discovery is what mathematics is all about. Pythagoras regarded 'the intoxicating delight of sudden understanding that mathematics gives' as something spiritual, and that is how it is with me.

CALCULATORS Marion Stubbs

M500-*WHICH*? REPORTS ON CALCULATORS: NUMBER 1 - HP-55

HP-55 and HB-25 are now displayed in Dixons (at least hereabouts). They look similar, with various obvious differences:

<u>HP-55</u>	<u>HP-25</u>
£215	£107
10 visible digits	8 visible digits
2 applications books at $10 each extra	1 applications" book supplied free
Metric conversion keys	Fewer keys than HP-55

I wrote to Hewlett-Packard requesting information about the non-obvious differences and also asking for free HP-25 applications book with an HP-55 if I bought it by direct mail. (Well they could only say no ---!) I quote sections of the reply:

"The real advantages of the HP-55 compared with the HP-25 are:

- 20 addressable stores compared with 8
- Additional statistics functions (linear regression, estimate, &c)
- H.MS addition and subtraction
- Metric conversions
- Crystal controlled timer.

"The HP-25 on the other hand has these advantages:

- Fully merged keystroke programming
- Engineering notation
- Pause function
- Additional conditional statements."

The letter went on to offer me an ex-demonstration model HP-55 of which they had several in stock at £175.44, being less 15% ex-dem discount, less 4% cash-with-order discount - plus 8% VAT, together with a free HP-25 applications book for my cheek. Completely refurbished with full one year guarantee.

I now possess my HP-55 - a long-held dream incidentally and more use to me than any Hi-Fi gear at similar price. It seems to correct most of the faults I have complained about when testing other calculators and is worth its price. It has four working registers, called `X` (displayed), `Y`, `Z` and `T` (= top of the working stack). These rotate upwards and downwards automatically during arithmetic operations, or can be rotated down (towards `X` and visibility) by keystroke `R↓` and also the usual `x ↔ y` interchange facility is available.

It could definitely be improved by a fully-merged keystroke programming since one keystroke = one instruction and this uses up more of the 49 instructions available than the HP-25 would do. However, I am coping well enough. My prize program - indeed my first - is Clenshaw-Curtis Integration with $n = 8$, which will be old hat to M351 types. This uses 17 of the 20 registers and 98 instructions in 2 pages. The first page is entered and run, calculating $f(\cos(k\pi/N))$ for $k = 0$ to N and storing results in register a .0 to .8. Then follows the main program entry and run. Due to lack of indirect addressing facility both programs have to stop intermittently, show me what k is, and register .k is then recalled or stored by hand. Tedious. M251 please note!

The 20 addressable registers, by the way, are labelled 0 to 9 and .0 to .9, It takes three instructions to recall .0 to .9 (`RCL . 7` for example) so one soon learns to use them for data storage leaving the 2-address registers (`RCL 7`) for intermediate and final results.

Anyone with an HP calculator might like to exchange this and other programs, but you really need M351 Unit 11 before using the method so there is no point in giving further details here. Like all numerical methods you need to know when how and why it is likely to succeed, and tests for the error bound.

On the whole I give 9.666666667 out of a possible 10 marks to the HP-55. It loses 0.333333333 for lack of fully-merged keystroke programming. Nothing deducted for lack of indirect addressing as I doubt if any calculator has this yet - but surely will soon.

FOUNDATION

Steve Murphy

Tony Brooks's article was very enjoyable and it may well have been good philosophy, but was it good physics? I always thought that the aim of physics was to find properties that were invariant (absolute) under transformations in space and time. To abolish a particular temporary view of the absolute may be possible, but to abolish the concept of the absolute would seem to abolish physics. As for the Schrödinger and Heisenberg approaches being different - why am I learning all this junk about Hermitian Operators (SM351 Units 9-11)? I've always been told that such methods were introduced to show that the two were completely equivalent.

Then there was the bit about Set Theory being an adequate basis for "mathematics". It either says nothing very much like "Mathematics is what we can deduce from the axioms of Set Theory", or really is saying something worth saying like "We have an external definition of mathematics and can deduce it all from the axioms of Set Theory". If it is the latter then it is a technical question which has either been answered or is still being worked on. Perhaps someone could tell us whether the question is a sensible one to ask, and if it is, whether the answer is "Yes" it isn't obvious that we can start from anywhere else in what we call mathematics and deduce the Axioms of Set Theory. We may be able to do so though it is perhaps very inconvenient. After all you may explore the world from anywhere but sensible people start from Heathrow.

The age of chivalry is gone. That of sophists, economists, and calculators has succeeded, and the glory of Europe is extinguished for ever.

Burke

OBITUARY: Professor Allan Birnbaum,

Allan Birnbaum, who was Professor of Statistics at the City University since 1975, died suddenly on July 1.

He was the man who "made crystal clear the likelihood principle", and wrote in 1962 On the foundations of statistical inference. L J Savage described that as an event "really momentous in the history of statistics. It would be hard to point to even a handful of comparable events."

Briefly: born SanFrancisco.1923; educated University of California and Columbia University. PhD Columbia (mathematical statistics) and lecturer there. 1959 associate professor at Courant Institute of Mathematical Sciences, New York; then full professor there in 1963. In England: 1956-7 Imperial College, London, 1972-3 Cambridge, 1973-5 University College London. Then 1975 City University.

In *The Book Collector* for Mar there is a piece of archaeological scholarship by Eric Thompson, who died last September.

The subject is the recent "Mayan" discovery, the *Grolier Codex*. This has been accepted as only the fourth Mayan book to have survived. Like the other three it is a Venus Table, enabling priests to predict the natural disasters associated with the appearance of the Morning Star.

Since its appearance at the Grolier Club, New Vork, in 1971, it has been published in facsimile and exact colour, thus giving Eric Thompson his opportunity. This he took with devastating effect. The hieroglyphs, astrology and iconography of the codex, though ingenious, would have baffled - even terrified - any Maya.

In the first place the vigesimal system of numeration used is an impossible mixture. Then the ring at the top of numbers (which indicates the distance before the Mayan era) is misused, to make statements as meaningless as "from appearance of Venus as morning star to its disappearance is 236 days BC". It has Venus hurling pestilential darts (rays) indiscriminately whereas all other sources are agreed that he did this only at his first appearance.

There are many other anomalies, such as a god capturing prisoners (when it is well known it is not possible to survive meeting a god), another carrying an unknown ("belt- and-braces") combination of weapons, and - particularly disastrous - the God K with the wrong teeth. Altogether this is an impressive piece of detective work which deserves to be read in full.

From Michael Gregory - The missing item from near the bottom LH corner of "Imperfect Indicative" (M500 Special 1976) is "transference from continuous problems to discrete".

I am glad to see that my anonymity is being preserved (M500 34 6) in the tradition of the BBC and numerous magazines; the usual domain (the person himself) is broadened in this case.

EUREKA!

E Kent

What Archimedes said, according to Vitruvius Pollio, was εύρηκα. (I have found!) He is also said (Pappus Alexandr., *Collectio*) to have told Hiero, King cf Sicily, "δώσε μου, αλλά μία σταθερή σημείο ..." (Give me but one firm spot on which to stand, and I will move the earth.) He was of course talking about levers, as every schoolboy knows. But do schoolboys in general stop to consider how long it would take to move the earth, say one inch?

Ozanam (in "Recreations in Science and Natural Philosophy", *Riddle*, 1854) did. He made the usual assumptions (usual to mechanical academics, that is) that any machine used would be frictionless, without mass and in complete equilibrium. He supposed the earth to weigh 300 pounds per cubic foot and to be spherical with a diameter of 7930 miles. That is, the earth weighs 11 530 342 879 148 611 584 000 000 pounds.

He further supposed that Archimedes was equal to an effort of about 30 pounds sustained for eight or ten hours moving with a velocity of 10 600 feet per hour. One of the laws of mechanics tells us that whatever the construction of the machine "the space passed over by the weight is to that passed over by the moving power in the reciprocal ratio of the latter to the former". So we have Archimedes cranking away at 30 pounds at 10000 feet per hour and by the time the earth has moved one inch the moving power has passed over 384 344 762 638 287 052 800 000 inches taking 3 202 873 021 985 725 440 hours.

This of course is 3 653 745 176 803 centuries. But he ought not to work more than eight hours a day and he should have time off occasionally to avoid getting stale. So twelve billion centuries should be ample time to get the job finished.

ONLY CONNECT

Willem van der Eyken

I remember that Dr Ian Kettley (the man you love to hate) made the point not all that long ago (M500 15! - Ed) that OU maths students have difficulty in relating one course to another as they climb the weary mountain of knowledge called M. Poor man, he looked like St Anthony after the Romans had finished using him for target practice - not so much a lively correspondence as a way of death! Never attack an OU student, Sirrah; we have no time to take ourselves lightly - the TMAs won't allow us to develop a sense of perspective. Anyway, what Ian Ketley was saying had, despite all our pained yelps, a good point, and I notice that to make connections between OU courses is by no means easy, even if you really set your mind to it.

Take M231 (yes, please do take M231; quite nice when you've stopped banging your head against the wall). In unit 9 there is a rather arbitrary discussion of characteristic functions which clearly has a lot to say about "chi squares", that somewhat mysterious formula which we all use and which appears briefly in MDT241, Unit 14 7, for testing expected against observed frequencies. But nowhere in M231 does it suggest that there is such a connection and one presumably waits for the heralded course on Mathematical Statistics to see just what the connection is. Until then can anyone write a neat little link between the two so that I can be put out of my misery? Incidentally, M231 also has something to say about isometries which clearly relate to the work of M201, to the units on groups in M100, and to M202; but nowhere does it make this link and the hard-pressed student is left to believe that this is something quite new. Does this suggest that there is a case for producing a few units (spare time reading) which do create the necessary bridges which will amplify and connect the various courses?

CREATIVE COMPUTING

M Stubbs

Creative Computing now has a UK address, in Southampton, due to some efforts and introductions by yours truly. My friend, Hazel Gordon, is now *CC*'s British Agency from amidst her many nappies because she has, with the utmost diligence and efficiency, organised herself and other friends into a clerical agency which undertakes every kind of office work for anyone who will employ them in their own homes. They don't yet have an enormous clientele but enough to make some pin money and it gives them a non-nappy interest while keeping them in clerical practise until such time as they can return to work.

I recommended Hazel to David Ahl, who edits and publishes *Creative Computing* in his spare time and makes me feel totally exhausted to watch. *CC* is now in its second year and is real big business from a basement or something in New Jersey. He even finds time to go to meetings of his beloved New York Star Trek Society (no comment!) It has taken several months to set up the British Agency, partly because of all the certification which is legally necessary not to mention the exchange control act, but now they have most of the details sorted out. Meanwhile, beating the gun a bit, David advertised the British address in various British advertisements, leaving Hazel with orders coming in by the dozen and no means of applying them. Hopefully this period is nearly at an end but she is still coping with some tariff barrier at the USA end which merely means that he cannot at this moment despatch *CC* publications in bulk to Hazel for her to distribute to individual customers; she is temporally forwarding orders to him for individual despatch.

The subscription to *CC* is £5 to be sent to Creative Computing, 60 Dorchester Road, Southampton, Hants. This is a magazine computer enthusiasts cannot afford to miss. Published six times a year, it usually runs to about 70+ M4 pages full of ideas, articles, programs, computer games, computer art, and anything except commercial computing. Pleases allow time for delivery from USA at present.

SOLUTIONS

33..4 *THE PROFESSOR*

A professor told his assistant that he had given a party for his wife and two nieces. “The sum of my wife and nieces ages”, he said, “is twice your age and the product of their ages is 2450 years. How old are my nieces?” The assistant complained he had insufficient information so the professor said “I was the oldest person present.” Then the assistant gave the answer. How old was the professor?

Bill Shannon and Brian Woodgate: the professor was 50. $2450 = 2 \times 52 \times 72$. (49 ,10, 5) and (50, 7, 7) are the only triples which sum to the same even number. If the professor was the oldest one present he must be at least 50. If he was 51 or more the ambiguity would still exist for the assistant.

34.2 *MORE BALLS*

One red pair, one white pair, one blue pair of balls. One ball of each pair heavy. All heavy balls of equal weight, as are all light balls. Find the heavy balls in two weighings on a balance.

Tony Crilly, Datta Gumaste, Krysia Broda and Mervyn Savage all had basically similar solutions. Here is a digest: Label the balls *r*,*r**,*w*,*w**,*b*,*b**. First weighing put *w*,*r** in the left pan and *b*,*r* in the right. For the second weighing put *w*,*r* in the left pan and *w**,*b** in the right in case it was the right hand pan went down. If neither pan went down then for the second weighing put *w**,*r** in the left and *b*,*r* in the right. It is easy to see that this method distinguishes all eight possibilities. The important thing is that the first weighing must allow three different outcomes or equivalently: after one weighing there must be at most three possibilities left.

34.3 *GOAT AND FIELD* II

The goat is tethered to a point on the perimeter of a circular silo (which he cannot enter). The length of the rope just permits him to graze to the diametrically opposite side of the silo. What is his grazing area?

Solutions from Mervyn Savage and Steve Murphy, confirmed by Richard Ahrens. Perhaps if anyone would like a full solution they could contact one of the above since I don’t understand it at all. If the silo has radius r then the area grazed is $5\pi^3 r^2/6$.

34.4 *NEXT TERMS*

17. Brian Woogate has pointed out that the next two terms can be constructed from the difference triangle, so we print that.

1	1	1	3	3	15	15	105	105	945	945	10395
2	2	4	6	18	30	120	210	1050	1890	11340	
4	6	10	24	48	150	330	1260	2940	13230		
10	16	34	72	198	480	1590	4200	16170			
26	50	106	270	678	2070	5790	20370				
76	156	376	948	2748	7860	26160					
232	532	1324	3696	10608	34020						
764	1856	5020	14304	44628							
2620	6876	19324	58932								
9496	26200	78256									
35696	104456										
140152											

solutions continued

34.1 *MAGIC SQUARES*

L J Upton, our new Canadian member, contributed the following with the heading “Knight’s Tours”:

1	30	47	52	5	28	43	54
48	51	2	29	44	53	6	27
31	46	49	4	25	8	55	42
50	3	32	45	56	41	26	7
33	62	15	20	9	24	39	58
16	19	34	61	40	57	10	23
63	14	17	36	21	12	59	38
18	35	64	13	60	37	22	11

It has been proved that fully magic squares (where diagonals also add up to the magic constant) are possible only with squares whose sides are multiples of 4, but none has been found for a square with side of 8.

The square on the left was published by William Beverley in the *London, Edinburgh and Dublin Philosophical Magazine and Journal of Science*, August 1848, and is understood to be the first published.

If the square is quartered there are four magic squares as regards rows and files. If it is further quartered each order-2 square contains four numbers that add up to 130. The square is not re-entrant, that is you cannot go by Knight’s move from 64 to 1 as you can in the solution published in M500 34.

L J Upton also sent in solutions to *MORE BALLS, GOAT AND FIELD* II as well as to the original *BALLS*

PROBLEMS

36.1 *THE* 196*th ROOT* - MS

The problem below is taken from *Games and Puzzles* 24, May 74, - before our member David Wells became puzzles editor! The solution is given quite simply and in one line as the most famous number you are likely to think of, perhaps. (It has one digit which limits you to your ten most favourite famous digits for guesswork!)

Being totally thick the only way I can devise to do it is by calculator. I have now several alternative solutions. M351 should enjoy it. Question for others is just how many solutions are there? Proof required.

Problem: Calculate the value of the expression shown here, not using pen, pencil or paper, and doing it within a minute.

$$\sqrt[196]{10^{59}\cdot\left(\frac{1025}{1024}\right)^5\cdot\left(\frac{6560}{6561}\right)^3\cdot\left(\frac{9801}{9800}\right)^4\cdot\left(\frac{15624}{15625}\right)^8\cdot\left(\frac{1048576}{1048575}\right)^8}.$$

36.2 *FIND THE NEXT TERMS* - the end of our collection from N J A Sloane’s *J Recreational Mathematics* article. Rule wanted.

21 1, 2, 2, 3, 2, 4, 2, 4, 3, 4, 2, 6, 2, 4, 4, 5, ...,

22 4, 6, 9, 10, 14, 15, 21, 22, 25, 26, ...,

23 1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025, 175099, ...

problems continued

36.3 *POLYGONS* - Richard Ahrens

(a) Consider an *n*-sided polygon. Join every pair of vertices by a straight line. How many pairs of lines intersect other than at the vertices of the polygon?

(b) Show, without actually drawing the figure, that no three diagonals of a 7-slded polygon meet except at vertices.

(c) Can you prove that in a regular polygon with an odd dumber of sides, at most two diagonals meet at a point other than a vertex. (Looks hard - meaning 'I can't do it'.)

36.4 *LOGS* - Bill Shannon

We all know that $\log_{10} n < n$ where n is a positive real number. How big does x have to be for $\log_x n$ to be less than n (>0)?

36.5 *THE BLACK ACE* - Jeremy Humphries

A pack of cards is shuffled until the first black ace appears. Where is it most likely to be?

This problem was given in the *Mathematical Gazette* December 1969 by A E Lawrence.

EDlTORIAL

Not much to say this month except to hope that everyone feels the outlay for the new machine was worth it. It certainly makes life a lot easier for me. We can now have symbols with our mathematics and even Greek though purists among you will notice that the epsilon on eureka! is the wrong one and the sigma on δός is a curious shape. That is because it is in fact the upper half of a left-hand curly bracket moved slightly down. From which you will gather that we only have ε and σ (and yet both δ and ∂). Also there is not a square bracket in sight. We are going to get some italics and some large headline stuff in time for the next issue and then I think that will be enough - unless anyone thinks we should vary the typeface for variety's sake.

I would like to mention that there is very little on hand for the next issue. It's lucky there was a two month gap between 35 and 36 or I would not have been able to fill this one. We are short of everything - articles, letters, cover designs, even problems (which I thought would never run out). Of course, I am writing this during the last days of August and you are reading it after the Mathematics Weekend Work in and presumably, the M500 Society's AGM, but even if I get the sack I'll still need lots of stuff to pass on to the next incumbent.

And now, if you'll excuse me, I'd better get on with some work.

Eddie Kent.