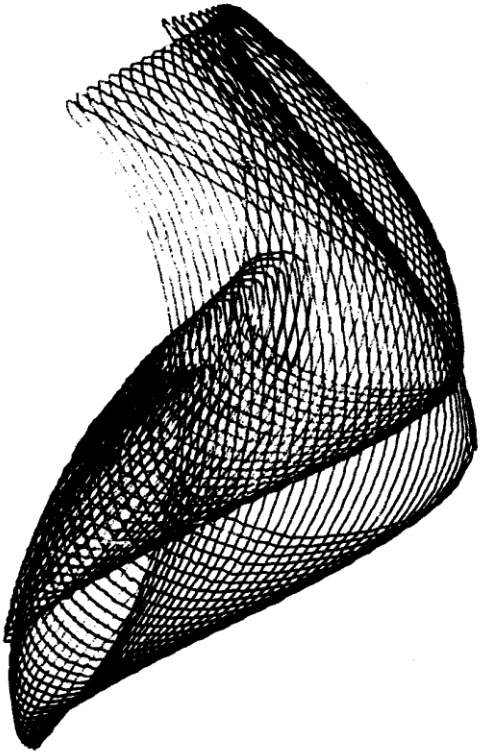

M500 37

M500 is a student operated and owned magazine for Open University mathematics students and staff, and for anyone else who is interested. It is designed to alleviate student academic isolation by providing a forum for public discussion of Individuals' mathematical interests.

Articles and solutions are not necessarily correct but invite criticism and comment. Articles submitted for publication should normally be less than six hundred words. Anything longer ought really to be split into instalments.

MOUTHS is a list of names, addresses, telephone numbers and previous and past courses of voluntary members by means of which private contacts may be made, to share OU and general mathematical interests, or to form telephone or correspondence self-help groups.

The views and mathematical abilities expressed in M500 are those of the authors and may not represent either those of the editor or of the Open University.

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Please send anything for inclusion in M500.

MEMBERSHIP SECRETARY
Peter Weir.
Send membership applications, subscriptions, change of address, MOUTHS *data*.

TREASURER Austen F Jones.

Cheques and postal orders (which should go to Peter Weir) should be made payable to THE M500 SOCIETY and crossed *a/c payee only, not negotiable* for safety in the post.

(*Anything sent to any of the above will be considered for possible publication in* M500 *unless otherwise specified.*)

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SUMMER SCHOOL HANGOVER

Bob Margolis

There are several nice things about Summer Schooling at Reading. There is, of course, the stimulating company of real live students; there is the beer - the real stuff at last in Bridges; there is also, most unexpectedly, a bookshop with occasional real bargains. A couple of years ago I found Halmos’s *Measure Theory* (a sort of prehistoric Weir for the M331 buffs) for 60p and a little book by Kaplansky on Lie Algebras and other things for 40p.

One of the yawning gaps in my mathematics that I’m actually conscious of is labelled Lie Algebra. I’ve been looking for a long time for a book that begins “a Lie Algebra is a set with binary operations ... satisfying the following axioms ...” that was cheap and readable. Kaplansky certainly starts in the desired way and was cheap - the readable bit I’m not so sure about! However, it’s left me with a two year hangover in the shape of an early exercise I can’t do. On the principle that if I’ve got problems you can suffer too let me expose my ignorance and plead for help!

First you’ll need the background; not much because it’s exercise 4 of the first set. A *Lie Algebra* is a vector space v over a field F with a multiplication of vectors defined. The multiplication is not associative (nothing so simple!) but is distributive over addition etc., and satisfies

(i) $v^2 = 0$, “$v \in V$;

(ii) $(uv)w + (vw)u + (wu)v = 0$, “$u,v,w \in V$.

A consequence of (ii) obtained by putting $v = x + y$ is $xy = -yx$. A *finite dimensional* Lie Algebra is one where v has a finite basis (*à la* M100, M201 etc).

There are some Lie Algebras around. Try taking 2×2 real matrices with multiplication $*$ defined as $A * B = AB - BA$ (usual multiplication). This is a four-dimensional algebra with basis

$$\begin{pmatrix}1,0\\0,0\end{pmatrix}, \begin{pmatrix}0,1\\0,0\end{pmatrix}, \dots .$$

One more trivial example, take any vector space and define $uv = 0$, “$u,v \in V$. This is called *the abelian* Lie Algebra of dimension whatever $\dim V$ is. (*The* because all vector spaces of a given dimension look the same ... .)

Last definition - then you can have my problem! If V is a Lie Algebra then the square of V, called V^2, is the set of all sums of products of elements of V. (V^2 is a subalgebra of V.)

PROBLEM V is an n-dimensional Lie Algebra with the following properties:

(1) V^2 is one-dimensional;

(2) $\exists x \in V^2$ and $y \in V - V^2$ such that $xy \neq 0$.

Prove that there exists a basis $x_1, x_2, \dots, x_n$ with the properties

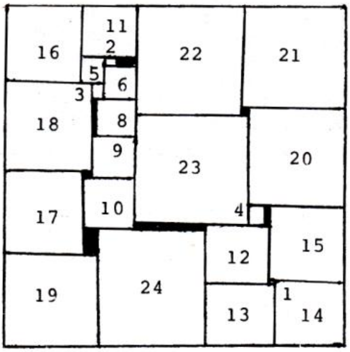
$$x_1 x_2 = x_1; \quad x_i x_j = 0 \;\; i,j > 2.$$

(Or: prove V is the direct sum of the 2-dimensional nonabelian lie algebra and the $(n-2)$-dimensional abelian one.)

TANTALISING TRIANGLES C W Pile

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}(n+1)(2n+1) = N^2;\ n,N \in \mathbb{Z}^2$$

If n = 24 the sum is 4900, giving N = 70; and this solution is unique, apart from the trivial n = N= 1.



The problem of fitting a set of squares of sides 1 to 24 inside a square of size 70 was posed in *Scientific American* several years ago. The best arrangement (shown here) omitted only the 7-square. leaving 49 unit squares uncovered (shaded). As far as I know this arrangement has not been improved upon nor shown to be minimal.

Searching for a perfect arrangement led me to consider triangular elements. An equilateral triangle of side n yields N^2 unit triangles. Can a set of 24 equilateral triangles of sides 1 to 24 be fitted inside an equilateral triangle of side 70? If not what is the best arrangement of a subset of 24 triangles which leaves the minimum number of uncovered unit triangles. Is this minimum the same as for the square problem?

So far my best arrangement leaves 149 unit triangles uncovered, omitting the 10-triangle and the 7-triangle. (This will be printed in M500 38 - *Ed*.)

The problem can be tackled as an entertaining puzzle by ruling a large equilateral triangle on stiff card. The side length should be 70 units and it is useful to have a coarse triangular grid of, say, 5 units. The set of 24 triangles can be cut from this card. A convenient unit length is 1/4 inch. The smallest members need not be included as it is usually obvious that these can be fitted.

Of the N^2 elementary unit triangles there are N more upright than inverted. Thus a perfect arrangement (if one exists) must produce a difference of 70 unit triangles. Therefore the sum of the sides of the upright members of the 24-triangle set must exceed the sum of the sides of the inverted members by 70 (i.e. the sum of the sides of the upright triangles for a perfect arrangement is 185) .

"We doubt that anything has been gained by the formula language." (Hoppe);
"... indulges in the Japanese habit of writing vertically ... monstrous waste of space", (Schröder).

Two contemporary views of Gottlieb Frege's *Begriffsschrift*.

LETTER FROM BELGIUM

Tony Brooks

I thought M500 readers might be interested to know how I am settling in here in Belgium. I have been over here now for seven months and I feel no urgent desire to return to England. There are a number of advantages in living in Belgium, such as much higher salaries (which is why I came over here in the first place) which more than compensates for the higher prices, low tax - as an *étranger* I get a special tax concession, - double payments, under Belgian law one gets paid double salary in June and December; good food - even the smallest corner restaurant beats all but the very best British restaurant; holidays - no problem now about an expensive channel crossing. This year my family and I drove to Switzerland for a couple of weeks.

There are of course some disadvantages. If like me your French is non-existent then one obviously has some communication problems. I work for an American company (Westinghouse Nuclear Europe) where most people speak English so I have no problems at work. However I have tried to learn some French so that now je parle Francais une petit peu. I find that I can now understand much of what I hear but I still can't think of the correct words to string together to say something! The Belgians love paperwork even more than the British. It has taken 6½ months and endless amounts of form filling to legally import my motor caravan and acquire Belgian numberplates for it. Also I am still trying to obtain a permanent identity card; at the moment they keep renewing my provisional card. However, overall I am glad I made the move here, it's a great experience. I hope that Westinghouse will be able to find plenty of business in Europe for a few more years at least.

I was most interested to follow the Max Bramer saga. If you want to see what can go wrong when OU staff try to run a magazine for you then you need look no further than *Open Mind* - the new philosophy journal produced by the OU for their philosophy students and staff.

THE OPEN SCIENCE SOCIETY

Colin Mills

Some months ago, after moving into a new job in a different town, I enrolled on M251, MDT241 and MST282 (TM221 subsequently abandoned) and became involved with other OU societies, besides helping the local Labour Party. I then made the rather rash statement that M500 ought really to be represented in the Open Science Society because it is a science in its own right and constitutes both a basis and a source for science and technology; Science, Technology and Mathematics are intimately connected. To my amazement I found myself being coöpted onto the OSS committee as M500 rep. This is the reason for the entry in *MOUTHS* 5, citing Walter Dalton (not Bill - my mistake) and myself as contacts.

I shall try to begin a discussion on what OSS and M500 might have to contribute to one another: comment, criticism and suggestions please! I'm not excusing any lapses in advance, but committee members are liable to slide into putting their own preferences first without ferocious prodding from their members.

The aims of OSS include the fostering of the amateur tradition in Science; the linking of the older scientific institutes with the OU scientific societies; demonstration of the value of science to society, and the promotion of the interests of the members of the OU - especially those involved in scientific disciplines.

What do mathematics students want (and need) from their societies?

What should the relationship be between M500, OSS and OUSA? Lectures, visits, relations with other mathematical societies at national and branch level can be arranged, but as Marion Stubbs pointed out in MOUTHS 7, in organisations such as ours which do not have a system for filtering the opinions and wishes of its members, all members must be consulted. If some of you have some free time to write articles on mathematics we would be able to publish them in our newsletter and I will gladly pass on details of our meetings and plans for regional branches.

THAT WEEKEND

Nick Fraser

Strange that I should be writing in favour of the Mathematics Weekend 1976 when I have probably the biggest grudge against it. Namely having my car stolen. I got it back the following Wednesday (found in South Brum) with the driver door window smashed. Silly really as it is a sports car and all they needed to do was unclip the hood! One consolation they left me the hammer.

M100 is perhaps the least likely course to benefit from such a weekend as we had, basically because most do have contact with other students in their area during the course. With 2nd and 3rd level courses this is far from true. However, even though I am “privileged”, a bit more contact is useful, especially with those who have experienced it before. It helps clarify what I want to do (and with the inside info dished out by a certain Dean, a clarification of what I can’t do). So I enjoyed it but reckon I will enjoy it more next year.

The other thing to come out of the Weekend was my agreement to act as Public Relations Officer (assistant to Marion that is because of the constitution). Here as with the Treasurer experience does count: I am a Product Manager in a Marketing department with experience of advertising and PR campaigns. I am a member of the Institute of Marketing and have access and knowledge of information needed. One point however: For all the work I may do the best publicity machine is word of mouth. More members means more ideas and viewpoints and it’s up to you to see if you can’t persuade just one person to join. If we all did that then we would double our membership. How’s that for a geometric relationship?

MATHEMATICS AND LIFE

Alan Slomson

“Mathematics, rightly viewed, possesses not only truth, but supreme beauty - a beauty cold and austere, like that of sculpture, without any appeal to our weaker nature.”

“For the past three centuries we of the so-called advanced civilizations have been living in a world dominated by the intellect, in which mathematics is king, a world in which we are removed, stage by stage, further from and outside our organic body. ... So alien to us is this abstract world that we are incapable of understanding it, we are at the mercy of an elite whose purposes and their results we deeply distrust. There is real danger in this intense, supreme cultivation of the intellect both for the individual and society. For the individual it can involve a schizophrenic split in the mind that spells withdrawal from ordinary life and human relations, for society deep divisions of class and values. The arch-schizophrenic - and magician - is the mathematician.”

“[T]he atmosphere is very informal and it takes only a short time in their company to realise that mathematicians are not dry people, but on the contrary are warm and lively”

It would be interesting to offer a prize for the best guess as to the source of these quotations, but I shall reveal to you that the first quotation is from an essay *The Study of Mathematics* written by Bertrand Russell in 1902. The impression it gives is that mathematics is cold and aloof. It is therefore interesting to set it alongside the second quotation which comes from the autobiography of Russell’s second wife, Dora Russell, and occurs in the middle of a description of Russell’s character. (Dora Russell, *The Tamarisk Tree*, London 1975. See especially pages 290-293.) It confirms the view that mathematicians are cut off from ordinary life.

At first sight this impression is contradicted by the third quotation. It comes from a local newspaper description of a conference on mathematical logic held in Denmark in 1971. But it is clear that the reporter expected mathematicians to be dry people, and there is no doubt that mathematicians are commonly regarded as apart from other men in the way Dora Russell indicates.

Indeed it is true that mathematics, as usually presented, almost alone of disciplines seems cut off from human life. It is impossible to imagine history, philosophy, law, economics or medicine being presented without any consideration being given to the nature of human beings, their needs and their relationships with each other. Like these other subjects mathematics is also the creation of human beings responding to problems raised by their own existence, but it is usually presented in a formal way without any serious consideration of why or how it has developed.

As Russell says, in some ways this aloofness of mathematics has its attractive side. Abstract mathematics has a beauty unsullied by grim reality. The study of mathematics can be a means of escape from the world, and it is not surprising that many people welcome an escape from the reality of the world today. Also the divorce of mathematics from other human activities makes it easier for mathematicians from different social and cultural backgrounds to communicate and co-operate with each other.

At the same time however this divorce of mathematics from life can cause uneasiness in students of the subject. It is often difficult, especially in pure mathematics - the choice of the adjective “pure” is significant - to see the justification for what one is studying. The problems of mathematics seem remote from the problems of everyday life. The unique difficulty in explaining one’s subject to non-mathematicians reinforces this feeling of isolation.

Furthermore because of the way mathematics is usually taught there is a danger that in studying mathematics you develop only one aspect of your personality. In learning mathematics you learn nothing about relationships between people, very little about how to communicate with other people or about how to handle inexact concepts and questions of value. And although there is plenty of scope for creativity and imagination in mathematics it is difficult to teach the subject in such a way that students experience the joys of inventiveness.

It may be that these problems are not so acute for Open University students who have the opportunity to study many non-mathematical courses. It may also be that the *History of Mathematics* course helps to place mathematics in the context of other activities, and I gather that an attempt is also being made to do this in the rewrite of M100. The views and comments of readers on these points are welcomed.

P331: INTEGRATION AND NORMED SPACES

Philip Newton

Whilst I recognise that a course tutor is entitled to defend his course if he can, the remarks of an Occasional Correspondent concerning my comments on this course appears to be an ill-founded attack upon all my comments in the Special Issue. Firstly, my views on the course were made on the format it was then in, which has since been changed because, as usual, the course team were on the ball and ready to incorporate refinements in what was after all a brand-new course. Basically it seemed to cater for students working together on TMA, especially on the project to which great weight was to be attached. My point about this was if a student got behind due to pressure of work or not having long summer holidays then they were going to lose the advantages of the self-help group and be penalized. Since the Handbook description of the course did not emphasise this I was pointing it out for the benefit of future students.

Secondly your correspondent claims that only two units deal with integration in more than one dimension and that the concept of unitary space does not enter the course. Now after the “leisurely introduction” the set book commences on page 22 with “functions forming a space over $\mathbb{R}$;” this space has more than one dimension as we soon find on page 25, proposition 1. From exercise 9 on page 30 we find that the integral of the product of functions (i.e. product of elements of the vector space) is defined. Therefore we can deduce that the space of step functions is a (finite) dimensional space over the field ℝ and satisfies the requirements of a linear (associative) algebra. Meanwhile the Lebesgue integral operator leisurely introduces a norm with a unit on pages 26-32. Since we are dealing with ℝ as the field we now have a unital normed algebra over the field ℝ. *This is the definition of a unitary space*! Subsequently L^{inc}, L^1, etc. are built up to form this space with appropriate engineering operations on the algebra. Thus given a function we are told that there is a four-step process which can verify the existence of the integral operator upon it, by relating it to one of the hierarchy of spaces. Effort saving theorems follow depending still upon the conditions of the hierarchy. It is only then that we discuss multiple-dimension field domains for the functions. It is these which your correspondent has confused with vector spaces!

Your correspondent says that I am “much given to giving advice”. Let me point out that I have an official printed request to submit my impressions and advice for inclusion in the special issue.

WEEKEND CONTINUED

Ann Jamieson

It amazed me to hear from Professor Pengelly’s talk at the Aston Weekend that only 1% of M students were there and I am torn whether or not to spread the gospel to the other 99%. On the one hand I would like others to enjoy the marvellous tuition and companionship we had - on the other hand if I rave about it too much we may end up with rationing the places.

Everyone I spoke to were more than pleased with their tutors and for my part Jeremy Gray and Peter Thomas were marvellous. Once again we all owe a terrific thank you to Marion for organising it.

> *Following your departure, our cleaners found a blue cardigan and an anorak on the fourth floor. Unfortunately they did not make a note at that time which came from which room, but the two rooms were probably* B2 *and* B6. ...

From a letter sent by Miss J A Limb, Administrative Assistant, Residence Section; The University of Aston in Birmingham, Gosta Green, Birmingham, B4 7ET. (Reference RES/JAL/PJB.)

Technological progress has merely provided us with more efficient means for going backwards.

Aldous Huxley

THE SOCIAL SCIENTIST AND THE COMPUTER

John A Wills

Those of us who have done D100 all have at least a suspicion that intellectual (never mind moral) standards may not be very high among social scientists. I help maintain for the TR440 computer a language specially designed for social scientists (although biologists and musical theorists have also found it of use); I also have to advise the users at my own computing centre on the use of this statistical package for the social sciences. I am therefore in a position to confirm the suspicion.

The language has a construction to specify a list of values on which to perform operations to be specified. A continuous series a values may be indicated with the key-word `THRU`. A typical list might be: `2.5, 7 THRU 11, 32, 55 THRU 57, 100`. Around the `THRU` the left value must be lower than the right. If it is not the interpreter gives a fairly clear error message.

A user came to me with the plaint that he did not understand the error message, which is in a human language other than his own first language. Fair enough. “You have here a larger number before the `THRU` than after it; the computer will only work when the first number is smaller. Turn them round then it should work.” The next day he caught me again and claimed that he had done exactly as I had said but still had the same error message. I examined his listing and found that he had exactly the same kind of mistake as the day before. One of his lists was now `-99,27 THRU -72,33`. Negative numbers are allowed.

What had it been the day before? What kind of transformation was he using to recognise the justice of the error message the day before but to claim that he should now be without the error? Answers: `-72,33 THRU -99,27`; he was using decimal commas instead of decimal points. What has Dijkstra to say about this man? “if I were choosing programmers, I would choose them for excellent command of their native language”.

If this user had given any thought to what he was trying to tell the computer; if, that is, he had had a clear idea of what his research was about, he would have realised that the same symbol cannot be used to divide the elements of a list and subelements of those elements. He might not have thought of the decimal point (although there are examples in the Handbook), but he would have realised that he had something to find out.

We worked things out when I made him describe in human language what he wanted. He was lucky that his silliness led to a syntax error, for otherwise he might have had reams of tables based on false transformations, and he would probably never have understood what was wrong (he might have blamed the computer). Dijkstra’s remark applies to scientific investigators as well as to programmers.

It is more important to have beauty in one’s equations than to have them fit experiment.

Paul Dlrac.

M500 - HANSARD WEDNESDAY 13 OCTOBER 1976

The use of calculators in school exams

Miss Betty Harvie Anderson (*East Renfrewshire, C*) asked the Secretary of State for Scotland to publish the analysis of examination results as announced in July and considered by the Scottish Examination Board in the light of the use of calculators.

Mr Frank McElhone, Under Secretary, Scottish Office (*Glasgow, Queen's Park, Lab*) - This is a matter for the Scottish Certificate of Education Examination Board, but since it was raised by Miss Harvie Anderson on July 21 we have been in touch with the board and they are pursuing the matter.

Miss Harvie Anderson - That is a wholly unsatisfactory reply. Is it not possible to make an analysis of the recent results?

If the examinations show, as I suspect they will, that there is an in-built benefit to some, will he undertake either to supply calculators to all pupils taking such examinations or to say that calculators must not be used in examinations?

Mr McElhone - This is a matter for the board. They are keenly aware of the need to ensure equity and fairness in examinations where some candidates use calculators. The board believe that their arrangements in the way they present examination papers do not give a significant advantage to anyone using a calculator.

The board are today reexamining their attitude on this matter.

THE M500 SOCIETY

Milada Mitchell

Following up points mentioned at meeting at Aston Weekend:

(1) SALE OF BOOKS The question asked was: Could we as a society purchase set books cheaply or in bulk at a discount? Answer: No! I have investigated this thoroughly and it would contravene too many Acts - Nett Book Agreement, Booksellers' Charter, etc. If we wanted say 10000 × 1 set book from one publisher it may have been possible (but illegal) but for more than one book definitely impossible. Subsidiary question: Could we (M500) operate a swop/sale/loan of books? Answer: If there exists a demand. Would people like to send a list of requirements and/or books for sale/loan. I don't mind volunteering to coordinate lists and put people in touch - if demand exists. (*Ed* - Milada had some harebrained idea about using subscription renewals to work this scheme and send vast quantities of books bumbling about the country but I don't agree at all. If you want to send books Milada is in MOUTHS, but do remember you can advertise in M500 - free to members!)

(2) DAYSCHOOLS Conversation, late at night, with Professor Pengelly, Marion Stubbs, Philip Newton and others about M500 'dayschools' throughout the academic year. Answer: Is there a demand? Philip Newton has volunteered for NW and I suppose I volunteer for regions 03 (SW), 10 (Wales) and 04 (W Midlands) as I live more or less on the boundary of all three. If a demand exists I see it being a Saturday all day, once in March/April, then in June/July in Bristol, Gloucester or Birmingham for this area; and there would have to be some

fee - for tutors, rooms etc., as it would supplement regional arrangements and would have to be self-supporting.

Following on from this, Philip Newton (not me) thinks there exists a demand for social activity between now and next academic year. He was thinking of a meeting in a pub (yes Eddie, a *pub*) and a general mathematical discussion to save oneself from the boredom of this otherwise-revolting time of year. Location of pub would be difficult geographically and timing definitely awkward. Is there a demand???

CALCULATORS

Richard Stearn

I am a G-year M100 student, G0267760 to be precise - but just plain G02 to my friends. Although I have not yet officially started I extracted the first dozen M100 Units from (or rather in spite of) the system in mid-September, and have been plodding away steadily ever since.

The remark in the last paragraph of *Which?* by Marion Stubbs, M500 36 12, is no longer correct. There are now at least two pocket sized calculators available with indirect addressing: the TEXAS SR-52 and the HEWLETT-PACKARD HP67.

Both have 224 program steps and use magnetic cards to record or playback programs. They also have subroutines, decrement and skip on zero, umpteen conditional tests and many other goodies. The HP67 has the usual ‘reverse Polish’ logic. The SR-52 is algebraic with enough levels of parentheses to drive any reasonable mortal insane. They are both priced so that no numerate oil sheik would ever want to be without one!

There is also a version of the HP25 (designated HP-25C) with non-volatile data and program memory. It is the same as the HP25 except that the program is retained when the device is switched ‘off ‘. For a button programmable machine this is quite an advantage. By just switching on you can repeat on Monday morning the bug-filled program devised the previous Friday afternoon.

M404

Brian Woodgate

A few thoughts on a possible fourth level course. (1) One full credit only, to be a project type course; (2) A choice of streams, e.g.: Algebra, Analysis, Computing, etc.; (3) Prerequisites: at least one post-foundation M-credit in the chosen stream; use of a good library or a computer terminal; an interest and willingness to get as much out of it as possible. (4) Limited to 100 in its trial year and perhaps run from Walton Hall. (5) With a Summer School.

NUMBER REPRESENTATIONS CONCLUDED Krysla Broda

(*There are three similar systems which might provide some enjoyment if you like playing with numbers.* (i) *Fibonacci systems;* (ii) *Factorial systems, were both described in issue* 36, *and*)

(iii) Ternary systems. Of a slightly different nature. If we could have 3-state electronic devices, perhaps computers would use the system.

Represent an integer $N = \sum_{i=0}^{m} a_i . 3^i$ where $a_i = 0, \pm 1$.

> EK - For convenience after this I have used T to represent Krysia's $\bar{1}$

e.g. 15 = $1\bar{1}\bar{1}0$ where $\bar{1}$ is the coefficient of –1.

Negation is easy: –15 = $\bar{1}110$.

Addition table

sum

	1	T	0
1	T	0	1
T	0	1	T
0	1	T	0

carry

	1	T	0
1	1	0	0
T	0	T	0
0	0	0	0.

Multiplication is as simple as in binary; e.g. 15 × 16 = 240 :

```
        1 T T 0     15
        1 T T 1     16
        -------
        1 T T 0
      T 1 1 0 0
    T 1 1 0 0 0
  1 T T 0 0 0 0
  -------------
    1 0 0 0 T 0    240.
```

We can use the addition tables to define two associative and distributive operations which obey similar laws to the De Morgan laws for Boolean logic. Call them `TAND` for carry and `TOR` for sum.

<u>Problem 6</u> Prove these facts.

<u>Problem 7</u> Extend these ideas to perhaps design ternary `and/or` gates out of conventional `and/or` gates.

Another item from *The Times* Diary; August 9 1976:

'The editors of *Computing* magazine have become pleasingly entangled in their own obfuscations. A statement of editorial policy in the issue of July 29 reads: "The misinformation of readers by too early an expectation of a definitive solution to the computer security of operating systems must remain the first priority of a responsible publication". If this seems misleading they are only doing their job.'

M202 RECRUITMENT

Ken Wogan, Pauline Gerrard, Viv Lucas, Phil Littlechild, Geoff Senson

Rumour has it that M202 may he discontinued soon due to the fall in numbers of students wishing to take it. Previous students of the course will no doubt agree with the present crop that this is a shame. Therefore, courtesy of Stirling week 4, we present our own TMA and CMA designed to show that maths, and particularly M202, has a lighter side to it. All the terms are genuine (even if they are used slightly out of context) .

TMA M202 09

1. *Assessment Question* Consider the sets M and F.
   Consider the function $f{:}M\to F$, defined by $f{:}a\to b$, $a \in M$, $b \in F$, such that f is one-one and onto.

i. Is this function continuous? Prove by exhaustion.

ii. Consider the Cartesian product $\{a\} \times \{b\} \cong C_2$. Under what conditions can the function f generate new elements of the group?

iii. How frequently are the elements of $\{a\} \times \{b\}$ conjugate to each other?

iv. Define the injection map i which maps M into F.

CMA M202 99

U1 Given the following conditions select the options which are TRUE.

Consider the sets M and F and the set $P = M \cup F \cup \{0\}$, $(M \cap F = \varnothing)$.

Define the operations $\oplus$ and $\otimes$ as follows:

i. $m_1 \oplus m_2 = 0$, $f_1 \oplus f_2 = 0$, $m_1 \oplus f_1 = m_1/f_1$; $m_i \in M$, $f_i \in F$.

ii. Since multiplication may be considered as repeated addition
   $m_1 \otimes m_2 = f_1 \otimes f_2 = 0$; $m_i \otimes f_i = \{\{x\}{:}\, x \in P\}$.
   Define the function $f{:}M \to F$, such that $f(m_i) = \frac{m_i}{f_j}$.

Options

A $(P, \oplus, \otimes)$ is a ring.

B P is commutative under multiplication.

C The function f is one-one.

D The function f is onto.

E There exists an inverse function

F $(P, \oplus, \otimes)$ has zero divisors.

G The mapping $f{:}\ M \to M$ is a homomorphism.

In philosophizing we may not terminate a disease of thought. It must run its natural course, and slow cure is all important. (That is why mathematicians are such bad philosophers.)

Wittgenstein: *Zettel* #382.

(From Tony Brooks, who comments “which must explain my progress on A402.”)

ADVICE ON THE SELECTION OF ELECTRONIC KITS - Philip Newton

Once upon a time if you wanted a radio set you started off by buying a quantity of cotton covered wire, ebonite and so forth, and made each item of a simple crystal set and then assembled it. There was no other way. Later you could buy 'ready to use' radios from the shop and it was at this time that the KIT was started. Some people found that it was fun to assemble the parts provided someone else who had the equipment saved you the time involved in winding hundreds of turns of fine wire on to a reel.

Later kits were supplied to make almost every piece of electronic apparatus, and the success of the electronics industry owes much to the keenness of the constructor and his deep pocket.

Even later, with increasingly heavy purchase tax on finished goods a new type of kit was supplied. This was for the non-constructor and was as near complete as regulations allowed. Usually all the soldering was done and only fitting together of major assemblies with a few nuts and bolts was needed. Sometimes a few inches of solder was supplied and one or two non-critical joints were left to be done. Since the tax on parts was much less than on the finished product substantial savings could be effected by the customer.

If you are a constructor no advice is necessary since you will either know what you are doing or will want to learn by experience. So these notes are intended for the non-constructor.

1 Make sure the kit has had the hard work done - this is hinted at by phrases in the advert such as 'needs no soldering' 'can be completed in only two hours' (sometimes an exaggeration) - and is COMPLETE.
2 Ensure that you are dealing with a supplier with premises in Gt Britain, or that any replacement parts can be obtained easily, i.e. manufactured or supplied in Gt Britain.
3 For the absolute novice, get advice. Usually there is a College of FE or a technical college not too far away and an electronics instructor on the staff who will be willing to help. (Unlike science and maths teachers all tech teachers are wildly enthusiastic about their subject and none more so than electronics teachers with a kit.) Failing this try laboratory technicians (electrical) or even your local TV repair man.
4 If the supplier advertises in one of the electronics magazines such as *Radio & Electronics Constructor, Practical Wireless, Practical Electronics, Wireless World* then you can order with confidence relying on the vetting of thousands of customers who are very vocal if anything goes wrong.
5 Actually kit construction is normally very logical and a lot easier than knitting. The writing of assembly instructions is a profession in high regard.
6 Finally: you get the quality you pay for, as in all things.

I have heard Mr Hobbes say that he was wont to draw lines on his thigh and on the sheetes abed, and also multiply and divide.

Aubrey: *Brief Lives*. (JH)

OBITUARY - PROFESSOR GILBERT RYLE

Gilbert Ryle, Waynflete Professor of Metaphysical Philosophy in the University of Oxford from 1945 to 1968, died on October 6 at the age of 76. He had suffered a stroke while on holiday in Yorkshire.

He was the son of R J Ryle MD and was educated at Brighton College, then as a Classical Scholar at Queen's College Oxford, 1919. He gained First Classes in Mods, Greats, and the School of Philosophy, Politics and Economics; was captain of the College Boat Club and rowed in the University Trial Eight.

He was Tutor in Philosophy at Christ Church from 1924 till 1945 being first Junior then Senior Censor and in 1939 Junior Proctor to the University. During the war he worked in intelligence and then succeeded Professor Collingwood as Waynflete Professor and Fellow of Magdalen where he stayed until he retired.

He saw his task as that of redeeming Oxford philosophical studies from the disputations over unreal difficulties that had been their outstanding characteristic in his early days there. To this end he contributed many papers to philosophical societies and journals in the thirties although no book of his appeared until 1949.

"Systematically Misleading Expressions" (1931) and "Categories" (1937) were the papers most important in defining the direction he would move in.

The notions put forward in *The Concept of Mind* (1949) were quite new, though some of the ideas in it had been discussed in the thirties, especially around Wittgenstein. Basically he pointed out that most problems in philosophy are the result of "category mistakes" - confusions between the grammatical functioning of words and the logical functioning of concepts. It had been assumed that a relation existing in the former necessarily induced an analogous relation in the latter; that if two sentences are similar then the objects they are talking about are similar also. He was able to criticise, systematically if not entirely convincingly, the mind–body dualism of Descartes (which he called "The ghost in the machine"). Ryle was much misunderstood and it was claimed by many of his critics that he was aiming at some kind of behaviourism.

In *Dilemmas* (1954) he distinguished between formal and informal logic for the first time but this book was much less influential. He was really much happier writing papers than books and his output numbered some 80 titles.

He was editor of *Mind* from 1948 to 1971 and while there did much to encourage beginners, often publishing their articles in preference to those of established writers as a deliberate policy. Then in 1959 he laid himself wide open by declining to have a book by Ernest Gellner reviewed In *Mind*. It was supposed to have been an attack on linguistic philosophy but included some sociological analysis of the milieu of its practitioners. Bertrand Russell wrote to *The Times* about Ryle's "partisan view of the duties of an editor". Ryle replied that in the book "about 100 imputations of disingenuousness are made against a number of identifiable teachers of philosophy". And so it went on. Ryle ended up badly singed.

He spent a lot of time travelling and while at Oxford concerned himself more with affairs of his own faculty than with those of the University. He was primarily responsible for introducing the new degree of BPhil which was designed to get students to work together rather than in isolation as for the DPhil. Because of his position at a time of expansion his influence on philosophical appointments throughout the country was unusual. His most distinguished pupil was A J Ayer who carried on similar work to Ryle's at London.

In 1966 Ryle brought out *Plato's Progress* which contained many ingenious if unorthodox theories. The book is certainly entertaining and bears out the belief that if one was a student of Ryle's it was better to be obtuse than pompous.

SOLUTIONS

35.2 *RIFFLE*

The pack of cards is split exactly in halves and the two halves are interleaved with each other. Show that 8 *'out' riffles are needed to restore the pack to its original order while* 52 *'in' riffles are needed.*

The 'in-riffle' on 2*p* cards can be expressed as the function

$$N_{2p}: x \to \begin{cases} 2x, & x \le p \\ 2x - (2p+1), & x > p \end{cases}$$

Thus $N_{2p}(x) = 2x \pmod{2p+1}$. From which we deduce that $2^m \equiv 1 \bmod 2p+1$. For $2p = 52$ we have $m = 52$ as the smallest solution so that 52 in-riffles are needed to restore the order of a 52 card pack.

An 'out-riffle' on 2*p* cards is equivalent to leaving the two outermost cards unchanged and doing an in-riffle on the central $2p–2$ cards therefore the order of the 2*p* cards will be restored in *m* riffles where $2m \equiv 1 \pmod{2p–l}$. But $2^8 = 256 = (5 \times 51) + 1$ so 8 riffles are required.

Steve Murphy.

35.3 *NEXT TERMS*

18: *1 2 4 8 16 3 2 6 4 1 2 8 2*; 5 6. Nick Fraser
19: *1 3 5 8 12 18 24 30 36 42 52 60 68 78*; 84 90. $u_n = p_n + p_{n-1}$, where p_i is the *i*th prime. NF
20: *0 1 1 2 4 7 13 24 44 81 149 274*; 504 927. $u_n = u_{n-1}+u_{n-2}+u_{n-3}$. NF.

35.4 *INFLATED RUGBY*

Under the old Rugby Union points system where only multiples of 3 *or* 5 *points could be scored there were four different scores which could never be achieved*: 1,2,4,7. *If there had been* 35 *impossible scores, one of them* 58, *what would the two basic values have been*?

The two basic values are 8 and 11. More generally if *a* and *b* are positive coprime integers then except for $\frac{1}{2}(a-1)(b-1)$ numbers all nonnegative integers can be written as non-negative linear combinations of *a* and *b*. Someone might like to prove this.

David Asche.

<u>36.1 THE ONE HUNDRED AND NINETY SIXTH ROOT</u>

$$\sqrt[196]{10^{59} \cdot \left(\frac{1025}{1024}\right)^5 \cdot \left(\frac{6560}{6561}\right)^3 \cdot \left(\frac{9801}{9800}\right)^4 \cdot \left(\frac{15624}{15625}\right)^8 \cdot \left(\frac{1048576}{1048575}\right)^8}.$$

Calculate the value of this in one minute without using pen pencil or paper. The answer is a single digit. Or using a calculator how many solutions are there?

The answer is 2 as is obvious from the given information that the solution consists of a single digit. All the factors under the radical sign except the power of 10 are very close to 1 and can be ignored for an approximation. Thus we have $10^{59/196} \approx 2$. Perhaps this is not really what is meant by calculate - it does not seem likely that it can be done. To quote from one of the letters which flooded in "If it is interesting are there any other results like it and do they rest on any worthwhile facts in arithmetic." C W Pile, Percy Sillitto and Alan Slomson factorised each factor and ended up after heroic cancellation with $\sqrt[196]{2^{196}}$. Alan also pointed out that 59/196 = 3.0102... ≅ 3.0103... = $\log_{10} 2$. The third method of attack was by the use of calculators. This is where the alternative solutions came in. Marion Stubbs has suggested the following table:

Let A = $(1025/1024)^5$ ... $(1048576/1048575)^8$; let B = 1/196 which varies according to the calculator used; then

calculator	B	Range for which $(A\times10^{59})$ [Y^x] B = 2
Rockwell 63R (M351 kit)	5.10204 08 E 3	1.0038426 E 59 – 1.0048368 E 59
CBM SR-36 (Alan Slomson)	5.10204 0816 E 3	1.004336168 E 59 – 1.004337171 E 59
HP-5 5 (Marion Stubbs)	5.10204 0816 D 3	1.004336238 E 59 – 1.004336337 E 59
Texas SR52* (John Owen)		1.00433627870 E 59 – 1.00433627870 E 59

* Although the SR52 only displays 10 digits, by subtracting 1 E 59 you can read the 11th and 12th. With the 10-digit display the range in column 3 is:

|11.0043362288 E 59 – 1.0043363269 E 59

36.2 *NEXT TERMS*

21: *1 2 2 3 2 4 2434 2 6 2 4 4 5*; 2 6. Number of positive divisors of the integers starting with 1. (All primes have 2.) Krysia Broda.

22: *4 6 9 10 20 14 21 22 25 26*; 33 34. Ordered list of products of primes (2×2, 2×3, 3×3, ...) KB

23: *1 5 19 65 211 665 2059 6305 19171 58025 175099*; 527345 1586131. $u_n = u_{n-1} + 2^{n-1}$. KB & EK

36 . 3 *POLYGONS*

(a) *Join every pair of vertices of an* n*-sided polygon by a straight line. How many pairs of lines intersect other than at the vertices of the polygon*?

Every crossing identifies 4 vertices - the ends of the crossing diagonals. Also every 4 vertices correspond to exactly one crossing - there is only one way to join pairs so that the diagonals cross. Hence the number of crossings is equal to the number of ways of choosing 4 from *n*: $\binom{4}{n}$. Richard Ahrens.

36.4 *LOGS*

How big does x *have to be for* $\log_x$ n *to be less than* n? $\exp\frac{1}{e}$ = 1.4446 ... . C W Pile.

36.5 *THE BLACK ACE*

A pack of cards is shuffled and dealt until the first black ace appears. Where is it most likely to be?

It Is most likely to be the first card. The probability is 51/1326. (The probability that it is the nth card is $(52 - n)/1326$.) By symmetry the second black ace is most likely to be at the bottom. For a pack of a cards containing two black aces the probability that a black ace is on top is

$$\frac{n-1}{\sum_{i=1}^{n-1} i}$$

Jeremy Humphries.

PROBLEMS

37.1 *PAIRS* - Jeremy Humphries.

a. Place eight points in the plane so that the perpendicular bisector of the line joining any two points passes through two other points.

b. Can you find any other finite sets which have this property, (the bisector can pass through more than two points if necessary) or prove there isn't one?

This problem was proposed by Hallard Croft of Cambridge in the form of part b. A solution for eight points was found. I haven't seen any other.

37.2 *PAIRINGS* - Max Bramer.

There are $2N$ teams in a competition. Is it possible to arrange all possible pairings between them in only $2N - 1$ rounds? If so prove it. If not give the correct formula for the number of rounds needed.

Give a constructive method (an algorithm) for generating pairings in each round - how many different possible choices are there?

Example: for 4 teams one solution in three rounds is

1 v 2, 3 v 4; 3 v 1, 4 v 2; 1 v 4, 2 v 3.

(Note that a v b and b v a are to be counted as the same.)

37.3 *POWERS* - Tony Crilly.

Find all positive integers m, n ($m \neq n$) with $m^n = n^m$.

37.4 *PLANES* - Steve Murphy.

Given four distinct parallel planes show that it is always possible to construct at least one regular tetrahedron with a vertex on each plane and show that the length of the side of any such tetrahedron is unique.

37.5 *PSHAW*! - Jeremy Humphries

How many commutative groups are there?

EDITORIAL

37 comes to you after the second consecutive big gap, this time to allow exams to sneak in. Must try to arrange things better next year. I hope everyone did slightly better in the exams than they expected even if not as well as they hoped. M500 is beginning to surround itself with helpers: Colin Mills to represent us elsewhere; Nick Fraser to try and get us some publicity; Milada Mitchell is beavering solidly away at the index. Any more ideas? By the way, do read Milada's piece and let us have your views.

Talking about views, as I was typing away at Alan Slomson's article on page 5 I thought to myself "!" and "?" and then I thought "Well!" I'll give you an example of what I mean. Quote: "... it is difficult to teach [mathematics] in such a way that students experience the joys of inventiveness." While not disputing the sentence on purely logical grounds I take extreme exception to the hidden implication that one can so teach students of other disciplines. Creativity, in any sphere, is born not made, or if it is made then by University age it is too late. Have you ever tried talking ("communicating") to the average man-in-the-pub? On *any* subject!

I see the four colour map problem (How many colours are needed to make a map in which no two countries are the same colour?) has finally yielded. No hoax this time. Four colours are enough! It has been proved by a "highly inelegant" proof, by Wolfgang Haken and Kenneth Appel of the University of Illinois at Urbana-Champaign. They claim that this is the first ever problem that has been solved in an intrinsically inelegant way; that is, there is no possibility of a proof which does not require a sledgehammer approach and man-centuries of work. Now I suppose some of the other famous problems are going to be attacked in the same way. Why bother: let us at least have elegance, even in our ignorance.

The suggestion came up at the Weekend that M500 should have its own device. What does the membership think? You may well believe that such a concept would lower the tone irretrievably and ought to be avoided at all costs. On the other hand your views might be less extreme than that. In which case why not design one? If something really good comes in we might be able to use it as a cover design; or even have our own cufflinks. Two thoughts of my own:

The first is supposed to look something like a bird (or at least a wol) and is based on an early Indian (red, I think) symbol for man (where "man", one hopes, embraces "woman"). The second, though dignified, might be thought to lose something of the special significance of our number 500.

Finally, the usual plea. You know what I am going to say and I know that you know that I always say it but still manage to fill the magazine so am obviously crying wolf. But what would you say if I told you that my hair is completely grey now, with worry. You've no excuse - the exams are over and you can't do anything about next year because you don't know if you've passed yet so PLEASE send in a contribution to M500.

Eddie Kent.