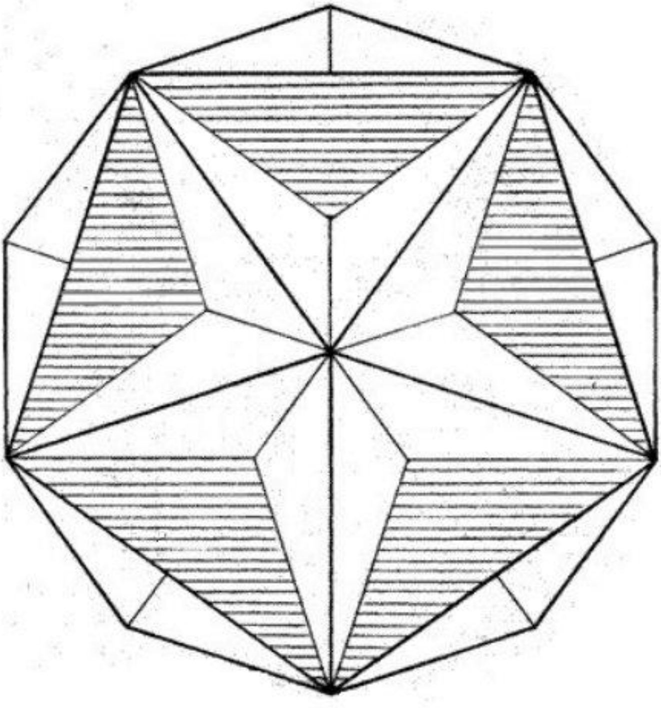

M500 38

M500 *is a student operated and owned magazine for Open University mathematics students and staff, and friends. It it designed to alleviate student academic isolation by providing a forum for public discussion of the mathematical interests of individuals.*

Articles and solutions are not necessarily correct but invite criticism and comment. Nothing submitted for publication ought to be longer than about 600 *pages - if longer it will probably be split into instalments.*

MOUTHS *is a list of names and addresses with previous and present courses of voluntary membes by means oj which private contacts may be made, to share OU and general mathematical interests, or to form self-help groups by correspondence or telephone.*

The cover design is by Chris Pile and is a diagram of the Great Dodecahedron.

PUBLISHER	Marlon Stubbs
EDITOR	Eddie Kent *Please send anything for inclusion in* M500
MEMBERSHIP SECRETARY	Peter Weir *Send membership applications, subscriptions, change of address,* MOUTHS *data.*
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ANYTHING SENT TO ANY OFFICER OF THE SOCIETY WILL BE CONSIDERED FOR POSSIBLE INCLUSION IN M500 UNLESS OTHERWISE SPECIFIED

KNOW YOUR SPACE

Stanley Collings

It is ironic the way M100 would have nothing to do with Geometry. A strong point was made of historical mathematics being divided up into the water-tight compartments like isolated islands popping out of the sea. The ethos of the course was then to expose the common continental shelf underlying and unifying all these separate topics. About the only topic specifically mentioned was Euclidean Geometry, and in the very next breath it was expressly denied that this would be one of the islands we would be visiting.

Whether the above denial was a good thing is a matter of opinion. What is pursued at any moment is a matter for personal predilection - and these days predilection may be heavily governed by the vagaries of fashion. Under these changes, geometry has slumped heavily both in schools and universities. Euclid was once decried as containing fallacies, one of these being the way in which movement was allowed to establish conditions for congruency of triangles. Yet what branch of geometry is comparatively flourishing in schools today? Motion geometry. More irony!

It is a pity if, for any reason, some source of beauty and excitement is cut off; and for me the excitement is increased by the fact that the theorems generating it are properties of the 3-dimensional space in which we live, rather than of some artificially contrived axiomatic system. Who can fail to be thrilled by the fact that for any triangle drawn in a plane the three altitudes concur at a point *H*, that the feet of the perpendiculars lie on the circle *C* passing through the midpoints of the sides of the triangle, and that the following four points are collinear:

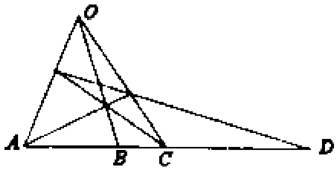
the centre of the circle C
the centre of the circle circumscribing the triangle
the centre of gravity of the triangle
the point *H*.

Moving away from *Euclidean Geometry*, a thin straight rod sticking out of the ground in sunlight will cast a shadow, and a point halfway up the rod will project under the sun's parallel rays into a point halfway along the shadow. More generally, for any three points ABC, and their projections A' B' C', it is fairly obvious that AB/BC = A'B'/B'C'. But suppose the sun is replaced by a lamp at a finite distance. What relationship then holds? Between *A B C* and *A' B ' C'*, there is no invariant metrical relationship. Adding a fourth point *D*; it is not true in general that the ratios *AB*/*BC* and *AD*/*DC* are respectively equal to *A'B'*/*B'C* and *A'D'*/*D'C*. In other words, ratios are not preserved. What is preserved is the ratio of ratios; thus

$$\frac{AB}{BC} \Big/ \frac{AD}{DC} = \frac{A'B'}{B'C'} \Big/ \frac{A'D'}{D'C'}.$$

This fact is an essential property of the space in which we live, yet how many know about it? People in the street know more about numbers which are constructs of the human mind, or about gravity and the motion of the planets which inhabit the space we are talking about. The invariant expression $\frac{AB}{BC} / \frac{AD}{DC}$ is called a '*cross ratio*'. Is this just another example of useless abstract mathematics? No, As already stated, it is a property of our space. It has applications in crystallography. There is even a paper on *The use of the cross-ratio in ætiological surveys* by the Professor-elect of Medical Statistics at Oxford University.

In conclusion, let us take the points *A C D* on a line, and a point *O* outside, and any line through *D* meeting *OA*,*OC*. Join up as shown:



then the cross ratio (*AB/BC*)/(*AD/DC*) is necessarily equal to –1. In working this out for the above figure, you must count the distance *DC* negatively, as it is traversed in a negative direction.

BOOK SWAPS

Norman Lees

In response to Milada Mitchell's article in M500 37; how much demand would there be for book swaps? I, for one, tend to keep the books. It is the course units which tend to breed in dark corners, and living in a small house I would prefer that they did it elsewhere. There are others that I would like to get. Perhaps the easiest idea would be all wants (get/rid) to be sent to a coördinator by a certain time and have a list published as a supplement to the late summer/autumn edition of M500 and let individuals sort it out for themselves. This would enable the enthusiasts to get ahead with next year's courses.

Another problem which some of your experienced readers might care to suggest is preparatory reading. There was a little in the last M500 supplement but perhaps a fuller coverage would be of use to us weaker brethren.

Ed - There is a useful idea here. Anyone retaking a course (*for whatever reason*) *will end up with two sets of course units which are similar if not indistinguishable. I have a spare* MST282 *for instance. We could certainly publish a monthly list of what is available, and where - but nothing need move until it is asked for thus avoiding double movements. I don't think there ought to be any charge made, except that the recipient should pay postage of course. Anyone with a spare set of course units send details to the editor and when they are finally disposed of someone can give me a ring to knock them off the list. How's that*?

HOW MATHS RESEARCH IS DONE *Rosemary Bailey*

(“*Another item for your scurrilous rag. Obviously marking* M334 *exam scripts has turned my mind not a little. Of course you may well feel that* M500 *readers have heard enough of my doings this year. However I know I have one avid reader. He's in the Forestry Dept here, with a PhD in his own subject, and friendly with many of our department. We were all astonished when he started telling us about our research till we found out that he's an* M100 *student and had read it all in* M500.”)

It’s Monday morning. I’ve got a hangover and a cold. It’s windy as it can be only in Edinburgh, with driving rain to add to the fun, and I’m soaked. Of the two articles I’m supposed to be working on, one I’ve left at home and the other my colleague has failed to return to me. I cast my mind dully over the papers on my desk and feel uninspired.

Eventually I idly pick up a paper to which I referred on Friday, when I located a minor mistake in it. This paper takes an approach, say approach *A*, to a longstanding problem; I haven’t studied it in much detail because I believe the approach to have been completely superseded by approach *B*, as expounded by another author. Approach *A* is listed in full tedious detail for all systems with $n \leq 4$. I look at this in some horror but at last decide that, as my mind isn’t feeling capable of anything more intelligent, I’ll make a list for approach *B* to show how much simpler and superior it is.

n = l, n = 2, no trouble. n = 3 getting more elaborate but still *B* is demonstrably superior. n = 4: there are sixteen cases here, can I bear to check them all? Headache, sleepiness, general sense of fedupness lead to bloodyminded decision to persevere with this mindless task. Can find only eleven cases by approach *B*. Aha - that’s because I’m still half asleep; I’ve missed some out. But wait: even after a most comprehensive checking of approach *B* it still fails to give one of the approach *A* cases. I christen this “the anomaly”.

Convinced that the anomaly must be an error, probably following on from the minor mistake I had noted earlier, I work through all the theory associated with it to locate the exact place where the mistake occurs. Funny - it seems to be all right; I just cannot find that mistake. The headache wins and I go home.

It's Tuesday morning. I feel able to tackle anything. Twelve hours sleep has convinced me that I can find no error in the anomaly because there isn’t one there. In a flash I realise that not only does approach *B* not deal with the anomaly but that for n >5 there must be many cases that lie outside the scope of approach *B* but which I can deal with by generalising the central theorem in approach *B*.

To work. Set to writing a note on the subject. Compare the approaches, explain the anomaly, state the new general theorem. I take a virgin sheet of paper and boldly write PROOF at the top of it. Then it comes to me that there are a lot of holes in the proof I had sketched out in my mind. For three hours I struggle with ideas on scraps of paper, convinced of the truth of the theorem but unable to complete the proof. I go home.

It is Wednesday morning. I can see this proof is going to need a disciplined attack to be beaten. I plunge in, regardless of complications and dirty work,wading through notation and ugly formulas. At last I have a set of little results which I can put into a logical sequence to complete the proof. With any luck I can simplify it in three months time; for now it is enough that a proof of some sort exists.

I collect together the manuscript and hand it to my typist before my nerve breaks and I have doubts. Tomorrow is Thursday - who knows what I may disbelieve then? I don’t care; just now I’m going for a drink.

CALCULATORS

John Owen

The Texas SR52 is a magnetic and programmable calculator with 20 memories, 224 program steps with merged prefixes and an algebraic operating system (rather than the reverse polish notation or RPN of the Hewlett Packards including the HP55). It cost me £225. Its smaller brother the SR56 which is without the magnetic card unit and has 10 memories and 100 program steps is offered by Comet at £55. Having played with Marion's HP55 at the maths weekend I would prefer the SR56 to it. At its price it makes the HP55 look ridiculous. I believe the SR52 is in a class of its own.

The algebraic operating system means that entering 2 + 5 × 3 = gives the answer 17. The SR56 allows 7 pending operations, the SR 52 allows 10. In the above equation 2+ is stored until 5 × 3 is evaluated. In addition there are parentheses and these can be nested nine deep. Operators take priority over × and ÷. 2 × 3 y^x 2 = gives 18. This makes entering an equation whether as a calculator or in program mode as simple as possible. It is very similar to entering an equation in `BASIC`.

In program mode both SR56 and SR52 allow direct addressing. In addition the SR52 allows labelling and provided all `GOTO` instructions are to a label it is possible to insert and delete instructions without changing `GOTO` address. Also the SR52 allows indirect addressing and has 10 ‘user defined keys’ whose use is best illustrated by a program segment: volume of a sphere.

LBL[1] | A[2] | ([3] | STO[4] | Y^x | 3 | × | π | × | 4 | ÷ | 3 |)[5] | RTN

[1] label; [2]A; [3]bracket; [4]dummy (brings display value inside brackets); [5]evaluates function - does not affect pending operations.

The above program could be anywhere in the program memory. Pressing key A would take the value in the display register as a radius and return the volume of the sphere. `RTN` operates as a halt in this case but also allows the program to be called as a subroutine by another program.

I would suggest that anyone purchasing a programmable calculator should try one of these Texas machines before going for anything else. With the SR52 memories are numbered from 00 to 19 and you can Store, Recall, Sum, Subtract, Multiply, Divide and Exchange Display with any memory. Also you can clear all memories. What Texas does not tell you is that locations 60 to 99 will also store numbers. 60 to 69 is the stack for Pending operations - not suitable for use as memories. 70 to 97 is the program memory. These registers can be used as memories in the calculator mode but must be left blank if programs are going to be run. 98 and 99 seem suitable as general purpose memories - they allow all memory functions except that they are not cleared by the clear memory key.

There is a printer available which the SR56 and the SR52 will plug into but at £180 it is not good value. A 4-function printing calculator is only £99 so the printer should come down. There is also a SR60 with printer, alphanumeric display, program and data read/write on magnetic cards, 1920 program steps, 100 memories and £1500 - only the price of a Mini.

CALCULATORS FOR COURSES

PhilipNewton

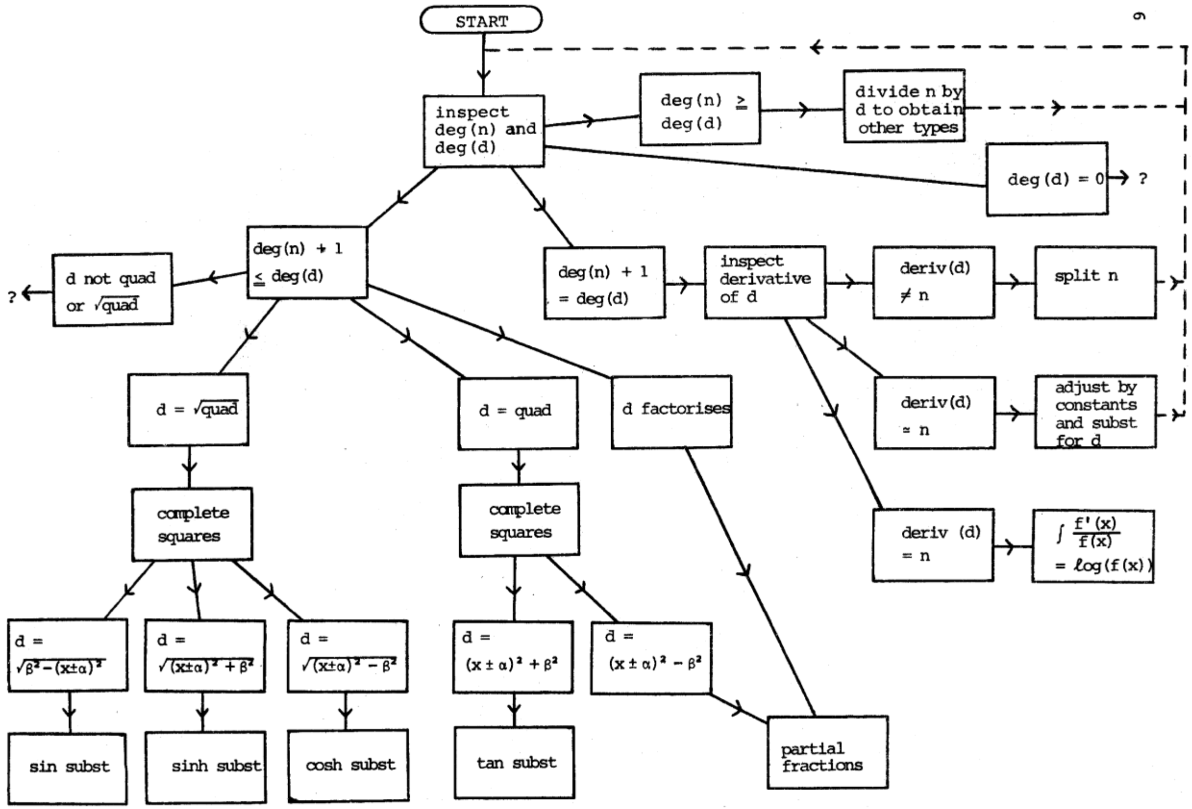
I always look forward to the arrival of the kit when one is supplied with a course. I am a sucker for kits and admit they influence my choice of subject. When I started with the OU I purchased a couple of good slide rules which I have hardly used. This has surprised me and begs the question “Is the lack of calculations due to the lack of calculators?” I believe this is the case for second and third level courses.

In industry calculations are usually carried out to arrive at a numerical result and quite a lot of students are in industry so they will be pleased that teachers are not getting all their own way. So welcome to the 63R for the M351 course.

Having been in the office equipment field for twenty years the facts of calculator life have not passed me by. So I rate the Rockwell 63R as a nicely constructed middle market machine. Rockwell announced recently that they will not be manufacturing the 63R in Great Britain in the future. Presumably the OU knew this when they purchased, and will buy other machines in the future (how about the Corvus 500 which retails at the same price as the 63R but has nine memory registers, vector addition and subtraction capability, direct entry of hyperbolic functions, ---? The 63R could be used with benefit on ST285 (especially Summer School), M231, M201, and is a good machine for second level work. It would not pass the ‘Weinberger test’for M321, however. The lack of direct entry of hyperbolic functions alone guarantees this.

Finally, why don’t the Technology Faculty make oodles of money by designing special calculators using standard components, such as calculators for matrix manipulation, group operations, and general set work? The education market (including exports) should be quite large.

They who introduced images removed fear and added error. Augustine


START
inspect deg(n) and deg(d)
deg(n) ≥ deg(d)
divide n by d to obtain other types
deg(d) = 0
?
deg(n) + 1 ≤ deg(d)
d not quad or √quad
?
deg(n) + 1 = deg(d)
inspect derivative of d
deriv(d) ≠ n
split n
deriv(d) ≃ n
adjust by constants and subst for d
deriv (d) = n
∫ f'(x)/f(x) = log(f(x))
d = √quad
complete squares
d = √(β² − (x±α)²)
sin subst
d = √((x±α)² + β²)
sinh subst
d = √((x±α)² − β²)
cosh subst
d = quad
complete squares
d = (x ± α)² + β²
tan subst
d = (x ± α)² − β²
d factorises
partial fractions

INTEGRATION FLOWCHART *Richard Shreeve*

This chart is based on notes from a sixth form class I was preparing for the Oxford and Cambridge Scholarship papers. I have added a little rigour and hope it might be of use to some M500 readers. I had intended developing the ideas further by adding some other sheets to cover the types of function omitted here. Unfortunately I never got that far.

NOTES

1. *Glossary*:

deg — degree of a function; a function mapping the function F to its degree. deg:$F\to \mathbb{R}$.

n — numerator, *and*

d — denominator of the function to be integrated,

deriv — the function obtained by applying the differential operator to any function that is differentiable.

quad — any function Q that is applied to the set of all real functions F. Q:$F\to aF^2 + bF + c$; $a, b, c \in\mathbb{R}$.

2. *Sin (etc)subst*: substitute sin(etc.)u for $(x \pm \alpha)/\beta$.
3. *Complete squares*: convert $ax^2 + bx + c$ to $a(x^2 + bx/a + c/a)$ and then write as $a\left((x + \frac{b}{2a})2 + \frac{c}{a} - \frac{b^2}{4a^2})\right)$ or $a\left(\left(x+\frac{b}{2a}\right)^2\right) + \left(\frac{\sqrt{4ac-b^2}}{2a}\right)$. i.e. convert $ax^2 + bx + c$ to form $k((x \pm \alpha)^2 \pm \beta^2)$.
4. *Partial fractions*: convert the fraction $a/(x + b)(x + c)$ into $\frac{A}{x+b} + \frac{B}{x+c}$. Hence $A = -B = \frac{a}{c-b}$.
5. *Split numerator*: example: $\frac{4x-7}{x^2+3x+5}$ splits into $\frac{2x+3\ +\ 2x-10}{x^2+3x+5}$ and again into $\frac{2(2x+3)}{x^2+3x+5} - \frac{13}{x^2+3x+5}$
6. *Adjust by constants and substitute for denominator*: e.g. for $\frac{5x^2+3x}{x^3+x^2-17}$. $\frac{d}{dx} = (x^3+x^2-17) = 3x^2+2x$. Adjust to give $\frac{5(x^2+3x/5)}{x^3+4x^2-17}$ and then $\frac{5(3x^2+9x/5)}{3(x^3+x^2-17}$ which becomes $\frac{5(3x^2+2x-x/5)}{3(x^3+x^2-17)}$ and finally $\frac{5(3x2+2x)}{3(x^3+x^2-17)} - \frac{x}{3(x^3+x^2-17)}$.
7. *By the way*: Checking through the MOUTHS list I discovered that > 30% of subscribers have taken are are taking M251. Perhaps it would be an open-ended challenge to M500 readers to develop “page 2” or the other ? (on the left of the diagram).

It is a basic principle in the study of mathematics, and one too seldom emphasised, that a proof is not really understood until the stage is reached at which one can grasp it as a whole and see it as a single idea. In achieving this end, much more is necessary than merely following the individual steps in the reasoning. This is only the beginning. A proof should be chewed, swallowed, and digested, and this process of assimilation should not be abandoned until it yields a full comprehension of the overall pattern of thought.

George F Simmons *Introduction to Topology &c.* (D Gumaste)

MOUTHS *Marion Stubbs*

There is an increasing number of enquiries about the meaning of the title “MOUTHS” and I am urged to explain. Perhaps a little history will be suitable.

In the winter of 1972/3, as a member of the Solent OU Association Committee, I was charged with the task of organising self-help groups for the Solent mathematics students. Other committee members took resposibility for Arts, Social Science and possibly Science. My view was that the mathematics students known to me did not want group meetings, but wanted telephone contact urgently. Within days, the other Committee members were reporting a certain hostility from their “groups” at “wastage of precious tutorial time spent discussing SOUA matters”: My tutorial was the following weekend, so overnight I typed 24 copies (by hand) of my manifesto, inviting any of my M202 tutorial group who wanted telephone contact to let me know, and I would produce a list. Our course tutor (Phil Goble) was very helpful and sent copies of the paper to those not present at the tutorial. That historic paper was headed “Solent M202 Newsletter No. 1 February 1973” and was, effectively, M500 1. Six students had already been contacted before I issued it and their names were attached; so let's record the founder-MOUTHS: Jill MacKean, Alan Nichol, Dave Turner, Joy Dickens, John Bennett (later creator of the Hoops problem) and myself. Dave and I are the only ones remaining. The rest have graduated - or lost interest!

By the time of the next tutorial the list contained 17 names, now including Michael Gregory, Geoffrey Yates, Riki Rickard and Tony Brooks from M202 plus two M100 students, and the Solent *M202 Newsletter* was duplicated — I think this may have been the first time I ever used a duplicator. Simultaneously Phil Goble resigned and our group was merged with the Bournemouth tutorial group, thus doubling potential membership overnight. The *Solent M202 Newsletter* No. 3, April 1973, ran to all of 3 pages; and the Solent OU mathematics Telephone self-Help scheme was formally dubbed M.O.U.T.H.S. by Brian Hernen, then doing A301 and A302, who had a thing about mnemonics. It has remained M.O.U.T.H.S., or rather MOUTHS, ever since, with no objection until now.

When we reached number 6 I picked up Peter Weir from a letter of his in *Sesame*, and he was the only person to suggest a better title than *Solent M202 Newsletter* - which may have been because he was doing M100 and living in Coventry. Number 7 was titled M500 7 in consequence. Numbers 6 and 7 caused no little stir at my 1973 (and last) Summer School, where some WH staff present objected rather strongly to poor little M500, perhaps because it contained some home truths about poor M202-as-it-was-in-l973! I was decidedly squashed and indeed feared for my exam results - no kidding! But Stubbs valiantly put M500 first - Heroine, Martyr, etc., etc. and continued to battle (against all odds at the time) for the survival and expansion of this baby which was saving all our lives down Solent way since M202 was no joke in 1973. (It is better now, I hear and hasten to add!)

By M500 8, October 1973, I was all set to fold up the whole enterprise, not knowing then in my editorial innocence that students don't write anything except revision during September/October; but suddenly along came a letter from the Maths Faculty saying they had a mind to institute a Remote Mathematics Student Service, and what advice had I to offer. My advice was to give me the support I needed by publicising M500/MOUTHS,which already had an existence, and weren't we all 'remote'since we had no means of knowing who or where other OU students were, even if in the next street? They concurred, and M500 has grown continuously since then. The Faculty idea of the R.M.M.S. was that it should be informal and growing in accordance with students' needs, also that it should be student-operated. The whole set-up seemed to be tailor made for their needs and mine.

Winter 1974/5 seemed ripe to check whether members liked "M500" as a title, or would they prefer a more dignified one. "Open Set" had been suggested and I liked it. However, members were overwhelmingly in favour of "M500" and quite a few were vitriolic about any attempt to change the title. Still nobody questioned "MOUTHS" as a title for the other side of the system.

1975 saw our first Weekend Work-in, or Conference if you like, at which we conferred little and worked-in much. The conferring part, however, produced Peter Weir and Eddie Kent volunteering happily to take over their respective sections of my ever-increasing load. 1976 has seen the first Constitution of THE M500 SOCIETY and now there are some small rumbles about "MOUTHS" as a title for the directory part. Well, we are briefed to "grow in accordance with students' needs", so if a majority is unhappy with "MOUTHS" as a title and anyone comes up with alternative suggestions, we can vote it out; presumably next winter.

One wonders, still, about the future of THE M500 SOCIETY, just as one has always wondered in the past. When we had 17 members I wanted it to, but scarcely dared hope that it could, become a nationwide scheme - but even my optimism and fanaticism never thought it could reach the stage of needing printing. When each issue had been typed and sent out I always wondered where the material for the next would come from - and Eddie still has the same anxieties! But it always does come in, and nearly always in the correct sort of balance of maths and chat which discussions during 1974 revealed were what readers truly wanted. People write to me and ring me with happy messages saying "Keep up the good work". But without written contributions from the membership, M500 would die overnight, so I reciprocate with the grateful and happy message: "Keep it up, authors all! Many thanks for all you have done in the past to make M500 what it is."

GRAFFITI *Philip Newton*

At the Aston Weekend the next M500 SPECIAL ISSUE was discussed and it was agreed that the front cover was to be a collage made from photocopies of tutors comments when returning TMA.

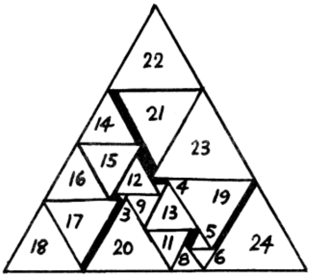
The idea is to send Marion a photocopy of the actual comment which can be from any maths course. Please mark the course number on the sheet since the overflow will be used at strategic places inside the cover.

I am entering at least two comments; one from M351 - *A very lucky choice of branching strategy has materially shortened your work* - and one from another course (which shall be nameless at the moment) - *What can we say about X*? There are some others.

Comment from M.S.: There may not be a "next M500 Special Issue" and anyway we do already have one cover design for it, but GRAFFITI welcomed for *ordinary* covers, please.

TANTALISING TRIANGLES CONCLUDED *C W Pile*

Last month we showed an old Scientific American *square packing which left out the* 7-*square. C W Pile asked if a similar packing could be produced for the* 70 *unit sided equilateral triangle:*



"Can a set of 24 equilateral triangles of sides 1 to 24 be fitted inside an equilateral triangle of side 70? If not what is the best arrangement of a subset of 24 triangles which leaves the minimum number of uncovered unit triangles. Is this minimum the same as for the square problem?"

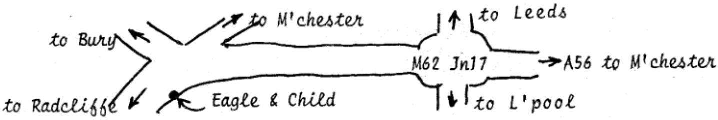
This is the best he has managed so far. It omits the 10-*triangle and the* 7-*triangle, so that* 149 *unit triangles are uncovered. These are shown shaded*. (*The* 1- *and* 2-*triangles are not displayed for obvious reasons*.)

THE LOCAL

Philip Newton

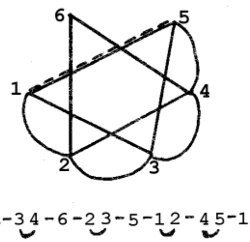
The reason I suggested a pub for a meeting place for a social chat before the commencement of the new academic year in March (M500 37) was two-fold. Firstly it is quite cheap, our local commercial travellers association has met on the first Monday in most months for years now at The Eagle and Child in Besses O' Th' Barn area of Whitfield at a cost (now) of £l for the evening, for a room quite capable of taking 50 or more persons. Secondly there is a good precedent for meeting in a pub; apart from the Carlsberg sponsored physicists do which resulted in the Copenhagen convention and its startling pronouncement of the Uncertainty Principle (I make no comment) there was also a pub in Salford where many used to gather some years ago. Cavendish, Dalton Grey, ---.

The Eagle and Child is ½ mile from the M62 motorway and Hull, Abergele, Carlisle and Worcester are all within a two hour driving radius of the place. So I would like to suggest this homely pub initially for an evening meeting on the second Monday (10th) in January 1977 when we shall also have our results to discuss. The commencing time will be eight in the evening. Anybody arriving in Manchester by train let me know in advance so that a lift can be arranged for them. Also contact me directly if you have any problems on this.

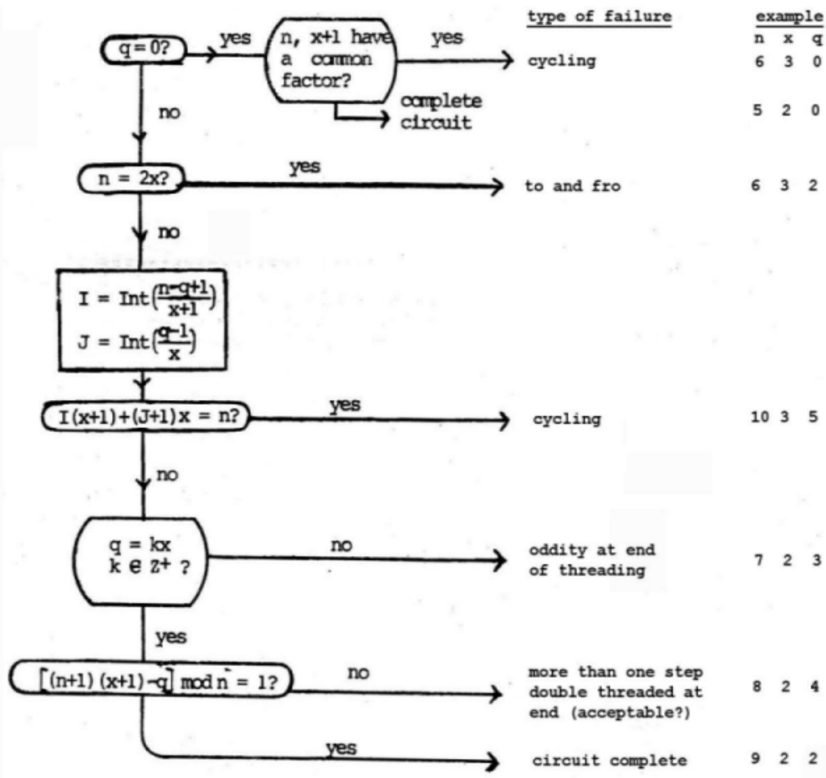


CONSTRUCTIONS ANALYSED Michael Gregory

In Constructions, reprinted in M500 28 I described 3-D ruled surfaces can be made from perspex and thread. Using the same convention: n = number of holes; x = step length; and allowing a double threading for the first $n - q$ holes, the $(n - q + 1)$th hole was considered to be special. We had for $n = 6$, $x = 2$, $q = 2$ the "complete" threading as shown on the right.



I have developed a series of tests for failure to give a complete circuit. Briefly these are:



Here Int(y) is the greatest integer $\geq y$ and $|y|$ mod n is the remainder in $|1, n|$ when n is repeatedly subtracted from y.

IMPORTANT - PLEASE READ

PRISONERS AND MOUTHS Peter Weir (*Membership Secretary*)

Recently, for the first time two prisoners from Broadmoor who are taking M100 in 1977 have applied to recieve M500 and the MOUTHS list. M500 they can have, but as for the MOUTHS list ---.

After a quick emergency council session (Marion and I) I put forward the following: if any one person objects then the prisoners shall not receive the MOUTHS list under any circumstance. M500 officers don’t count - the prisoners may already have their addresses as they have mine.

As it is a dead cert that someone will object, Marion and I agreed to propose a ‘sub-MOUTHS’ list. We cannot just ignore these people: they particularly want the MOUTHS list, they are truly isolated. Anyone who wishes to be on this list, which will be specifically for prisoners etc., must state their wish VERY CLEARLY IN WRITING and repeat their wish each year at renewal. Write to me or Marion (or phone) about the subject. Or an article to Eddie, or both. We await your feedback. Until then **NO MOUTHS LIST WILL BE SENT.**

Unlike next year’s OU fees and prices generally, the M500 subscription remains fixed at £3 for 1977, which I consider to be excellent value for money (speaking as a recipient).

However, as you may already know, the Society has a new typewriter which has resulted in the depletion of the Equipment Fund. Therefore any donations will be gratefully received, so that we can start planning future equipment requirements.

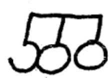
Finally may I wish you all the best with your examination results.

Austen F Jones (*Treasurer*)

THE EMBLEM Chris Pile

I would be in favour of having a device or emblem for M500 (M500 37) and I think this should consist of some arrangement of the characters M500:

or the more symbolic



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BROTHERHOOD OF MAN Joyce Moore

Did you know it is theoretically possible to trace your ancestry back to the time of Edward III, that is, to the 14th century? Consider this then:

Each child has two parents, four grandparents, eight great-grandparents, and so on, doubling up with each generation. Assume a generation to be 25 years - four to each century - and double each set of ancestors till you get back to the 14th century. That's around 24 generations, which comes out at 16 265 216 ancestors each.

BUT

The population of England during the 14th century (immediately preceding the visitation of the Black Death) was between four and six million souls. It reached a peak at that time, then fell, recovering only in the 18th century. In other words, these few million people have to be the joint ancestors of most of the present population of Great Britain (excluding recent immigrants), not to mention a hefty proportion of the populations of the former 'white' colonies (USA, Australia, etc).

If we have virtually all our ancestors in common can it be that, to paraphrase Kipling, the Colonel's Lady and Rosie O'Grady really are sisters under the skin - well, cousins anyway? You might care, as a small diversion over Christmas, to work out just how closely you may be related to your next door neighbour, the local postman, or the Lord of the Manor! Incidentally, if you went even further back, to 1066, you could expect something in the region of 68 thousand million ancestors from an even smaller overall population. Put that on your family tree, if you can!

M201, THE MORNING AFTER

Peter Weir

Guilt
I never looked at that unit
Or this, nor that assignment -
Quiet lies the untouched page.

Laplace Transforms:
A solid name, that.
Still as the day it came
Waiting for action
Waiting to impart words
On Laplace Transforms.
Tales of how, and why
and where
And Laplace Transforms.
Calm.
It waits
Passed by in the rush
For success
For degrees
Laplace Transforms
Waits its turn
But waits in vain ---.
Laplace Transforms
Informs
But not me.
Laplace Transforms
Transforms dark into light
But not tonight.
Laplace Transforms
Reforms
Maybe.
Not me.
Not now.

A SMALL DITRIGONAL ICOSIDODECAHEDRON FOR CHRISTMAS

Marion Stubbs

The dodecahedron, as anyone reading this journal will recall from M100 Unit 30 TV Notes sketches, has twelve faces, each being a regular pentagon. For the Small Ditrigonal Icosidodecahedron, cut a regular 5-pointed star-shape template from card, and also a similar card template for an equilateral triangle with sides equal in length to the length of the star sides.

Place each template on the material to be used, such as computer waste cards or even old wallpaper if strong enough, prick a mark on the material at each vertex of the template, and join up the pricks with a scoring knife or pencil. Then cut round the shape, leaving about quarter inch tabs all round for gluing. Fold the tabs inwards along the scored lines.

You need 12 stars and 20 triangles. Start with one star, surrounded by ten triangles. It is easiest if the triangle pairs are glued together first and then glue them between the star arms, as dihedral grooves. Then you can immediately add the next five stars, followed by the remaining pieces. As usual, the final star is the most difficult to insert. It is best done slowly, in stages, gluing only one tab at a time, and using a suitable instrument as a probing needle to work in where needed.

The result is a pretty Christmas decoration which can be hung from a length of wool or string glued into one of the vertices where three stars meet. It can be decorated as desired, sprayed with silver or gold dust, and used for experiments with the Four-Colour Map Theorem!

Meanwhile, keep children occupied with simpler models of the tetrahedron (four equilateral triangles), octahedron (eight equilateral triangles) and dodecahedron (twelve pentagons) and supply them with large quantities of old magazines, scraps of fabrics, poster paint and whatever comes to hand for decoration of their finished models.

I confess freely that I last made the SDI in 1972. At that time there were no problems, so I hope that I have not introduced any into this description of an old favourite. Full instructions, including very detailed plans for colour arrangements, can be found in *Polyhedron Models* by Magnus Wenninger (Cambridge UP 1970, costing £5 at that time), to whom I shall always be indebted for making polyhedra into a happy hobby.

He (Thomas Hobbes) was 40 yeares old before he looked on geometry; which happened accidentally. Being in a gentleman's library ... Euclid's *Elements* lay open, and 'twas the 47 El. libri 1. He read the proposition. "By G—", sayd he, "this is impossible:" So he reads the demonstration of it, which referred him back to such a proposition; which proposition he read. That referred him back to another, which he also read. *Et sic deinceps*, that at last he was demonstratively convinced of that truth. This made him in love with geometry.

Aubrey *Brief Lives*. (JH)

SOME LETTERS

From Doreen Mitchell - I am very much a 'remote student' - my first glimpse of a tutor and other students being at Stirling at the end of July. Summer School was terrific.

Am wondering who would be the best person to contact if I get stuck? (which is frequent:) I need someone to translate units into primary school standard.

From Bill Midgley - Why I do not contribute. Fact is, most of what is in M500 is still a bit of a mystery. I have just done M100 in a sort of daze - sessions of working like mad to catch up or get ahead interspersed with periods of not being able to get on owing to being posted hither and yon and being sent off on courses. I went to Summer School having convinced myself with the quizz that I knew nothing (and kept proving it in lectures). Fortunately the examiners managed to find a few things I knew a bit about so I might be all right.

The general plot is M201 next year and the two computer second levels (½ credits) the year after, but again I shall be a bit pushed as I shall be doing HNC Business Studies at the same time.

From Russell Brass - I had intended to take a break next year; I've been at it 4 years; but I've crossed my tutor on one of this year's courses so to be on the safe side I've put down for a couple of ½ credits next year.

Actually I haven't completed my exams yet, they scheduled me for two exams at the same time on the same day! (Thinks: the OU couldn't organise a good time in a house of ill-repute.) As you will no doubt guess, the exam I am waiting for is the subject I've had continual trouble over. This proves Murphy's Law that when things start to go wrong they will get worse.

Re the M500 monomark on the last page of 37, how about an M500 tie? If anyone will be interested I can probably get the address of a firm that will do club ties in dozen lots.

DEPARTMENT OF VICTORIAN PARLOUR TRICKS   Eddie Kent

This used to be a very popular trick and is described (badly) in *The Young Scientist* for November 1880 - together with a beautiful illustration which I can't unfortunately reproduce.

Obtain a hollow tube (the inside of a kitchen towel-roll is ideal, anything shorter won't work properly). Hold it in the left hand to the left eye. Keep both eyes open. Place the right hand, palm forwards, against the right side of the tube at the end furthest from the eye. You will then be able to look at objects through a hole in your right hand.

"The result is startlingly realistic, and forms one of the simplest and most interesting experiments known."

SOLUTIONS

35.2 RIFFLE *Not a solution but a note from David Asche.*

Since riffle shuffles seem to be of interest, readers may care to know about about a paper entitled 'The Generalsied Faro Shuffle' by S B Morris and R E Hartwig appearing in *Discrete Mathematics* volume 15 number 4, August 1976.

36.3 POLYGONS

(b) *Show, without actually drawing the figure, that no* 3 *diagonals of a* 7*-sided* (*regular*) *polygon meet except at vertices.*

Krysia Broda: Assume that 3 lines do intersect somewhere. By symmetry we have either 7 points of intersection or 1 point. 1 point is impossible, it would have to be the centre (*RA: If you rotate a regular* 7*-gon by* $\frac{1}{7}$ *of a revolution it looks exactly the same as before rotation. Hence if* 3 *diagonals pass through one point* (*not the centre*) *then there must be a similar point* $\frac{1}{7}$ *of a revolution away from this spot*). We have 14 diagonals altogether. A diagonal which cuts out one vertex (7 of them) cannot intersect any others except the four from the vertex which it cuts out - and these cut the diagonals at different points. Thus we have 7 diagonals left for our 7 points, each with 3 diagonals. Hence each diagonal goes through 3 points (*RA: This, is not quite obvious to me although I would believe that some line contains at least* 3 *points and if the* 7 *points are at the vertices of a regular* 7*-gon this is certainly impossible*) which is impossible.

Richard Ahrens: Using the result published in M500 37 that there are $\binom{n}{4}$ (Ed: not $\binom{4}{n}$ as actually appeared) crossings inside an *n*-gon we have $\binom{7}{4}$ = 35. Krysia has pointed out that some intersections cannot possibly be triple points - a diagonal which “cuts off” one vertex contains 4 “double” points. There will be 21 of these double points. If there is a triple point there must be 7 of them by the argument above. Each triple point means 3 pair of intersecting diagonals - another 21 intersections. But 21+21 > 35 - contradiction.

PROBLEMS

38.1 *THEREFORE FIRE ENGINES ARE RED* - Marion Stubbs

This problem is taken verbatim from *Creative Computing*, the wow mag for computer addicts perpetually promoted by M Stubbs - with apologies to those who already have a sub to *CC*.

It is not easy. It could swallow up a day of your time, even a week. But it will take more than an hour. The following fifteen facts are all you need to solve it:

1 There are five hunting cabins on a lake. Each cabin is a different colour and is inhabited by a man of a different nationality, each drinking a different kind of liquor, firing a different kind of shotgun shell, and shooting a different duck.

2 The Englishman lives in the red cabin.

3 The Pole shoots only bluebells.

4 Bourbon is drunk in the green cabin.

5 The Finn drinks beer.

PROBLEMS continued

6 The green cabin is immediately to the right (your right) of the brown cabin.
7 The hunter who uses Winchester shells shoots mallards.
8 Remington shells are shot in the yellow cabin.
9 Brandy is drunk in the middle cabin.
10 The Norwegian lives in the first cabin on the left.
11 The man who buys Federal shells lives in the cabin next to the cabin of the man who shoots redheads.
12 Remington shells are used in the cabin next to the cabin where the canvasbacks are shot.
13 The hunter who fires Western shells drinks gin.
14 The Irishman loads up with Peters shells.
15 The Norwegian lives next to the blue cabin.

Your mission, should you decide to accept it, is to figure out who drinks the Scotch and who shoots the teal. Good luck.

38.2 *BERWICK'S SEVEN SEVENS* - Jeremy Humphries

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               **7**
       ****7*) ** 7 *******
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            *****7*
            *******
             *7****
             *7****
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38.3 *NATURAL EXPANSION* - E Kent

It is a well-known fact that $e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!}$.

It was (until now) less well-known that, if n is a natural number then the nth and $(n+1)$th terms of the expression for e^n are equal. Why?

(Examples: $e = \underline{1} + \underline{1} + ½ + \ldots$; $e^5 = 1 + 5 + 13.5 + 20.83 + \underline{26.041\dot{6}} + \underline{26.041\dot{6}} + 21.7\ldots + \ldots$.)

38.4 THE ECCENTRIC CARPENTER - ? (If you sent it in please let me know.)

A carpenter adds legs at random on the circumference of a circular table. On average how many legs must he add before the table will stand up without toppling over?

38.5 ALMOST PERFECT - M S Klamkin, *J Rec Math*, 1960.

A perfect number is of the form $2n = \sum_{d|n} d$. If this expression is actually equal to $2n \pm 1$ it can be called *almost perfect*. Are there any almost perfect numbers other than those of the form 2^m?

EDITORIAL

This is my Problem Editorial: perhaps a kind of continuation of the previous page.

First a pair of footnotes: (a) to problem 36.1 196th ROOT. More replies are trickling in about the general problem and correspondents will forgive me for not including them (unless anything startling turns up). But Chris Pile and some friends tried out Continuous Reversed Addition (CoRA) on the number 196 and had still found no palindromic number by the 43rd iteration when enthusiasm and writing materials ran out. For those who didn't read M500 35 1 and for those who have forgotten it, CoRA takes a number, reverses it and adds the two together; reverses the sum, adds, and so on until eventually a palindromic number appears. There seems no way of disproving this assertion but I would be grateful if someone with access to a machine could run it on 196 to see what palindrome turns up, and when.

(b) to 36.4 LOGS. More solvers, including John Owen who points out that the solution depends on finding the maximum value of the *n*th root of *n*. Foundation people might like to try this - they have all the information necessary and the techniques required could be useful in solving that other perennial: which is the greater, e^{π} or π^{e}?

Perhaps some M500 readers noticed the article by Martin Gardner in the October *Scientific American* on the No-Three-In-A-Line problem, where amongst others 'D Craggs of the university of Kent' was credited with solving for $n = 12$. Ever jealous o£ the honour of our Society I wrote pointing out that publication in M500 21 (is M500 a publication?) preceded that cited: *J Comb Theory*, May 76 and that Dorothy Craggs was at the OU not Kent U. Mr Gardner sent a pleasant letter back with some information: "One reader has found (by computer program, not exhaustive) 29 solutions for the order 13, and one for the order 15. I'll publish details at the end of Jan column. So far, no order 14 sol. has been found". Let us see if we can scoop the world agaln. That is, place 28 points within a 14×14 array so that no three are in a straight line in any direction.

Talking of which, no one ever solved these: 31.3 COMBINATIONS, M Bramer: Prove that $^{n}C_{r}$ is an integer. 32.5 RATIONAL TERMINATION, MB: What rationals have a finite *N*-cimal expansion in base *N*? 33.1 VECTOR SUBSPACES, R Bailey: If *v* has three subspaces how many possibly distinct subspaces of *v* can be obtained by repeated use of + and $\cap$? NEXT TERMS, 33.3(6): 1 21 21000 101 121 1101 1121 21121 ... . 34.4(11): 1221121221221121122121121122112 11212212211... . 35.1 SHUFFLE, R Ahrens: Prove that $2^{m+1}x_m \equiv \pm (2x_0 - 1)$ mod $4p$+l where x_i is the posltion of a particular card after *i* Monge-shaffles. 35.5 RATIONAL TERMINATION II, K Broda: Find a number system in which *all* rational fractions have finite decimal expansions. And 36.3 POLYGONS, RA: (c). For this see page 16, where it is shown that at most two diagonals meet at any point within a 7-gon. (c) requires a generalisation of this result to any regular $(2n+1)$-gon although the methods for the 7-gon probably won't generalise. And that's all unless I've missed anything. What we need is a Problems Editor!

If anyone is interested in 'almost perfect' numbers there is a paper by R P Jerrard and Nicholas Temperley, 'Almost perfect numbers', *Math Mag* 46 (1973). Cf problem 38.5.

What is the longest word.in the English language. How about pneumonoultramicroscopiesilic-oivolteanoconlosis? From Chambers *Words for Crosswords and Endgames* (Chambers, 1.50) - which sorts words according to their number of letters; §45 has one entry. One day you can produce it to clinch an argument with all the authority of M500 as backing.

Merry Christmas — Eddie Kent.