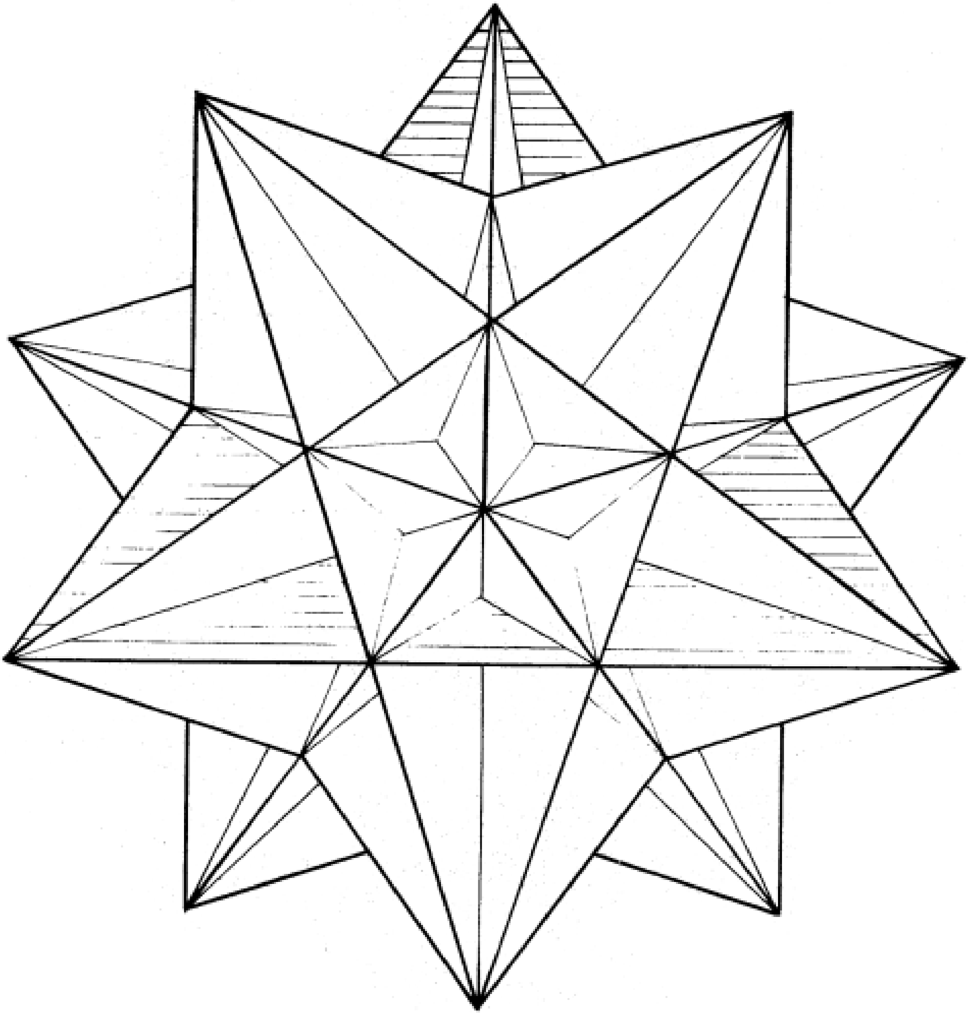

M500 40

M500 *is a student operated and owned magazine for Open University mathematics students and staff, and friends. It is designed to alleviate student academic isolation by providing a forum for public discussion of the mathematical interests of readers.*

Articles and solutions are not necessarily correct but invite criticism and comment. Nothing submitted for publication ought to be more than about six hundred words long; otherwise it may be split into instalments.

MOUTHS is a list of names and addresses and telephone numbers with previous and present courses of voluntary members, by means of which private contacts may be made, to share OU and general mathematical interests, or to form self-help groups by correspondence or telephone.

The views and mathematical abilities expressed in M500 are those of the authors and may not represent either those of the editor or the Open University.

The cover design is by Chris Pile and is a diagram of the Great Icosahedron. Chris has written on the subject of polyhedron models in this issue, page 3.

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FOURIER FUN

Krysia Broda

Given the functions $f_i(x) = x^i$, $x \in (-\pi, \pi)$ we can find the Fourier series expansions for them. e.g.

$$f_1(x) = x = 2\sum_{k=1}^{\infty}(-1)^{k-1}\,\frac{\sin kx}{k^2},$$

$$f_2(x) = x^2 = \pi^2\left(3 + 4\sum_{k=1}^{\infty}(1)^k\,\frac{\cos kx}{k^2}\right),$$

$$f_3(x) = x^3 = \sum_{k=1}^{\infty}\left(\frac{-2\pi^2}{k}(-1)^k + \frac{12}{k^3}(-1)^k\right)\sin\ kx,$$

$$f_4(x) = x^4 = \frac{\pi^4}{5} + \sum_{k=1}^{\infty}\left(\frac{8x^2}{k^2}(-1)^k - \frac{48}{k^4}(-1)^k\right)\cos kx.$$

Parceval's equality gives us that $\frac{1}{\pi}\int_{-\pi}^{\pi} f^2 = \frac{2}{\pi}\int_0^{\pi} f^2 = \frac{a_0^2}{2} + \sum\left(a^{(i)}k^2 + b^{(i)}k^2\right)$ where $a^{(i)}k$ and $b^{(i)}k$ are the Fourier coefficients of f_i . Having got this far we notice that we get expressions such as (for example) $\sum_{k=1}^{\infty}\frac{1}{k^4} = \frac{\pi^4}{90}$. For large n

$$\sum_{k=1}^{\infty}\frac{1}{(2k-1)^n} - 1 = \sum_{k=1}^{\infty}\frac{1}{(2k)^n}. \qquad (\dagger)$$

In fact $\lim_{n\to\infty}\frac{1}{k^n} = 0$ for all k except $k = 1$. (*sic*)

Given $\sum_{k=1}^{\infty}\frac{1}{k^n} = \frac{\pi^{\mathrm{n}}\mathrm{p}}{q}$ we can use (†) to get an approximation to π. $\sum_{k=1}^{\infty}\frac{1}{k^n} - \frac{1}{2^n}\sum_{k=1}^{\infty}\frac{1}{k^n} = \sum_{k=1}^{\infty}\frac{1}{(2k-1)^n}$ (can be proved since $\sum_{k=1}^{\infty}\frac{1}{k^n}$ is convergent). Using (†) we have $\sum_{k=1}^{\infty}\frac{1}{k^n}\frac{(2^n-1)}{2^n} - 1 = \frac{1}{2^n}\sum_{k=1}^{\infty}\frac{1}{k^n}$ where = means equal in the limit, as $n\to\infty$.

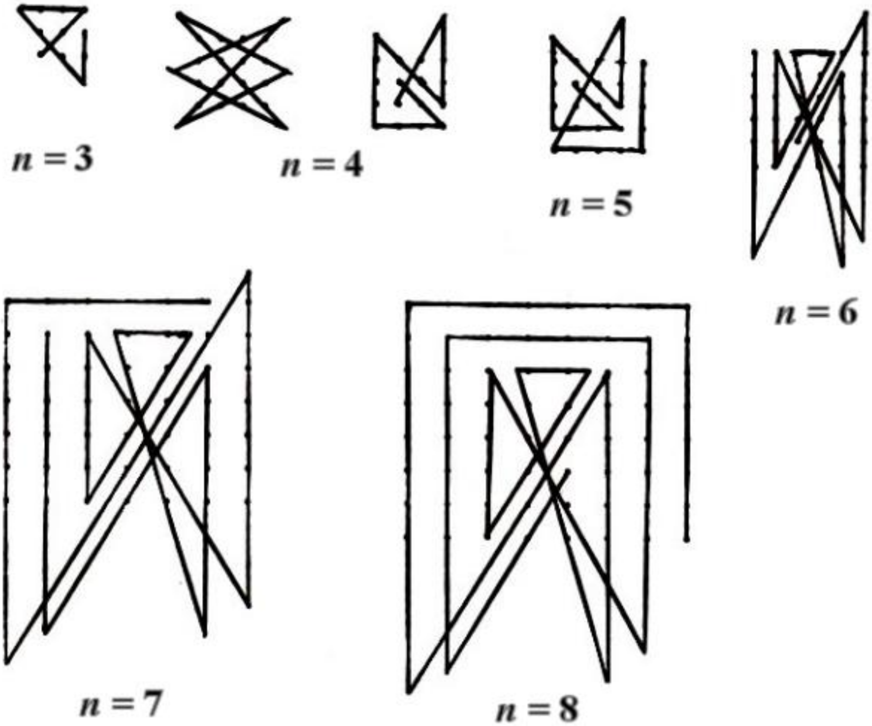
i.e. $\sum_{k=1}^{\infty}\frac{1}{k^n} = \frac{2^{n-1}}{2^{n-1}-1}$ $\therefore$ $\pi^n\,\frac{p}{q} = \frac{2^{n-1}}{2^{n-1}-1}$ and $\pi^n = \frac{2^{n-1}\,q}{2^{n-1}-1_p}$. I have only five-figure logs and using $f_5(x)$ we get π to 4 decimal places.

It might be noticed that to get f_3 from f_2 we just integrate f_2. But to get f_4 from f_3, we have to integrate f_3 and add in a constant. This is $a_0/2 = \frac{1}{\pi}\int_0^{\pi} f_4$. This is because f_4, is an even function and so its Fourier series is a cosine series, which includes the term $a_0/2$. For f_n where n is odd there are only sine terms and no constant. The justification for integrating the series term by term is part of M331 Unit 14.

... the feeling of mathematical beauty, of the harmony of number, of forms, of geometric elegance. This is the true æsthetic feeling that all mathematicians know.

Poincaré.

(Quoted by Koestler in his PEN opening address, August 1976.)



DOTTO

Jeremy Humphries

Does anyone know anything about drawing lines through an array of n^2 points? I've seen the problem for n = 3: draw 4 lines through a 3 × 3 array of points so that every point is on a line and you don't lift the pencil from the paper. Does it generalise - draw $2n - 2$ lines through n^2 points with the added proviso that lines may meet but not cross on a point? I've done it up to n = 9. I've got two versions for n = 4, one of which is closed. 4 and 5 are obviously similar. 6, 7, 8 and 9 are very similar in the middle. Can anybody find different versions (excluding trivial changes) or any higher values of n? Is $2n - 2$ necessary / sufficient for all n?

Ed - *The diagram for n* = 9 *is omitted from the above set for three reasons*. (i) *I felt I'd done enough,* (ii) *it would have taken up a lot of space and* (iii) *I thought it would make a nice problem hence see Problem* 40.1(a).

POLYHEDRA

Chris Pile

The pursuit of mathematics can take many forms and one of the more attractive can be illustrated by a collection of three-dimensional models of polyhedra.

A polyhedron is regular if all its faces are identical regular polygons. The five convex regular polyhedra (tetrahedron, cube, octahedron, dodecahedron and icosahedron) have been known since the earliest days of mathematics. The four non-convex polyhedra were discovered comparatively recently by Kepler and Poinsot. The Great Dodecahedron* has 12 intersecting pentagonal faces and the Great Icosahedron* has 20 intersecting triangular faces. The other two both have 12 pentagrams (five sided stars), the Small Stellated Dodecahedron* having 5 meeting at each vertex while the Great Stellated Dodecahedron* has three at each vertex.

Apart from the tetrahedron, which is self-dual, the regular solids can be arranged in dual pairs. For example the cube has eight vertices and six faces whereas the octahedron has six vertices and eight faces. These two solids can be arranged so that the vertices of one correspond with the faces of the other and the edges bisect each other at right angles. In this case the common solid contained entirely within both the cube and the octahedron is the cuboctahedron - one of the thirteen Archimedean polyhedra (convex polyhedra with regular polygonal faces which are not all identical).

The Kepler–Poinsot dual pairs yield more interesting common solids: Great Dodecadodecahedron and Great Icosidodecahedron*.

The large range of stellated and re-entrant polyhedra form a most attractive collection especially if the individual faces are coloured. All the polyhedra with regular polygonal faces are described in ‘Uniform Polyhedra’ by Coxeter, Longuet-Higgins and Miller, *Philosophical Transactions* A volume 246 (1954). The constructional details of some of these are given in *Mathematical Models* by Cundy and Rollett. There is a very fine display of polyhedra at the Science Museum.

Ed - *Those marked * will be used, intermittently as covers*. (*See* 38.)

NOUGHTS W M Dalton

I was a child when somebody said it
‘Nowt is not owt, neither debit nor credit,
it’s when you have none and nor do you owe.
I must have been ten, that’s a long time ago.

When I was twelve, much to my surprise,
I found that the value of zero just lies
no value of nought is commensurable.
at now here or nowhere, it was demonstratable;

At eighteen I learned that a j makes things sane
for X plus jY gave the size of a plane;
rotated through half pi this plane disappears
the Y side retreats, or is it it nears?

But now, what with binary, zeros commutative,
logic truth tables, noughts that are relative,
maxima minima, call them associative,
the kernel, forget it, the whole world is transitive.

For if we reflect we find for a surety
all maths has become a moronic fatuity,
never no numbers, sometimes a unity,
after which nothing but noughts to infinity.

THE DEVICE

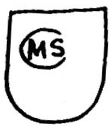
Some more suggestions:

Dave Diprose: This made me think of an open hand - the digits representing mathematics at a very basic level, five digits for M500 and the hand generally to represent contact.

Marion Davis-Stubbs: Re-reading M500 37 editorial for the first time I observe that our Ed has nattily incorporated the Founder's initials in his second suggestion, namely:

Obviously I *like* it! But, this egalitarian SOCIETY has no place for bigeds, so a little mathematical justification might be suitable.

Form the Cartesian product of {OU Maths students) × {OU Maths staff}where Students = $\{S\}$ and staff Mathematicians = $\{M\}$. Then $S \times M = C$ where C is an Open Set in OU^2. (Like an open circle, you see?) Now C = 100 according to the Romans while D = 500; and D just happens to be the maiden initial of the Founder: MD→MS. Let $C \subset D \Rightarrow C \subset OU$ and draw the appropriate diagram. We obtain:



Enuf said.

Ron Wheeler:

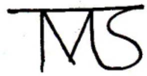
M for M; $\overset{M}{\wr\wr}$ for $\frac{M}{2} = \frac{1000}{2} = 500$; $\wr\wr$ for S (laterally inverted) for SOCIETY. On its side it can be read as "sum approximately equals" or just "sum approximately" which I offer as an apt motto for the SOCIETY.

Before finally adopting an emblem check with the Trade Marks Registry against possible infringement. (Registrar, TMR, 25 Southampton Buildings, London WC2.)

Richard Ahrens: I don't think we need one - if we want to distinguish ourselves I suggest that we make use of the Roman abbreviation for 500 and simply write M.D. after our names.

Richard MD.

Now that the supply of designs has dried up here are two more of my own:



One is obviously a modification of Chris Pile's in 38 12 while the other I see made up in brass.

Perhaps we can end this phase with some advice, culled by permission of Granada Publishing Ltd from *Gamesmanship* by Stephen Potter (Rupert Hart-Davis), p.69:

> "what a sensible notion it was to make the colours and pattern of this special tie precisely the same as that worn by the I Zingaris! This has the triple advantage of (1) doing away with the need of designing a special tie, (2) allowing the gamesman to be mistaken for one who has the very exclusive honour of belonging to IZ, and (3) irritating any genuine IZ against whom the gamesman happens to be playing."

HALLO SAILOR

MARION STUBBS

According to Chambers *Twentieth Century Dictionary* a sinnet or sennit is a "flat braid of rope yarns (origin uncertain)." A SOLID SINNET therefore is a solid braid of rope yarns - or in mathematical terms a three-dimensional permutation of rope yarns. Non-mathematicians might call it a rope. Most of us think of ropes as strong but fairly uncomplicated things, although those who work in rope- and cable-making industries probably have better ideas. Certainly The *Ashley Book of Knots** contains a host of patterns for solid and flat sinnets, each one worthy of mathematical consideration and experimentation, as well as an enormous collection of patterns for knots. The following piece contains nothing original on my part, but is really a précis of the basic *Ashley* details concerning some of the known solid sinnets. There is nothing like practical model making for improvement of one's understanding, so I hope that these ideas will stimulate others to play with lengths of coloured wools or similar materials to find out just how permutations work in 3-D, and some may even produce some erudite findings which the author of the book perhaps never suspected.

A sinnet is a 'shuffle' of a 'pack' of *x* strands of wools, string or similar threads. - To make a solid sinnet you will need some simple equipment, apart from the chosen strands.

First, you need a stand with a hole in the middle. Ideally this is a self-standing tall tripod but make do with, e.g., a flower pot standing on a stool. Flower pots are OK. The sinnet will need a weight on its starting

* Ashley, Clifford W., *The Ashley Book of Knots*. London: Faber. SBN 571 09659 X (about £10 in 1976)

point, to counterbalance the spools on which the strands are wound. Currently I have not found a useful balance but others may be more ingenious. Clothes pegs are good as spools. My ‘test’ strands are ten feet long and wind 70 times round a clothes peg, initially.

Next, draw a circle-diagram of 6 segments, for starters. The segments are numbered 1 to 6 counter clockwise. Stick it on top of the flower pot with sellotape. Now you need dividers between segments - ideally pins or nails but they won’t stick into flower pots, so I use the plastic strips from Mothers Pride bread bags folded into three. One end tucks under the hole in the middle and the rest of the strip makes a useful 3-D divider. You need as many strips as there are segments.

Attach your x strands to the counterbalance in the central hole (e.g. to a clothes peg, but experience will show that this is not enough - MST282 to the rescue please?). Drape each strand over its initial segment position as specified in the recipe for the chosen sinnet.

We now develop a few rules:

1. There are more strands than there are segments. Extra strands are introduced so that no segment is ever left vacant. Positions of extra strands are specified for given sinnets.
2. The ‘earliest’ strand into any segment is the first to be moved out of it. This unrigorous expression I dislike but in practice it is useful since all of us are human!
   (a) The ‘earliest’ strand in an odd numbered segment is called an ODD strand and is the right hand (RH) strand of its group. It is led to the right, counterclockwise until it reaches its destination when it is placed in the near, left hand (LH) position of an ODD segment or in the far, RH position of an EVEN segment.

   SUMMARISED: ODD: RH $\rightarrow$ LH (odd)

   (b) The ‘earliest’ strand in an even numbered segment is called an EVEN strand, and is the LH strand of its group. It is led to the left, clockwise, until it reaches its destination when it is placed in the RH position of an EVEN segment or in the LH position of an ODD segment.

   SUMMARISED: EVEN: LH $\rightarrow$ RH (even)
3. All strands are moved OVER all intermediate strands between origin and destination.

The diagram below shows two typical movements, 1 – 5 followed by 4 – 2, where segments 1 and 4 each have two strands initially.

We have started with a 6-segment sinnet and the least number of strands which will make it, following the rules, is 8. However one can use more strands with the same number of segments to obtain different sinnets with different shapes.

There are three different sequences by which a six segment sinnet can be made with 8 strands. One of these produces a round sinnet, two are barely triangular.

There are six different sequences for a 9-strand six segment sinnet. All six are triangular but irregular. By introducing extra strands where required all six irregular 9-strand sinnets become regular.

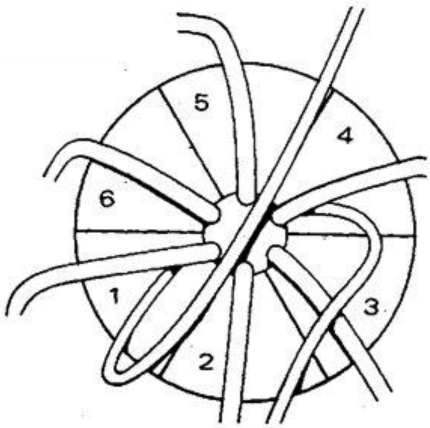
There are three different sequences for a 10-strand six segment sinnet. All three are triangular and symmetrical.

The shapes, symmetries, regularity and so forth of six segment sinnets with more than 10 strands are ‘left as an exercise for the reader.’

Now for a few sinnet recipes. The format is a three column table where column 1 shows the segment numbers, column 2 shows the number of strands initially placed in the corresponding segments. Column 3 is independent of columns 1 and 2 and shows the cycle of movements in the correct order. The five specifications on the right are all for 6-segment circle-diagrams.

At the time of writing I have no mathematics of sinnets ready for sharing but hope that these introductory notes may inspire others to start investigations. I feel that it should be possible to specify the cycle in terms of strand numbers, abandoning loose terms such as ‘earliest’ or LH or RH, and that this kind of notation could be fruitful.

segment	no. of strands	moves
1	2	1-5
2	1	4-2
3	1	5-3
4	2	2-6
5	1	3-1
6	1	6-4
8 strand round		
1	2	1-5
2	1	6-4
3	1	5-3
4	1	4-2
5	1	3-1
6	2	2-6
8 strand triangle		
1	2	1-5
2	1	4-2
3	2	3-1
4	2	6-4
5	1	5-3
6	2	2-6
10 strand triangle		
1	3	1-5
2	1	4-2
3	2	5-3
4	2	2-6
5	2	3-1
6	1	6-4
11 strand triangle		
1	3	1-5
2	2	2-6
3	3	3-1
4	2	4-2
5	2	5-3
6	1	6-4
13 strand triangle		



(diagram drawn by L. Seaton, Lecturer in Scientific Illustration, Southampton College of Art)

A HAIR OF THE DOG ... BY BOB MARGOLIS

In M500 37 I offered a problem on Lie Algebras (Summer School Hangover). On the day of publication we had a particularly boring Faculty Board (whoops, sorry Mike!) and as a result Chris Rowley gave me a proof and Richard Ahrens gave me a counterexample! In fact Chris' proof was OK and Richard's counterexample wasn't but there's a moral there somewhere ... .

As a reminder, here's the background and the problem.

A Lie Algebra is a vector space V with multiplication of vectors defined and satisfying $v^2 = 0$ and the Jacobi identity: $(ab)c + (bc)a + (ca)b = 0$ all $a,b,c,v \in V$. Multiplication is distributive over addition etc.

If V is a Lie Algebra, V^2 is the set of all sums of products of elements of V.

The dimension of a Lie Algebra V is its vector space dimension.

Problem: V is an n-dimensional Lie Algebra and V^2 is *one* dimensional, basis $\{x\}$. There exists a $y \in V$ with $xy = 0$. Prove that V has a basis

$$\{x_1, x_2, ..., x_n\}$$

with $x_1x_2, = x_1$, and $x_ix_j = 0$ for all $i,j > 2$.

[This says that V is the direct sum of a 2-dimensional algebra and an (n–2)-dimensional *abelian* algebra. The abelian bit comes from the fact that $0 = (u + v)^2 = u^2 + uv + vu + u^2 = uv + vu$ so that if $uv = vu$ both must be 0.]

My main difficulty was that I didn't sort the problem out clearly enough in my own mind. Once I'd written it out for M500 I solved it in about 10 minutes. *Viva Polya*!

Now for the solution (at last, I hear you cry!). I've had solutions from Chris Rowley and David Asche of a neat but non-elementary kind and elementary (i.e. first principles) proofs from Bob Escolme, Sue Davies, Percy Mett and Jeremy Gray. I've also had a proof for the special case $n = 4$ from Philip Newton.

Here's the elementary proof - Sue Davies' but the others are essentially the same. Level - M100 after the Linear Algebra Units really, but a bit of KK0P: p31 Theorem 1-7 comes in handy.

Given Lie algebra V of dimension n. $\dim(V^2) =1$. $x\in V$, $y\in V - V^2$ s.t. $xy \neq 0$.

Find a basis $x_1, x_2, ...,x_n$ of V s.t. $x_1x_2 = x_1$ $x_ix_j = 0$ all $i, j > 2$.

Proof $x \in V^2$ $x \neq 0$ so take $x_1 = x$. y e $V - V^2$ s.t. $xy \neq 0$. Since x_1y is a product it is in V^2. $\therefore$ $x_1y = k.x_1$ for some $k \neq 0$, $k \in F$. $\therefore$ Take $x_2 = \frac{1}{k}.y$ then $x_1x_2 = \frac{1}{k}(x_1y) = \frac{1}{k}k\, x_1 = x_1$. Because $x_2 \notin V^2$ x_1, x_2 are linearly independent. $\therefore$. x_1,x_2 can be extended to a basis of V:

$$x_1, x_2, z_3, z_4, ...z_n. \text{ (KKOP, p.31, Th. 1 - 7)}$$

Now, each x_1z_i is in V^2 ($i > 2$) hence $x_1z_i = m_ix_1$, say, for $m_1 \in F$.

If $m_i = 0$ take $y_i = z_i$

If $m_i \neq 0$ take $y_i = \frac{1}{m_i} z_i$.

Then, the $x_1, x_2, y_3, ... y_n$ are still a basis and $x_1y_j = \begin{Bmatrix} 0 \\ or \\ x_1 \end{Bmatrix}$ all $i = 3, ..., n$.

Suppose at least *one* of the y_i's has the property that $x_1y_j \neq 0$ *i.e.* $x_1y_j = x_1$. Renumber everything so that this y_i is called y_3. Relabel y_3 as x_3. Now have

$$x_1, x_2, x_3, y_4, ..., y_n. \qquad x_1x_2 = x_1 \qquad x_1x_3 = x_1$$

All the x_3y_j's are in V^2 so $x_3y_j = k_jx_1$ $\quad (k_j \in F,\ j = 4, 5, ..., n)$.

Take $x_j = (y_j + k_jx_1)$. $\quad j = 4, 5, ..., n$.

Then $x_3xj \quad = x_3y_j + k_j.x_3x_1$

$\qquad = k_jx_1 - k_jx_1 \qquad$ (Remember, $ba = -ab$).

$\qquad = 0 \qquad\qquad j = 4, 5, ..., n$.

Also $(x_3x_i)xj \ + (x_ix_j)x_3 + (x_jx_3)x_i = 0 \qquad i, j > 3$.

$\quad 0 \quad + (x_ix_j)x_3 + \quad 0 \quad = 0$

$\quad \lambda x_1x_3 = 0$ (for some $\lambda \in F$ because $x_ix_j = \lambda x_1$).

$\quad \lambda x_1 = 0$

$\qquad\qquad \lambda = 0$.

In other words $x_ix_j = 0$ all $i, j > 3$, and we've proved $x_3x_j = 0$.

If *all* the y_i's have $x_1y_i = 0$ the above breaks down. Instead take $x_3 = y_3 + x_2$, then $x_1, x_2, x_3, y_4, ..., y_n$ are still a basis and $x_1x_3 = x_1(y_3 + y_2) = x_1x_3 + x_1x_2 = 0 + x_1 = x_1$ as needed.

To finish with here's David Asche's alternative.

Let V^2 have basis $\{x\}$. Define $S = \{v \in V: xv = 0\}$ (**Aside**: S is the centraliser of x in V: those clever people who've done M202 should be able to deduce why. Hint: remember what abelian means in Lie Algebras.) We are given that $y \in V$ with $xy \neq 0$ and as in the previous proof we can arrange that $xy = x$. Define $T = \{v \in S; yv = 0\}$. Define U to be the subspace spanned by x, y. Because products of elements of V are multiples of x, we know U is an ideal of V. T is also an ideal of V: - suppose $v \in T$ and $w \in V$. By the Jacobi identity $(vw)y + (wy)v + (yv)w = 0$. But $wy \in V^2 \therefore wy = \lambda x$ for some scalar λ and $v \in T \therefore yv = 0$. Hence $(vw)y + \lambda x.\ v = 0$ but $v \in T \subseteq S$, $\therefore xv = 0 \therefore (vw)y = 0 \therefore y(vw) = -(vw)y = 0 \therefore vw \in T$ and T is an ideal.

Also $U \cap T = \{0\}$ (proof is an exercise for the reader!) Now, suppose $v \in W$, $xv = \lambda x$ and $yv = \mu x$ for some λ, μ scalars. Then $\left.\begin{matrix} x(v - \lambda y + \mu x) = 0 \\ y(v - \lambda y + \mu x) = 0 \end{matrix}\right\}$ (work 'em out and see!)

$\therefore w = v - \lambda y + \mu x \in T \quad \therefore$ Every $v \in V$ can be written as $x = (-\mu)x + \lambda y = w$

i.e. an element of U + an element of T. Hence $V = U + T$ (direct sum).

Now suppose $s, t \in T$. Now st is a multiple of x, Hence $st \in U$ but T is an ideal, hence $st \in T \therefore st \in T \cap U = \{0\} \therefore st = 0 \therefore T$ is abelian. $\therefore$ a basis

$$x_1 = x,\ x_2 = y,\ \underbrace{x_3, ..., x_n}_{\text{any basis of } T}$$

is as required. [Clever ain't it!] Chris' proof was similar.

Well there it is - hangover finally dispelled and egg on my face again! There wasn't anything exciting in last year's sale but perhaps 1977 will bring more bargains and problems. Meanwhile here's a long standing problem from Don Mansfield. What is the smallest finite group G with a quotient group G/N *not* isomorphic to a subgroup of G? Pick the bones out of that!

(*Ed - See Problem* 40.2)

LETTERS

From Eric J Lamb: I noticed that you published my comments about last year's M201 exam in M500 39. This came as a bit of a surprise but now it is done I don't want to retract anything - except one detail.

It was the M201 not M202 exam.

From Sue White: Problems such as the 'Fireengines are red' one in M500 38 appear regularly in the monthly *Quiz Digest* bought by a crossword-fan friend from the local newsagent. This admittedly supplies the matrix which makes the solution simple, with the instructions "Record in this chart all the info obtained from the clues by using an × to indicate a definite no and a √ for a definite yes. This leaves the solution a purely mechanical process taking certainly not more than an hour.

MS - *If people have subs to such journals they should send problems along. Problems are sent in by members; the editor does NOT expect to have to go and search for them. Anyone using such material must quote the source in full if it is from a magazine, plus address of publisher. We already have blanket permission to quote things from Games and Puzzles, Creative Computing and Scientific American provided the source is quoted in full but otherwise permission should be rightly applied for every time material is used.*

From John Carter: To be honest I had no intention of renewing my subscription because I've not read an M500 magazine for 2½ years (three years a student). However, as Marion Stubbs well knows (bless her), it was MOUTHS and M500 that got me through the first three months of OU studies. An experience that still sends tremors down my spine at the thought of the trauma my miserable brain was subjected to on M100. M500 helped me simply by revealing that all but the geniuses of this world suffered Ml00 as I did. However after finally surfacing from that initial surfeit of mental excess I found little enough time to study (most of my time was spent doing assignments!) let alone read M500. Hence the earlier statement that M500 was not read but quickly despatched to the bottom drawer of my desk.

As a further consequence of this lack of familiarity with M500, and because I had now finished with the OU (what a relief) I had decided not to renew. But this demands some action, especially in the manner (after all M500 holds a special place in my affection) of parting. So I turned to issue 38 to find the address to write my farewell to. Surprise, three possible addresses, and a publisher, an editor and a membership secretary; my curiosity excited I flicked the pages. Yes this was a magazine that appealed to me. Although I missed Marion's stamp of authority in its pages, but that's a personal preference. (*Chortle, chortle! - M*)

The magazine has changed a lot since the old days (obvious statement) but has retained much of its charm. So please send me the 1977 copies and I'll undertake to finally start to read M500 regularly. Best of luck to you all in 1977.

From Michael McAree: As a new subscriber to M500 I would like to congratulate you on your ‘mixed bag of tricks’, it is this fact alone which appeals to me most. It seems to show that people can have fun with mathematics (I certainly can). This brings me to the comments made by Sue Davies (M500 39) when she complains about problems 38.1 and 38.3. Sue says that these problems were excessively easy; I think that this would depend upon one’s ability and insight. Now it seems that at least some of M500 subscribers (like myself) have only completed M100 and therefore cannot be expected to be as far advanced as some higher level students. On the other hand these higher level students could not expect to find these trivial problems stimulating. As a compromise then how about graded problems or even one elementary problem per issue for the lower level student.

From Richard Pinch: Although not myself a member of the OU, I came across an advertisement for M500 in a copy of *Sesame* (My parents are both Open students). I would be grateful if I could receive details of membership of the Society.

In another direction I wondered if there is a local branch of M500? Any members who live in the Cambridge area might be interested to know that there are no less than five college mathematical societies, as well as the University society which publishes an annual magazine which I edited a couple of years ago. These societies hold meetings, roughly fortnightly in university term - an average of about two a week - and would doubtless be willing to admit OU students as members. I am sure they would be willing but cannot be certain as I no longer have any formal connection except as a member. (Most of the college maths societies, by the way, have a very small subscription fee, that of the Trinity Maths Society is only 40p for three years.)

Perhaps you could give me your reactions to this idea?

R G E Pinch, past-President TMS, Trinity College, Cambridge.

From Roger Elton: Thank you for sending the copy of M500 and the application form which I have lost! I have a horrid feeling I filed it in my special file for OU letters that don’t need reading; sorry!

I am a retired farmer. I actually teach mathematics at the local high school modern to O-level and traditional to O- and A-level. So if I can be of help to M100 people I am more than willing, as I am the sort who does get hung up and battles late into the night with very small details that I cannot understand.

From P F Minch: I am taking M351 this year and would like to be in touch with any M351 students.

The Sidcup self-help Study Group (M351, Numerical Computation) will have its first meeting at 10.30am Saturday 9th April at Committee Room, B Block Reception, Queen Marys Hospital, Sidcup, Kent; 01 300 2678 ext 4323. Or contact me.

From Marion Stubbs: Of course M500 is a publication. The British Library demands our copyright deposit, but regrettably doesn't have copies of M500 1 - 28. They or it knows, however, that master files are maintained by M500 itself and by the Mathematics Faculty at Walton Hall. They can refer researchers, if necessary, to those files.

I wrote to *Sesame* with comments for other OU editors/publishers ref the British Library demand for legal deposit of printed publications, particularly since the OU Library itself refused to file M500 long ago saying that it could not cope with the mass of OU internal material. Other OU publishers may think that their publications are too trivial for the British Library too, but this is not the case. Law demands deposit of printed publications in the National Archives - and the OU itself did *not*, apparently, deposit copies of our two Special Issues which they published such that I had to supply them to the BL myself belatedly. *Sesame* has (so far) declined to publish my note.

From Nick Fraser: Enclosed is a mathematical crossword I received from an M100 student called Don Mitchell. Not his but given to him! So I don't know how you can credit it. I have completed it myself and found a calculator helpful but tables would probably do. *Ed - OK, see Problem* 40.3.

I received it just before M500 39 so it coincided with the request from Beryl Brayshaw. I tend to agree with her to an extent. But the answer is for simpler problems to be supplied for publication. Also there is a spread of requirements from M334 to M100.

From Melanie Folkes: I have recently joined M500 and was impressed by M500 38, although I am reading Arts subjects with the Open University.

I have a sixteen year old brother who is in his first year at a sixth form college doing A-levels, with a view of going to Cambridge to read a combined Maths and Physics degree. He fell upon M500 38 with great relish, in fact I thought I had lost it before I had even had a real go at it myself. Anyhow, on to my reason for writing. In the Problem section mention is made of the publication *Creative Computing*; my brother would be grateful for details of where he can obtain this magazine as he has recently fallen in love with a computer! Thanks.

Ed - All together now: *CC, 60 Porchester Road Southampton*!

From Mrs D V Parker: I signed on for M100 with a great deal of apprehension being very much the wrong side of 40. (My husband and teenage children reckon I'm suffering from early symptoms of senile decay!) Having received my first parcel bomb (those large brown envelopes) I got really worried. Then met my tutor/counsellor who made it all sound easy but I still felt I needed help so I sent for M500. Now I really am worried - all those problems! Ray Tiver's letter both worried and pleased me. I was relying on those tutors and he says they're no help. I like his remarks re women teachers (M500 31 8-9) - yes I am one. I'm sure I wouldn't be of any use to him as I only teach infants.

However I'm still sticking to it so here's my subscription and if I ever survive the course perhaps I can look back through my copies of M500 and make sense of something.

I'm sure I'm not going to be of any help to anybody but if anybody needs a shoulder to cry on I can perhaps supply that.

From Anne Dennett: I find myself agreeing with Beryl Brayshaw in M500 39 when she asks for a few simpler problems.

My maths background is very sparse, consisting as it does of RSA I & II arithmetic (what do you mean, you've never heard of it ?!) and despite successfully grappling with MST281 last year, most of the problems are way above my head. My ego gets a terrific boost when I can understand a problem, let alone solve it, and I am sure there are others who feel the same. Why not have a 'simple corner' for idiots like me?

PS (to Ed) I'm glad you still use Tippex as the enclosed seems to need a home - it was a surplus purchase!

From Deryk Jenkins: I am not really a mathematician but have to dabble. I'm hoping that M500 will stop me from getting my feet too wet.

My problems are inexhaustible - I shall certainly need the Weekend. I just scraped through MST281 and am now engaged on MST282 to be faced with the conversion from axiomatic to Leibniz calculus. Since I am anyway a newcomer to the art (craft, science, practice!) of calculus, vectors and matrices &c I need all the patient tutoring available. I hope to retrain as an aircraft designer eventually.

Herewith £5. I have also sent a cheque to Peter Weir by way of subscription &c. However I notice that both are payable to the same account. It seems a little untidy for two cheques to be required.

Ed - *Who cares about tidiness? Peter and Marion are both kneedeep in M500 work and cannot get on with their course work. Is it unreasonable to split labour and ask members to go to the trouble of writing two cheques*?

From Henry Jones (*received before* 39 - *Ed*): The cover design by Chris Pile on 38 is very pleasing and by far the best yet in my opinion, and 'Know your space' by Stanley Collings was very absorbing. Like John Owen I'm a Texas Calculator fan, though my instrument isn't as expensive as his. That edition of M500 was not too difficult for me to read right through, but I appreciate that articles on subjects in which I am not interested are keenly read by other more versatile readers.

Having now read all the Histories of Mathematics that are available or have been these last fifty years it is clear that my logarithmic calculus was not anticipated, this being confirmed by the publicity kindly given it by M500 and so on. (Ed - M500 14 15 *where we have* $\log_e y = \int 1.ly$ *where ly is the logarithmic complement of dy*.)

I am entering my logarithmic calculus and a companion thesis 'Under the veil of nature' in the Rolex Competition for Enterprize, a step which might appeal to other members of MOUTHS. No doubt the latter thesis will be very faulty to begin with. For example one provisional statement it makes is "Any two discrete bodies in space have the same gravitational momentum relative to each other, and the total gravitational momentum of either body relative to all other bodies in space is the sum of its momenta relative to each of these bodies". The phraze "relative to each other" is a little obscure. Moreover from this 'law' I can prove the validity of both the 'big bang' theory and the 'steady state' theory which is a little disconcerting. Ian Ketley and Max Bramer could tear me to pieces on it but I'll enjoy having a go.

PRISONERS AND MOUTHS

SUE DAVIES

I was disgusted to see the meagre list of only ten names which it is proposed to send to M500 members in closed institutions. I agree with Nick Fraser that an *Open* University society is the last place one would expect to find members divided into first and second class citizens. As I understand it our Constitution states that full membership of THE M500 SOCIETY (which includes receipt of the MOUTHS list) is open to all interested in OU mathematics. Are we not then legally bound by the terms of the Constitution to send the full MOUTHS list to anyone who wants it? Anyone who does not want their name and address to be available to all can either withdraw their name from the MOUTHS list or campaign to have the Constitution changed to suit their moral prejudices.

FORTHCOMING EVENTS

From The British Society For The History Of Mathematics, 18 Wolseley Road London N8.

April 2. 10.45am to 5.30pm THE HISTORY OF PROBABILITY AND STATISTICS.

Chris Lewin: Richard Witt and the early history of compound interest.

Dr Ian Sutherland (MRC) : The life table is not quite dead.

Professor Philip Holgate (Birkbeck): The role of gambling games in the history of probability.

Dr Donald MacKenzie (Edinburgh U) and Dr Bernard Norton (UCL): Karl Pearson and the biometric school.

May 25. 6pm to 8pm at the London School of Economics: THE HISTORY OF ACCOUNTANCY AND ACTUARIAL SCIENCE UP TO 1820.
The speakers will be Professor Basil Yamey (LSE) on accountancy and Mr Peter Cox CB on actuarial science.

The Hon Sec of the BHSM is J J Gray.

Which of us has not held a half warmed fish in his breast? - Anon.

OBITUARY - MR EBENEZER CUNNINGHAM

Mr Cunningham, who died early in February at the age of 95, was born in London 7v81. Educated at Owens School Islington he went on an open mathematical scholarship to St John's Cambridge. Senior Wrangler 1902; placed in the second division of the first class of Part II of the Mathematical Tripos in 1903; Smiths Prizeman 1904; Fellowship 1904; lecturer at Liverpool 1904; lecturer at UCL 1907; mathematical lecturer at St John's Cambridge 1911; retired 1946. While at St John's and after his retirement he held some administrative positions and later he took on Congregational duties. He was always a pacifist and at one time flirted with the Oxford Movement.

In 1914 he wrote *The Principle of Relativity* which was the first book in English to summarise the researches of Larmer, Lorentz and Einstein which became known as the Special Theory of Relativity. In this book he recognised the connections between the special theory and Maxwell's electromagnetism; but primarily explained for English readers the brilliance of Einstein's work of 1905, in which he had forever destroyed the belief of Kant in the objectivity of simultaneity. Later commentators, bemused by the term 'relativity', came to believe that all Einstein had noticed was that some measurements vary for different observers; or even that time in a dentist's chair goes slower than the same time in happier circumstances (both of which concepts were considered self evident even to Cro Magnon). Ebenezer Cunningham had it right.

SOLUTIONS

31.3 *COMBINATIONS - Prove that* $\binom{n}{r} = {}^nC_r = n!/(n-r)!r!\ \ n \geq r \geq 0$ *is an integer*.

Let $(x)^*$ be the greatest integer not greater than x then if p is prime the highest power of p which is a factor of $n!$ is p^x where

$$x = \left(\frac{n}{p}\right)^* + \left(\frac{n}{p^2}\right)^* + \left(\frac{n}{p^3}\right)^* + \dots .$$

Because, considering the integers 1, 2, 3, ... , n, $\left(\frac{n}{p}\right)^*$ of them are divisible by p, $\left(\frac{n}{p^2}\right)^*$ by p^2 and so on.

Now we have to show that $n!/(n-r)!r!$, is an integer which is equivalent to proving that any prime p which is a factor of $r!(n-r)!$ is present in $n!$ with at least as high an exponent.

Now $(a+b)^* \geq (a)^* + (b)^*$ for all real a, b so if p is a prime and k is any integer we may write

$$\left(\frac{n}{p^k}\right)^* \geq \left(\frac{r}{p^k}\right)^* + \left(\frac{n-r}{p^k}\right)^*$$

and allowing k to take all integral values 1, 2, ... , adding, and using the first result gives a proof that ${}^nC_r$ is integral.

Steve Murphy.

Ed - This is certainly a proof, but one would like to see something simpler. My own thought on this subject starts like, this; Since $n \geq r$. then $n = r + j$ so ${}^nC_r = {}^{r+j}C_r = (r+j)!/(r+j-r)!r! = (r+j)!/j!r! = (r+j)(r+j-1)\dots(r+1)/j!$ which is an integer if the product of n successive terms is always divisible by $n!$ i.e. we must show that $\forall q,\ n! \mid (n+q)!/q!$

38.4 *THE ECCENTRIC CARPENTER* : *A carpenter adds legs at random on the circumference of a circular table. On average how many legs must he add before the table will stand up without toppling over*?

Three people tackled this one: Steve Murphy, Stephen Ainley and Krysia Broda. They all obtained the answer 5 although Stephen Ainley's method showed that half the time 4 legs will just suffice. I give Krysia's because it is the shortest and will retain the others on file.

Assume $n \geq 3$. Let the table balance when the nth leg is added but not before. Let the nearest two legs to the left and right make angles of θ and ϕ respectively with the nth leg. We have $\theta + \phi \geq ½$ else table would have balanced before.

Prob(table balances with n legs given θ and ϕ) = $(1 - \theta - \phi)^{n-3}(n-1)(n-2)$ since the $n - 3$ legs all lie in angle $1 - \theta - \phi$, and there are $(n-1)(n-2)$ possible configurations.

Adding up the contributions for all possible values of θ and ϕ we have Prob(table balancing for first time on n legs)

$$= \int_0^{½}\int_{½-\theta}^{½}(1-\theta-\phi)^{n-3}(n-1)(n-2)\,d\phi d\theta = \int_0^{½}\frac{-(1-\theta-\phi)^{n-2}}{(n-2)}(n-1)(n-2)\,\Big|_{½-\theta}^{½}\,d\theta$$

$$= \int_0^{½}((\theta-½)^{n-2}+(½)^{n-2})(n-1)\,d\theta = ((\theta-½)^{n-1}+(½)^{n-2}.\,\theta(n-1))\Big|_0^{½}$$

$$= (-½)^{n-1} + (½)^{n-1}(n-1) = (½)^{n-1}(n-2);\quad n \geq 3.$$

Therefore the average number of legs to be added is $\sum_{n=3}^{\infty}\frac{n(n-2)}{2^{n-1}} = 5$.

37.2 *PAIRINGS*: *There are* $2N$ *teams in a competition. Is it possible to arrange all possible pairings between them in only* $2N - 1$ *rounds*? *If so prove it. If not give the correct formula for the number of rounds needed.*

Give a constructive method (*an algorithm*) *for generating pairings in each round - how many different possible choices are there*?

David Asche - Here is my very partial solution.

There are $\binom{2N}{2} = (2N-1)N$ pairs. In each round we want each player to compete exactly once, so we need a $(2N-1) \times N$ matrix of pairs in which each integer from 1 to $2N$ occurs exactly once in some pair in each row.

For N=1 we have simply (1,2) and for N = 2 we get the matrix

$$\begin{pmatrix} (1,2) & (3,4) \\ (1,3) & (2,4) \\ (1,4) & (2,3) \end{pmatrix}.$$

What I am producing here is a construction for getting the required pairing for $4N$ players when the solution for $2N$ players is known. In particular it gives a solution for 2^m players since we have one for $N = 1$.

The construction is really very simple. Suppose we have the matrix for $2N$ players. Whenever we have a pair (i, j) we replace it by a 2×2 matrix:

$$\begin{pmatrix} (2i-1, 2j-1) & (2i, 2j) \\ (2i-1,\ 2j\ ) & (2i, 2j-1) \end{pmatrix}.$$

This gives a matrix of size $2(N-1) \times 4N$ which is one row short. So we put at the top the row (1, 2), (3 ,4), ..., $(4N-1, 4N)$. This gives us the required matrix of size $(2(2N)-1) \times 2(2N)$.

I won't burden you with a proof that this works. It is not difficult to find a solution for $N = 3$ and you can use this method to get one for $N = 6$.

The problem is thus reduced to finding solutions to the case of N odd. I expect that this problem has been thoroughly investigated so I doubt if this incomplete solution is worth putting into M500.

39.2 *WHAT'S INTERESTING ABOUT* 1977?

Two replies so far: Michael McAree says $1977 = 1^3 + 2^3 + 5^3 + 8^3 + 11^3$ which almost ascends in threes. (We should have had this problem last year - Ed)

Michael Masters: 1977 can be expressed as the sum of four squares in two different ways. $1977 = 44^2 + 6^2 + 2^2 + 1^2 = 31^2 + 30^2 + 10^2 + 4^2$.

39. 4 *TRUTH*: *A cynic suggested to me that* VERSATILITY is 1001 *times better than* VERACITY. *On this reckoning what is the value of* RELATIVITY?

RELATIVITY = 9410782876; David Asche (who asks "are there any other codes which work?")

9410237326: C S Evans.
9430126218: Ray Pountney.
0238541457: Sue Davies ("I don't particularly like the leading zero. I expect there are other solutions but I couldn't find any method of getting them other than trial and much error") and Steve Ainley.
8761903092: Michael McAree.

(Ed - this solution appears in 40 instead of waiting the customary two issues because of the variety of the replies. Perhaps someone would like to investigate how many solutions there are and how one would find them. See Problem 40.4.)

PROBLEMS

40.1 *DOTTO* Jeremy Humphries (see page 2.)

a) Draw 16 straight lines through 81 points in a square array (9×9) so that every point is on one and only one line except that two lines may meet (but not cross) on a point.

b) Draw $2n - 2$ lines through an $n \times n$ array, $n \in \{1, 2, ..., 8\}$ differing intrinsically from those shown on page 2.

c) Produce some theory. i.e. "is $2n - 2$ necessary / sufficient for all n?"

(Ed - $2n - 1$ is obviously sufficient but not necessary whereas $2n - 2$ is insufficient for $n < 3$. It would help if solutions to (a) and (b) were drawn in black ink on white unlined paper with the dots *no more* (i.e. *less* if possible) than ¼″ apart.)

40.2 *THE NON-ISOMORPHISM THEORY* - Don Mansfield. (See page 9.)

a) Find a finite group G with a quotient group G/N not isomorphic to a subgroup of G.
b) What is the smallest such group?

40.3 *EXPLAIN* - Alan Slomson

Get out your calculators and explain what is special about the following two numbers:
a) 56.96124842... ,
b) 0.692200628... .

(Ed - Sorry about the crossnumber promised for this spot. There just wasn't enough room in this issue but it should be alright for 41.)

40.4 *RELATIVE TRUTH*

If VERSATILITY = VERACITY $\times$ 1001 how many values has RELATIVITY? How do you find them?

40.5 *SAFETY* - M Hodgson

a) Place all the white pieces of a chess set on a board in such a way that no piece threatens another.
b) How many non-trivially-different solutions are there?

EDITORIAL

Some people complain that there is not enough mathematics in M500 while others think it is too difficult. One tries to keep a reasonable balance between chat and maths but actual letters ought to have priority because they are likely to be ephemeral.

There is no point trying to read a condensed mathematical argument in the way you attack the morning paper. If you are interested in the subject write it out for yourself and see if each step is justified. When the argument seems to break down ring someone; that is one of the things the MOUTHS list is for. You could even perhaps ring the author - he might be as confused as you are and you will both gain something.

I believe it was Professor Halmos in a television interview who said "Mathematics is not a spectator sport", and it certainly cannot be in a magazine this size. There has to be some condensing to get all the material in; although if enough people ask, I can go over to double spacing for mathematics. In fact I might be grateful to be able to do so later in the year when material begins to get sparse.

And may I point out once more that I can only put in what you send. If everyone sent in a problem he felt *he* could do I wouldn't have to fill up with famous unsolved problems or trivia.

To those who responded to my note in 38-editorial about maximising $n^{1/n}$ I can only beg their indulgence for another month; space dictates!

Eddie Kent