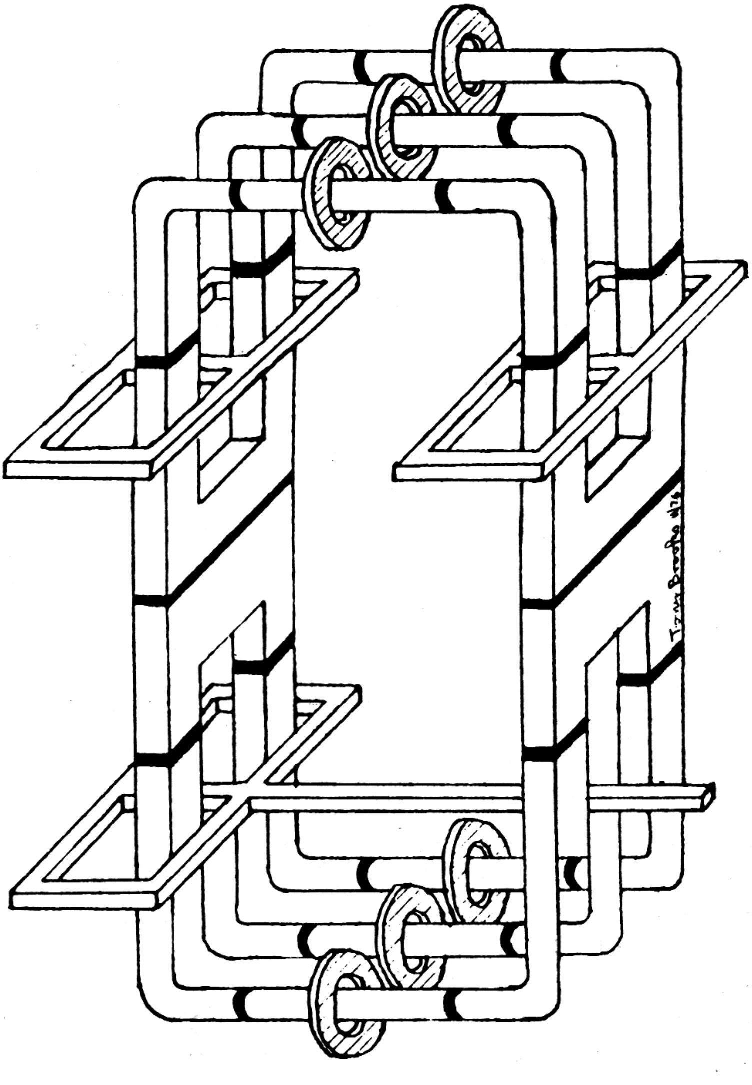

M500 41

M500 *is a student operated and owned magazine for Open University mathematics students and staff and friends. It it designed primarily to alleviate academic isolation of students by providing a forum for public discussion of the mathematical interests of readers.*

Articles and solutions are not necessarily correct but invite criticism and comment. Anything submitted for publication should normally be not more than 600 *words in length; or it may be split into instalments.*

MOUTHS *is a list of names addresses and telephone numbers, with previous and present courses, of voluntary members, by means oj which private contacts may be made - to share OU and general mathematical interests, or to form self-help groups by telephone or correspondence.*

THERE IS ALSO A SPECIAL LIST OF A SUBSET OF MOUTHS MEMBERS WHO HAVE EXPLICITLY VOLUNTEERED FOR THEIR MOUTHS DETAILS TO BE DISTRIBUTED TO MEMBERS IN CLOSED INSTITUTIONS, SUCH AS PRISONS

The views and mathematical abilities expressed in M500 are those of the authors and may not represent either those of the editor or the Open University.

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ANYTHING SENT TO ANY OFFICER OF THE SOCIETY WILL BE CONSIDERED FOR POSSIBLE INCLUSION IN M500 UNLESS OTHERWISE SPECIFIED

FUGUE RICHARD AHRENS

On a remark of Eddie Kent.

On page 15 of M500 39 Eddie gives the solution of Problem 34.4 (ii) - *Find the next term in the sequence* 1 2 2 1 1 2 1 2 2 1 2 2 1 2 2 1 1 2 1 1 ... . This sequence contains its own description in the sense that if we write down the lengths of the blocks as they occur we find we are writing the same sequence again.

```
1   22   11   2   1   22   1   22   11   2   11   ...
1   2    2    1   1   2    1   2    2    1   2    ...
```

We have reached the eleventh term in the second line - the twelfth term in the first line is 2 so we must continue the first line with a double symbol, namely double 2. Thus we can continue our sequence indefinitely. (Ed - *the property of self-generation was spotted by Anne Williams*.)

Eddie asks "When does the sequence start to repeat itself?" Presumably, since he worded the question as he did, Eddie would be surprised to learn that this sequence never becomes periodic. (Unless he was making a sneak attack on the computing types who cannot possibly pruduce the right answer here.)

PROOF Suppose the sequence does eventually start repeating itself with shortest cycle length l. Now write down the sequence of block lengths as we did above. This new sequence must start repeating itself when it is describing the periodic part of our original sequence but its repeating cycle will be shorter than l if the first sequence contains any double in its repeating cycle. But the new sequence is the old sequence so it cannot cycle with a shorter period. Hence our assumption of periodicity must have been false. (The possibility of no doubles in the repeating cycle is easily seen to be ridiculous.)

* * * * *

I have just been reading a fascinating article about non-periodic tilings of the plane in the January 1977 issue of *Scientific American* and this sequence seemed to be a one-dimensional analogy.

I propose the following rather long-winded definition:-

Let T be the set of doubly infinite sequences (i.e. infinite to both right and left) consisting of 1's and 2's. Let f be the function which maps any sequence in T to its description in terms of block lengths as we did above. Clearly T is not mapped into itself by f. e.g. if

s = ...   2222111221221112222...,

$f(s)$ = ...   4   3   2 12 3   4 .... .

Now let S be the largest subset of T which *is* mapped to itself by f. So S consists of' sequences of 1's and 2's and for any $s \in S$, $f(s) \in S$ also. S is not empty because the following sequence is in S:-

p = ...2112212212112212211212212 2112...
↑

You will recognise this as our original sequence from the arrow to the right and the mirror image of this sequence to the left. It is easy to see that $f(p) = p$ so $p \in S$. Here is a list of problems and conjectures about this set S.

Problem 41.1 *FUGUE*

1) Prove that no sequence in *S* is periodic or contains an infinite periodic segment.

2) The sequence *p* above is a description of itself but it is also the description of

$$q = ...122112112122112112212112112 21...$$
↑

i.e. $f(q) = p$ so *q* is also in *S*. Prove that *q* is not equal to *p*. (You must show that *q* cannot be made to coincide with *p* by a translation.)

3) Find infinitely many distinct elements of *S*.

4) Find one or more sequences in *S* which do not get mapped to *p* by repeated applications of *f*. i.e. a sequence $r \in S$ such that $f \circ f \circ f \circ ... \circ f(r)$ is never *p*.

CONJECTURE Any finite segment of a sequence in *S* occurs infinitely often in every sequence of *S*. (If this is true then it will never be possible to decide if two sequences in *S* are identical or not by examining a finite piece.)

CLANK ON BILL MIDGLEY

Oh, but surely the housewife in the supermarket is not simply multiplying six by eightpence halfpenny. She is trying to find out whether she ought to buy 81b 13 oz of soap powder for £2.30 or several smaller packets each containing 23¼ oz at 38½p. It is not always the giant economy size which is the better buy. The calculator is a useful weapon in the unending fight between ‘Us’ and ‘Them’.

The tiler is in an even worse situation. Time was, when the Sun never set on the British Empire, that there were thirty six six inch tiles to the yard super. But that was before the simple life brought about by metrication. Now tiles are fifteen centimetres (or rather 150mm) square and they are sold by the square metre. So you get 44 and lose a bit on the deal.

All of which has little to do with what Sue Davies was writing about. “I only do it to annoy ... .” My own aversion to running out and buying (or ‘investing in’ as the modern marketing experts say) a calculator was based on the fact that they were being improved all the time and the price kept falling. I had just got around to the idea of waiting till they were given away with the cornflakes when a globe-trotting brother-in-law brought me one all the way from Hong Kong. A CASIO Pocket Mini. Straight-forward eight digit job with square root and no memory. People ask me if it will ‘do’ percentages and I say it will and they look in vain for a percent key and I tell them to divide by a hundred and multiply by the number they first thought of and they look at me as though I were barmy.

I have a feeling that people like to chat about their calculators in the same way that other people like to go on about their cats, and what they will and will not do. Now my calculator sits on the table at meal times, goes with me all over the place - even to work, *never* stays out at night and won’t eat Kit-e-kat under *any* circumstances.

Some people think that not having a memory is a disadvantage. However I get by. I haven't got much of a memory myself. I find that I can use a pencil to write down a set of figures on a piece of paper. I like to use an HB pencil for this, sharpened to a fine point. At one time I preferred to use a 2B pencil but I found that I had to keep sharpening it. My favourite paper is Plus Fabric Bond despite its merry slogan 'The Paper with the Crisper Whisper'.

But what I am really looking for is a calculator which will tell me when I have pressed the wrong button.

PATSY AND PETER

PETER WEIR

Patsy is thirteen and goes to school in Harrow. Peter is older and works with Patsy's sister.

Peter, in OU maths courses, has studied integer programming (M351), group theory (M202), computers and programming (M251) and matrices and transforms (M201). Peter has not studied envelopes, though he knows they are in the University of Warwick MSc syllabus.

Patsy studies all these things this year, though not quite as deeply as Peter did.

This has shocked Peter somewhat. Poor old Patsy has missed out on all that exciting geometry that Peter so enjoyed for four terms. No matter; you can't have everything.

However, if you ask Patsy or one of her classmates to divide 36 into 808 she wouldn't know where to start. Likewise if you asked her which is the better value of 15oz for 34p or 21oz for 47p she wouldn't really know where to begin. Interesting. Peter con do these things if you hide the whisky.

Patsy doesn't do to well at maths. None of her teachers studied the maths that they teach her while at school themselves. Patsy's sister has to struggle hard to help Patsy with queries Patsy has. In maths Patsy is in the second stream out of five at a large comprehensive school.

Is Patsy typical? Is mathematical education being fair to her?

Footnote - from *The Times* January 17 1977, PHS;

A Crowborough reader has discovered one of the weirder consequences of metrication. The familiar size ten knitting needles, she finds, have, under the allegedly simplified system, become size $3\frac{3}{4}$.

SQUARE ROOTS

MICHAEL MASTERS

The answer to Peter Weir's problem (39 5) has been solved before and is known variously as compound interest, discounted cash flow and more recently, inflation accounting. (The problem was, *from five figures at yearly intervals calculate an average percentage increase which if applied four times to the first figure would have the same final total. The yearly figures were* 100, 110 (+ 10%), 121 (+ 10%), 140 (+ 15.7%) *and* 147 (+ 5%).)

The general formula is $100 \times \left(\sqrt[N]{\frac{\text{end sum}}{\text{start sum}}} - 1\right)$ where *N* is the period. The answer in Peter's case is 10.111% pa.

PROGRESS REPORT - SQUARE ROOT COMPETITION

PETER WEIR

Entries have been slowly gushing in in response to the competition published in M500 39. So far (March 5) attempts from Cyril Whitehead, Peggy Adamson, Dennis Hendley, Melanie Folkes, G V E Thompson, Penelope Taylor, Russell Devitt and our Eddie.

I will prepare the prize list about ten days after this issue is published, so if you haven't sent one in yet, hurry! You too could win a secondhand 2p stamp (only licked once) or whatever it is I choose for first prize.

Entries to me, Peter Weir.

PS, I haven't received any bribes yet. Hint, hint.

Ed - *For those who didn't see* 39, the *competition went like this: Write a routine* (*in English, a flowchart,* OU BASIC *or anything*) *to work out the square root of a real number from* 0 to 20 000 *to an accuracy* of 0.002, *using only add, subtract, multiply and divide.*

LETTERS

Malcolm A James : I feel I must write to you to say how much I agree with Beryl Brayshaw's letter in M500 397.

I have just begun M100 so can not speak from as much experience as she has to offer. I almost decided against joining THE M500 S0CIETY and the MOUTHS list, as reading through the sample copy I found there were several articles which might just as well have been written in Chinese as I did not understand the idea behind them. I am sure that many others would have been literally blinded by science and as bewildered as I was if they were in the same position as me. One reason why I joined was when I have obtained my degree I can look at old M500 magazines and with a bit of luck would be able to understand articles which are obviously written by more advanced mathematicians than myself.

May I suggest that one or two pages could be devoted to beginners in every issue? Some periodicals issued by professional institutes usually have an article written especially for students.

Having said all this I think it better to get off the soap box now! I was interested, however, in problem 39.5: Ethiopian Multiplication - not because I have any idea how to solve it but it reminded me of another problem which I first came across while still at school.

Can anyone explain how the Romans carried out simple arithmetic calculations with letters such as I, V, X, L, C, D and M? It seems they must have been able to do this, bearing in mind their magnificent buildings, collonades, aqueducts and other engineering structures which obviously called f or a high standard of mathematics.

Ray Partridge.: This is my first as a new member this month.

Has anyone who has previously to this year taken MDT241 any advice regarding the instruction or advice by the course team that purchase of a calculator is necessary?

They list CBM899D, CBM9R25, CBMSR7919D, CBM1800D, CBM1800R, CBM4148F, Texas 2250II, Texas TI30, Rockwell 24RDII and Rockwell 24KII. Could somebody say which to buy? I don’t mind higher price if it is going to bring useful features. I shall be involved with third level statistics (and non-OU physics).

Tony Brooks. In M500 39 8 Sidney Silverstone requests information about OU higher degrees. I have gone through the process of applying for an OU higher degree and I may be able to help others considering such a step.

I should add that I applied to do a higher degree in philosophy and not mathematics.

Sue Harling : M500 seems well worth reading, when I can get it away from my husband who keeps spiriting it off to work.

Willem van der Eyken. For those who, like myself, spend that in-between time from November to February making good resolutions about reading all the units we missed on the course and thus mastering the mysteries of calculus:

I have been using *Quick Calculus* by Daniel Kleppner and Norman Ramsey, from MIT and Harvard (good stable?), a programmed text which takes you, in about 400 frames, through all of M100 Units 7–15, and through quite a bit of M231. Using Leibniz throughout I found it a refreshing change and a confidence booster, and was surprised at the speed with which one can whip through the program. It has certainly helped with all those unresolved areas like logs, exponentials, integrating trigonometric functions &c which one is just supposed to know, but which I for one actually did not know. So I offer it - at £3.05 a throw - as one possible revision excercise. Or there are all those problems in Spivak which one didn’t quite get round to of course --- .

Ken May (*In reply to Graham Read’s request for feedback on* M231 *exam*; M500 39 8): To date I have acquired four full credits in mathematics and it was in 1975 that I studied M231 and M332. Some people were a little concerned about my studying the two analysis courses in the same year, but in all honesty I did not find that I was at a disadvantage by doing so. Although it has been said that analysis is a subject to be studied once and once only, all agreed that it should be contained in any serious course study in mathematics. In spite of this remark I found the course very interesting.

Like most people I do not shine at examinations. I need time to think and three hours is just not long enough - three days would be more acceptable. To prove the point, my aveeage mark for TMAs of four full maths credits is 8.23 (for M231 it was 8.59 and for M332 it was 8.33), yet I have only once attained the dizzy heights of grade 2 - surprisingly it was for M231. For all other courses I received a grade 3. I accept my fate - I am a slow thinking man. In my opinion therefore there is no justification for complaints - it is simply not true that M231 examination is a higher hurdle than other examinations. For those who think it is I suggest that they try M202 for size.

I would like to take this opportunity of congratulating Graham Read and all members of the Maths Faculty for the tremendous effort, foresight and ingenuity that has obviously been put into preparing these courses. I for one have been enriched and thank you all for giving me the benefits of your experience.

Joanna Burnet: I am currently tackling MDT241, TM221 and S266 and aim to finish with S333, S334 and M351 in 1978 - any help or advice on these would be gratefully received, particularly as we return to Germany in August this year and I will therefore be TV-less, Tutorialless and Computerless for the rest of my OU career.

I am the mother of four hefty sons, 14, 12, 10 and 8. We all ski madly, mostly for pleasure; and grass ski every weekend. We have a small-holding with pigs, calves, turkeys, ducks, geese and hens and in my spare time (apart from the OU) I teach at the local Grammar School - Maths (traditional), Geography, History and Drama.

PRONUNCIATION

C F WRIGHT

I quote some lines from M500 32 5 by John A Wills. ‘During the year OU-1 I heard an S100 lecturer trying to German “Einstein”. Someone had told him that “stein” is pronounced “shtein” and extended the principle to Einstein. Goofed again: st is sht only at the beginning of words .... By pretending we demonstrate our ignorance and in no way improve communication.’

Alas, I think the one who has goofed, pretended and demonstrated his ignorance is Mr Wills, whom apparently ‘someone has told’ that st is sht only at the beginning of words, without explaining that for the purposes of this rule compound words of German origin are treated as two words. Let Mr Wills tune his wireless to a German station, ask a German (not someone from the north where even a word like stein is commonly pronounced st-) or refer to a reputable book on the subject, eg Griesbach and Schulz, *Grammatik der deutschen* Sprache, 1967 edition, page four: ‘st und sp werden ... lm Anlaut oder nach einer Vorsilbe scht und schp gesprochen: Stern, verstehen, ... ‘.

I am not saying that the received English pronunciation of the word is -sht- (on this point one might consult, say, the Oxford Illustrated Dictionary, 1975 edition), but only trying to correct a rather elementary howler about German pronunciation.

A BYWAY: BILL MIDGLEY

Have you noticed how the labling of paragraphs (a, b, c etc.) is creeping into speech? I hear it all the time on the wireless - only folk usually get it wrong and start “a ..., secondly...”. I used to spend a lot of time interviewing a few years ago and sort of ‘collected’ these tricks of speech. My best was:

“a ...

secondly ...

fourthly and lastly ...

And there’s one more point ...”

The trouble is, I tend to do it myself, as a joke, and then find I’m doing it all the time. Similarly I got some odd looks at Summer School when I came out with such remarks as “I could do with this Maths business but people keep expecting me to think!”

CONTINUOUS ASSESSMENT

DAVE DIPROSE

To give a reply to Max Bramer’s article on this subject in M500 39 6, I spend about six hours per unit on the CMA and TMA.

For a particular unit, before looking at the CMA/TMA I go through the correspondence text (and set reading passages) making brief notes. I also ‘look at’ the self assessment questions. For a trouble-free unit this takes about six hours and I end up with three sides of A4 in notes. This is the worst part of learning - the raw study - and I generally split the work up into sessions of ten pages of text.

I do the CMA and TMA trying to use only my notes. This is as important as the first part. I hope that by doing a good assessment I can help offset a bad exam. Also it is a sort of short term review. (In as much as I found Polya useless I found Tony Buzan’s book and television series Use your head fascinating - but I only stumbled on it by accident. I wonder Why the OU doesn’t make students more aware of series like his.) And it redirects me if I have not understood the importance of some ideas in the unit. Finally it gives the enjoyment of actually solving problems.

I think this 50:50 split (for a trouble-free unit) is reasonable. I seem to recall from the Guide to the course that I should expect to spend 1½ hours on the assessed work associated with each unit so I am not the model student. But then I believe the continuous assessment is more important than the OU does.

STATUS

BRITISH COMPUTER SOCIETY

Here is the text of a letter received from Walton Hall, signed by S R Wallwin pp F B Louis.

There have been joint discussions between the Open University Mathematics Faculty and the British Computer Society for some time now, seeking inter alia, to obtain exemption from the Society's Part I Exemption for Open University students who have succeeded in a suitable combination of courses.

Accordingly, I should very much like to locate just one student who has passed PM951 *or* M251; TM221; and M351 (or who will have achieved this combination by the end of the current year) - and who would be willing to apply for BCS Part I Exemption, thus setting up a test case.

(Telephone: Direct line, Milton Keyes (0908) 6-3241.)

INSTITUTE OF MATHEMATICS AND ITS APPLICATIONS

Tony Brooks

Readers of M500 may be interested to know that I have recently been elected a graduate member of the Institute of Mathematics and its Applications on the basis of my OU honours degree.

Graduate membership of the institute is open to persons with a good honours degree in mathematics (or with mathematics as at least the main topic). It is clear from my acceptance that the Institute is willing to consider people with OU degrees; however they will only consider people on a case-by-case basis and are not prepared to issue any general guidelines for OU graduates.

Readers with OU ordinary degrees could try applying for the grade of Licentiate membership.

Further details of the Instutute of Mathematics, and of its meetings and publications can be obtained from: The Institute of Mathematics and its Applications, Maitland House, Warrior Square, Southend-on-Sea, Essex, SSI 2JY; telephone Southend-on-Sea 612177. Or readers can contact me.

LEIBNIZ THE LIBRARIAN

MILADA MITCHELL

In 1677, at the age of 21, Leibniz travelled to Nuremburg where he met Baron von Boineburg, statesman and scholar, who had an excellent library and who employed him as secretary, literary assistant and ‘librarian’. They lived first at Mainz, and then Frankfurt. Leibniz soon prepared a systematic subject catalogue of his books which was intended as an index to the contents of the whole library.

From 1670 Leibniz travelled, met Louis XIV invented the calculus, was elected FRS and, as librarian to the Duke of Brunswick-Luneburg, purchased the classified library of the physician and scholar Martin Vogel for 2000 thalers. He participated in state affairs and was involved in disputes and did more travelling: through Bavaria, Franconia, Swabia and other German provinces, to Vienna, then Venice Modena and Rome, examining books and manuscripts, archives and libraries. While in Rome he was offered the position of Keepership of the Vatican Library but declined because he was a Protestant.

About 1690 he was appointed librian to the ducal library at Wolfenbuttel, starting with 30 000 volumes. He retained this position till his death. Leibniz invented many schemes to increase the library funds - stamped paper and silkworms for instance - but soon ceased because of lack of encouragement and instead sold duplicate copies to raise money. He obtained high employment and expanded the library, paying 157 thalers for Lucius’s library and 2400 thalers for that of Marquand Gaudius.

Leibniz valued books by their merit as literature and set great value on pamphlets and dissertations, which often contained good and original work. He objected to ephemeral books. His system of classification was: 1. Theology; 2. Jurisprudence; 3. Medicine; 4. Intellectual philosophy; 5. Mathematics; 6. Physics; 7. Philology and literature; 8. Civil history; 9. Literary history and bibliography; 10. Collected works and miscellania. This was intended for the division of books in a library according to their subject, upon a simple and convenient method with no pretension to a logical sequence. His ideal was a complete, well-administered book collection with a strictly scholastic classification. He favoured an alphabetical catalogue and also chronological arrangement by year of publication, with subject indexes arranged by keyword. Whilst managing to control the library closely he was most generous in lending books to scholars.

Another item from *The Times*: Diary, Tuesday 15 February 1977, “The latest issue of *Computing* advertises a forthcoming article which asks whether its readers still feel as wealthy as they did four years ago. ‘It would not be surprising if you were beginning to feel the pinch, as the salaries of people have slipped behind those of other members of the population. Dave Grayston examines this unsettling trend.’ Unsettling indeed.”

M331: A COURSE FOR EVERYONE

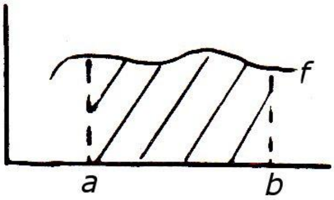
ALLAN SOLOMON

Like Caesar's Gaul, mathematics seems to be divided into three parts, Pure mathematics, Applied mathematics, and the rest. One can then separate Pure mathematics into Algebra and Analysis, among other things. And so on. It all seems very well organized. I suppose on this basis a course on Lebesgue Integration would fall neatly into the *Pure Mathematics*: *Analysis* slot. But does that give the whole picture? In many ways such a classification is both invidious and misleading. A lot depends on the importance - I know this is subjective but what the hell - and centrality of the material, on how well the field fits into the whole of Natural Philosophy.

An argument, analogous to that for the calculus, can be given to justify the centrality of Lebesgue Integration in much of today's mathematics and physics. This probably seems surprising: after all, we all have some Calculus but most of us have never even heard of Legesgue Integration. The reason is somewhat paradoxical: whenever the concept of integral occurs in modern mathematics and physics it is invariably the *Lebegue* integral which is meant, so the qualifying term is completely unnecessary.

First of all, what is the Lebesgue Integral and how does it differ from the elementary idea of an integral? The surprising answer is that the Lebesgue Integral *is* the elementary idea of an integral - it doesn't differ at all! Let me clarify that by reminding you of what the elementary idea of an integral is.

fig (1)



Geometrically if we draw the graph of a function f mapping real numbers to real numbers, then we may define the integral of f from a to b, $\int_a^b f$, as the shaded region - the area under the curve. This is the Lebesgue Integral of f (from a to b). You will recognise this as the basic pictorial description of the concept of the integral. The trouble with mathematics is that pictures aren't usually enough, and we need to give a formal analytical description of the process of finding the area in order to be able to evaluate it, or even to say which functions f 'have an area' in this sense and which do not. This is where the various techniques and definitions come in.

In the Lebesgue case (figure 2) we do the obvious thing: we approximate the area by a sum of rectangles - whose area is clearly easily obtainable - and take the limit. In the rather more complicated Riemann case (figure 3) we approximate both by rectangles lying under the curve as in the Lebesgue case, and also by the over-approximation given by rectangles lying above. We then proceed to the common limit, if it exists.

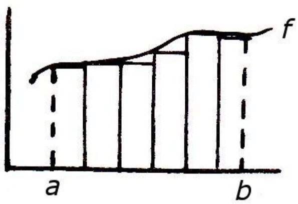


fig (2)

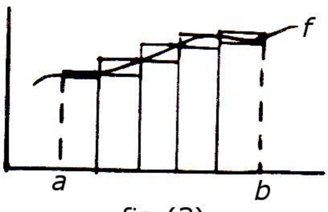


fig (3)

So the idea of integral is, like that of weight, pretty basic. And if you buy a kilogram of meat then you probably have a good idea of what you're going to get. But in order to evaluate the weight, i.e. weigh the meat, you can use a butcher's scales or a chemical balance. Both will give you the same answer to within their respective accuracies. But if you need to distinguish between two pieces of meat differing by a milligram you use the chemical balance. The Lebesgue integral turns out to be the chemical balance and the Riemann integral the butcher's scales. Every function which has a Riemann integral possesses a Lebesgue integral; the converse is not true.

But there is another reason why the Lebesgue integral has supplanted the Riemann integral in modern mathematics and physics, other than its greater generality, and this brings me back to my analogy with the differential calculus. When you want to differentiate a function it is very important that the function should be defined on the real number line $\mathbb{R}$, as distinct from on the rationals $\mathbb{Q}$, say. This is because differentiation involves taking a limit, and $\mathbb{R}$ behaves very satisfactorially with respect to taking limits, unlike $\mathbb{Q}$ which is full of 'holes'. We say that $\mathbb{R}$ is *complete* and $\mathbb{Q}$ is not. Now $\mathbb{R}$ is a space of points. Perhaps the most fruitful idea in the whole of mathematics is that of a *function*, so it is natural to go on to consider *spaces of functions*. And if one uses spaces of Lebesgue integrable functions it turns out that the spaces have this beautiful property of being *complete*, just like $\mathbb{R}$ while the analogous spaces in the case of the Riemann integral are more like $\mathbb{Q}$. It is this property of completeness that makes the Lebesgue integral the indispensable tool of, for example, quantum mechanics.

This brings me to the final and most difficult question: If the Lebesgue integral is so beautiful, so important, so easy, how come it's not taught in every first course in calculus? I don't know. The answer may be historic - after all Riemann came before Lebesgue. Certainly one of the aims of Alan Weir, the author of our set book, was to make the Lebesgue integral accessible to a student as his *first* introduction to integration theory. Perhaps there is a trend in this direction. In any case I hope what I've said will encourage you to read M331 for yourselves where you will find lengthier - and more accurate! - discussions of its properties than I've been able to give here. And if for some reason or other the delights of M331 leave you cold I hope that I've been able to take some of the mystery out of the Lebesgue Integral in this note.

CoRA ON 196

SUE HARLING

In the editorial of M500 38 you asked for someone with access to a computer to run CoRA on 196. My husband, who insists on reading M500 as soon as it arrives (a compliment for M500 if annoying for me!) spent some time doing as you requested and here are his comments:

1. If a palindromic number exists it is *big*. (That means larger than 10^{500} - at which point my computer blew itself to bits.)

2. The 400th iteration produces a number which starts 370..., has 171 more boring digits before finishing as ...963.

3. So what?

Ed - *CoRA is Continuous Reversed Addition, described in* M500 35 1. *Take a number, reverse it, add the two together: reverse the sum and add the two new numbers, and so on. I hypothesised that this process will always produce a palindrome - because it always had for me and anyway it looked likely. Well, I think the next thing is to look for conditions on numbers which stop them producing palindromes, if any.*

196→887→1675→7436→13783→52514→94039→187133→518914→938729→1866568→10523249→104755750→162313151→313626412→... .

CONGRUENCE CLASSES

JOHN READE

In M500 39 1 Bob Escolme has given a proof of the theorem

$$(m,n) = 1 \Rightarrow U_m \times U_n \approx U_{mn}$$

by exhibiting an isomorphism from $U_m \times U_n$ onto U_{mn}. I doubt whether there is a short indirect proof but I would like to offer a proof which is shorter and perhaps more direct than Bob's although it uses very much the same ingredients. In order to keep the notation simple I shall work with single representatives of congruence classes rather than with the classes themselves. Thus for any positive integer k, U_k denotes the group $\{x \in \mathbb{J}\colon 1 \le x \le k$ and $(x, k) = 1\}$, the group operation being multiplication mod k.

Let m and n be positive integers with $(m,n) = 1$. Let $U_m = \{a_0, a_1, ..., a_s\}$ and $U_n = \{b_0, b_1, ..., b_t\}$. For each pair (a_i, b_j) in $U_m \times U_n$ let r_{ij} denote the remainder of a_in+b_jm on division by mn ($0 \le r_{ij} < mn$). Clearly $(a_in + b_jm, mn) = 1$, so that $r_{ij} \in U_{mn}$. Thus $f(a_i,b_j) = r_{ij}$ defines a function from $U_m \times U_n$ into U_{mn}. f is one–one since $a_in + b_jm \equiv a_kn + b_lm \pmod{mn} \Rightarrow a_i = a_k \pmod m$ and $b_j = b_l \pmod n \Rightarrow a_i = a_k$ and $b_j = b_l$. For any $x \in U_{mn}$ there exist integers a and b such that $an + bm = x$ (this follows from $(m,n) =1$). Since $(x, mn) =1$, $(a,m) =1$ and $(b,n) =1$. Hence $x = a_in + b_jm \pmod{mn}$ for some $a_i \in U_m$ and some $b_j \in U_n$, so that $x = f(a_i, b_j)$. Thus f is one–one and onto. (This proves that $\phi(m)\phi(n) = \phi(mn)$, where ϕ is Euler's phi-function.)

Now let us assume that $r_{00} =1$, i.e. $a_0n \equiv 1 \pmod m$ and $b_0m \equiv 1 \pmod n$, and let us define $g\colon U_m \times U_n \to U_{mn}$ by $g(a_i,b_j) = r_{ij}^{(0)}$, (the remainder of $a_ia_0n + b_jb_0m$ on division by mn). Then g is also one–one and onto. Moreover, since $(a_ia_0n + b_jb_0m)(a_ka_0n + b_lb_0m) \equiv a_ia_ka_0^2n^2 + b_jb_lb_0^2m^2 \equiv a_ia_ka_0n + b_jb_lb_0m \pmod{mn}$, g is a homomorphism.

The progress of science is strewn, like an ancient desert trail, with the bleached skeletons of discarded theories which once seemed to possess eternal life.

Arthur Koestler: PEN opening address August 24 1976.

HOW MANY PRIMES?

BRIAN WOODGATE

Thinking about problem 28.5 (*Prove that the number of odd primes less than x is itself less than* $(x+2)/3$) the following developed.

Starting with a method of finding primes by dividing each integer by all primes up to its square root I reversed the process to take out all non-prime integers. The process follows in note form.

1. Reject all even numbers. 2 can be reinstated later.
2. Reject all numbers in the remaining half that are divisible by 3. Then reinsert 2 and 3. (We assume that 1 is not prime.) If P_N is the number of primes $\leq N$ then
$$P_N = N(1 - (½(1) + ½(½ \cdot 1))) + 2 = \frac{N}{3} + 2$$
as a first approximation.
3. $S_2 = ½(1)$; $S_3 = ½(1) + \tfrac{1}{3}(½ \cdot 1)$; $S_5 = S_3 + \tfrac{1}{5}(\tfrac{2}{3} \cdot ½ \cdot 1)$; etc.
4. Each value of S_i is useful up to the next prime squared; e.g., S_5 is used for $25 \leq N \leq 49$.
5. Also the number of series used must be added in to reinsert primes. Let K = number of series used.
6. So formula is $P_N = N(1 - S_i) + K$. (∗)
7. To find i and K : a. Take $\sqrt{N}$;
   b. greatest prime $\leq N = i$;
   c. number of primes $\leq i = K$.
8. Calculated values:

	S_i	$1 - S_i$	diff	K	valid up to
S_2	0.5000	0.5000	0.5000	1	9
S_3	0.6666	0.3334	0.1666	2	25
S_5	0.7334	0.2666	0.0667	3	49
S_7	0.7714	0.2286	0.0380	4	121
S_{11}	0.7922	0.2078	0.0208	5	169
S_{13}	0.8082	0.1918	0.0159	6	289
S_{17}	0.8195	0.1805	0.0112	7	361
S_{19}	0.8289	0.1710	0.0095	8	529
S_{23}	0.8364	0.1636	0.0074	9	841

9. Examples:

N	$\sqrt{N}$	i	K	i-Si	result	true P_N
20	4.47	3	2	0.3333	8	8
100	10	7	4	0.2286	27	25
500	22.4	19	8	0.1710	93	95
1000	31.6	31	11	0.1534	164	168

Hence the results seem a reasonable approximation.

10. With regard to larger values of N, calculation of S_i becomes difficult. We know that the quantity of primes is infinite, but that they become rarer as N increases. e.g. between 0 and 100 we have 25, between 1000 and 1100 we have 16, between 10000 and 10100 we have 11 and between 100 000 and 100 100, only 5.
11. I suggest a rough formula may be found in the form $P_N = ae^{-bx}$ for some a and b. As a first approximation try $1 - S_i = e^{-0.6 \log 10N}$.

Therefore $P_N = N \exp(-0.6 \log_{10} N)$. (†)

I have read that a possibility is $P_N = N/\ln N$. (††)

12. Comparison:

N	P_N by *	P_N by †	P_N by ††	true P_N
10^1	5	5	4	4
10^2	27	30	23	25
10^3	164	165	145	168
10^4		907	1085	
10^5		4980	8685	
10^6		27300	72411	

13. I realise the above is elementary, but perhaps interesting. Can someone suggest a readable book on prime number theory?

Ed - *I have something called* The factor book *which gives 'prime factorisation* 1 – 100000'. *It is by R L Hubbard BSc CEng MICE and someone has typed the whole lot out and printed from it. I shudder to look at it, but it does give, among other things, cumulative numbers of primes so I am able to fill in some gaps from the table above*: P_{10000} = 1229 *and* P_{100000} = 9592.

PRISONERS AND MOUTHS

Dear Marion, Thank you for your letters and the stationary. My M100 colleague here at the hospital says that he is writing to Peter Weir to give his name and address. I have passed on to him the literature you sent for him.

I am afraid that the reason I have been slow in replying to you is due to shyness and laziness. Before coming to this Hospital I rarely wrote letters at all, and I have to force myself to now. This probably affects my social behaviour. I have been told by the staff here that I must mix with people more, and engage myself with social activities within the hospital. They tell me that they are pleased with my progress so far, and stress that my OU studies must not interfere with this progress, that is, I must not cut myself off from everyone and simply study alone.

I have my own room in which I can do my studies, and in it I have my own private TV on which I watch the OU programmes. In coping with my activities and with the OU work I find little time to watch anything on the TV except OU programmes. However I suppose watching TV is antisocial anyway. I also have a privilege termed Parole. This means that I can walk about the Hospital grounds without a Staff escort, and can visit friends in other parts of the hospital, or other 'Houses' as they are called. I use this facility to visit Timothy, the other M100 student, to discuss the course with him. It also allows the Tutor who comes to see us to meet us together in 'Cornwall' - Timothy's House.

I have some previous qualifications in Electrical Engineering which I obtained about nine years ago, and which allow me two credit exemptions. Hence I only need to do one foundation course. Maths was always my best subject, and so I decided to try for an OU degree in it.

I obtained the course units for M100 privately last year and have worked through the lot. About half of it was completely new to me and the other half seemed strangely presented; in particular the notation.

My weakest points at the moment are Stastitics and Group Theory, neither of which have I studied before. I have already taken up the subject of Statistics with my Tutor who seems keen to help.

Your comments about the computer postal service have enlightened the minds of both of us, as we thought we might suffer a major handicap there.

Thank you for going to the trouble of making up a special MOUTHS list. I am already in contact with one of them, from Stafford. I believe Timothy is making contact with another. We appreciate your work and efforts.

Yours sincerely, Peter Johnson, Kent House, Broadmoor Hospital, 19th February.

* * * * * *

RON AITKEN :

I write in support of Nick Fraser (M500 39 11) . The great drawback of the OU is the relative lack of contact with one's fellow studerits. The isolation is even greater for those denied access to a phone and a study centre.

If you believe in the second chance offered by the *Open* University have the courage to stand up for what you are prepared to say and put your name on the list for particularly disadvantaged students.

Of course if you already hold equivalent qualifications and just enjoy taking the courses without being prepared to give something in return ---.

PS How about 'Closed Mouths'?

SOLUTIONS

34.4.17 *FIND THE NEXT TERMS* 1 2 4 10 26 16 232 764 2620 9496.

Michael Gregory: 35696 140152. $t_n = t_{n-1} + (n-1)t_{n-2}$.

39.1 *NEW YEAR RESOLUTIONS* 1977 Six kept:

2 3 5 8 10 11: Nick Fraser, Sue Davies.

2 3 5 7 10 11: Michael Masters.

2 3 $\begin{smallmatrix} 5 & 7 \\ 6 & 8 \end{smallmatrix}$ 10 11: Michael McAree.

39.3.i *THE BLACK ACE* II *A pack of cards is shuffled until the second black ace appears. Where is it most likely to be*?

If the first black ace is most likely to be the first card (see solution 36.5, M500 37 17) then by symmetry the second black ace will be the last card: Jeremy Humphries.

39.5 *ETHIOPIAN MULTIPLICATION* *Set the two numbers side by side and successively divide by two on the right, ignoring fractions, at the same time doubling up on the left. Strike out all lines with an even right number and add up what remains on the left. This sum is the required product. Why*?

Two people, Michael Masters and Michael McAree, have produced a table showing what happens. There are three columns: column 1 has the number to be multiplied, column 2 the multiplier and column 3 the remainder on dividing the number in the multiplier column by 2. At the end of the operation column 3 gives the binary form.of the multiplier. For example, with 13 as the multiplier

M	13	1
2M	6	0
4M	3	1
8M	1	1

and 13M = M + 4M + 8M.

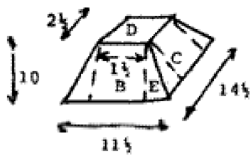
Sue Davies: All lines with even RH number are deleted so only lines which produce a rounding error in the next line are left. If $x \quad y$ is a line left then the next line will be $2x \quad y/2 - ½$. This gives a product of $x(y - 1)$ instead of the required product xy; i.e. an error of $-x$. This is corrected by the addition of the x from the previous row in the final sum.

PROBLEMS

41.2 FRUSTRATION - Michael McAree

The formula for finding the volume of a Pyramidic Frustrum is $V = \frac{h}{3}(A + a + \sqrt{Aa})$ where h is the vertical height and A and a are the areas of the two ends.

Consider this then. Divide the pyramid up with parallel cuts as shown. We have: volume of D = 11½ × 2½ × 10 = 37½. Volume of 2 × B = 6 × 10 × 1½ = 90. Volume of 2 × C = 5 × 10 × 2½ = 125. Volume of 4 × E (pushed together to make a pyramid) = $^{10}/_3$ × (12 × 10) = 400. Add these together: 37½ + 90 + 125 + 400 = 652.5. On the other hand, $V = {}^{10}/_3 \times (2½ \times 1½ + 14½ \times 11½ + \sqrt{2½ \times 1½ \times 14½ \times 11½}\,)$ = 10(3.75 + 166.75 + 25.01)/3 = 651.68 ≠ 652.5. Why?



41.3 OLYMPIAD I - Krysia Broda.

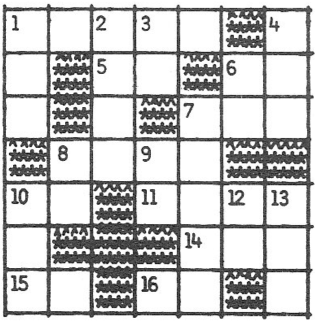
The 18th International Mathematical Olympiad was held in Lienz, Austria, July 1976. Britain were second to the Soviet Union. The competition is for teams of up to eight pre-university students and consists of six questions, three to be tackled in a four hour period on one day and three to be tackled in a similar session the next day. Perhaps M500 readers would like to try to solve the problems. Please note that in the competition correct answers are not enough. Rigourous proofs must be provided. Reprinted by permission of The Mathematical Association. So far I have only managed to solve two problems completely.

Ed - *These problem have been printed elsewhere but I don't think the solutions have. I will print them one per issue and will probably send any solutions straight to Krysia for checking, unless anyone objects.*

In a plane convex quadrilateral of area 32 cm^2 the sum of the lengths of two opposite sides and one diagonal is equal to 16 cm. Determine all possible lengths of the other diagonal.

41.4 CROSSNUMBER - Don Mitchell

For many years the Danks family have owned Little Piggley Farm. The farmer at the time of the story lives there with his three grown up children: Ted, Martha and Mary in order of age. As Mrs Danks is dead her mother, old Mrs Croisby keeps house.



On the farm there is a rectangular field called Dog's Head round which Farmer Danks walks $1\frac{1}{8}$ times before going to work.

From the clues below find the date of the story.

(It may help that only two of the clues are the same figure and also the number of roods in Dog's Head is the answer to another clue.

4840 square yards =1 acre,
4 roods = 1 acre,
20 shillings = £1.)

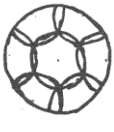
ACROSS

1 Number of square yards in Dog's Head.
5 Martha's age.
6 Difference in yards between length and breadth of Dog's Head.
7 Number of roods in Dog's Head multiplied by 9 down.
8 Date when Farmer Danks's family acquired Little Piggley.
10 Age of Farmer Danks.
11 Year of Mary's birth.
I4 Perimeter of Dog's Head in yards.
15 Cube of miles per hour of farmer's constitutional.
16 Difference between 15 across and 9 down.

DOWN

1 Value of Dog's Head in shillings per acre.
2 Square of Mrs Croisby's age.
3 Mary's age.
4 Value of Dog's Head in £.
6 Ted's age (he will be twice as old as Mary next year).
7 Square number of yards in breadth of Dog's Head..
8 Time of constitutional in minutes.
9 See 10 down.
10 Farmer's age multiplied. by 9 down.
12 Ted's age when Martha was six.
13 Years of occupation of farm by the Danks family.

41.5 DISC COVERING - Chris Pile



How many discs of unit radius are required to cover a disc of radius n? (n an integer.) Is there a strategy for placing the unit discs?

The answer is clearly 1 for $n = 1$ and looks like being 7 for $n = 2$.

EDITORIAL

To start with I must mention the replies to my point in 38 about maximising $n^{1/n}$ and using the technique to find the larger of e^{π} and π^{e}.

Max Bramer has $\log_e n^{1/n} = (\log_e n)/n$, ∴

$$\frac{1}{n^{1/n}}\frac{dn^{1/n}}{dn} = \mathbf{1/n^2 - (\log_e n/n^2)}$$

and $(d/dn)n^{1/n} = n^{1/n}/n^2(1 - \log_e n)$ which vanishes at $n = e$.

Leslie Upton says that since e maximises $x^{1/x}$, $e^{1/e} > \pi^{1/\pi}$. Raise both sides to the $(\pi \cdot e)$th power and $e^{\pi} > \pi^{e}$. One can also differentiate $x^{\pi e/x}$ with respect to x.

New readers may be puzzled by the items on pages 14 and 15, Prisoners and MOUTHS. This refers to an article in M500 38 12 by Peter Weir where he asked for volunteers to go on a special MOUTHS list which could be sent to people in closed institutions. I believe the list is about a dozen strong now and could probably do with filling out a bit!

From issue 42 we have a special problems editor. No, I'll do that again: we have a PROBLEMS EDITOR. He is Jeremy Humphries, joined 1977. He writes "I am not a mathematician, I'm an engineer; but I do know a couple of maths wizards at Hawker Siddeley who could probably help me with non-M100 bits. And there are the authors and staff members to turn to. I've got some time because I'm not doing a course this year. I was entered provisionally for 202 but didn't take it up. I've promised myself I shall get another job and move back to Birmingham this year, and I didn't want to get saddled with moving house etc. halfway through a course. However, I'm beginning to feel mathematically deprived, so this opportunity to do a bit is welcome." This could of course well mean yet another address to write to with the expense of a possible further 6½p but, as before, anything can be sent to any officer of the SOCIETY listed inside the front cover. It will get passed on.

Reference to the items in both the above paragraphs will appear on page 0 in future; this page is beginning to get a little crowded. At one point I was considering putting a contents list there as most other small magazines do. (I've been reading a lot of these lately as people send them along to give me some idea of the right way of doing things.) However, there's no possibility now. If we have one it will go on page 1 which I think would be a pity. Still I'd be interested in the views of others. The index for 1976, by the way, is 'in hand'. It has been compiled by Milada Mitchell and is merely awaiting processing.

As promised last month we have a crossnumber. I would be glad of some feedback on this, even from people who wouldn't dream of attempting it. Do you think it a waste of space?

There are a few lines left. Mention of page 0 above reminds me that I have decided to use some standard notation. In future the symbols $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$ will be used to represent the natural numbers starting with 1 (Halmos and Weir notwithstanding), the integers, the rationals and the reals. $\mathbb{R}_0^+$ for instance means 'the positive reals and 0' in the good old M100 tradition and to hell with newfangleness.

Eddie Kent.