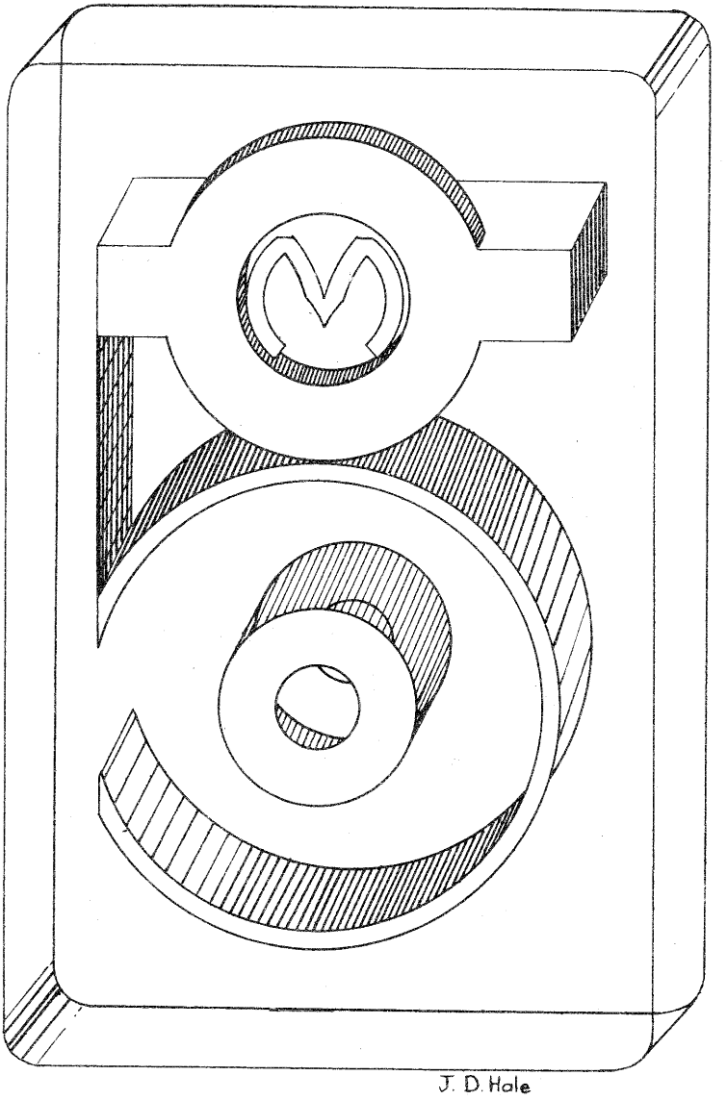

M500 42

M500 is a student operated and owned magazine for Open University mathematics students and staff, and friends. It it designed to alleviate student academic isolation by providing a forum for public discussion of the mathematical interests of readers.

Articles and solutions are not necessarily correct but invite criticism and comment. Anything submitted for publication ought to be not more than 600 words in length; otherwise it may be split into instalments.

MOUTHS is a list of names addresses and telephone numbers, with previous and present courses, of voluntary members, by means of which private contacts may be made, to share OU and general mathematical interests, or to form self-help groups by telephone or correspondence.

THERE IS ALSO A SPECIAL LIST OF A SUBSET OF MOUTHS MEMBERS WHO HAVE

**********EXPLICITLY********** VOLUNTEERED **********

FOR THEIR MOUTHS DETAILS TO BE DISTRIBUTED TO MEMBERS IN CLOSED INSTITUTIONS, SUCH AS PRISONS

The views and mathematical abilities expressed in M500 are those of the authors and may not represent either those of the editor or the Open University.

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ANYTHING SENT TO ANY OFFICER OF THE SOCIETY WILL BE CONSIDERED FOR POSSIBLE INCLUSION IN M500

RANDOM WALK
A PROBLEM IN TWO DIMENSIONS

GEORGE SANDERSON

From any position (x, y) in the Cartesian plane there are four equiprobable moves each of unit length:

(1) E: $(x, y) \longmapsto (x + 1, y)$ (2) W: $(x, y) \longmapsto (x - 1, y)$

(3) N: $(x, y) \longmapsto (x, y + 1)$ (4) S: $(x, y) \longmapsto (x, y - 1)$.

PROBLEM 1 Starting at the origin what is the probability P_{2r} of being at the origin after exactly $2r$ moves?

PROBLEM 2 Starting at the origin what is the probability Q_{2r} of being at the origin after exactly $2r$ moves without having previously been at the origin (except, of course, initially)?

PROBLEM 3 What is $\sum_{r=1}^{n} Q_{2r}$?

PROBLEM 4 Does $\sum_r Q_{2r}$ converge, and, if so, to what?

Problem 1 is fairly easy. $P_{2r} = \binom{2r}{r}^2 (¼)^{2r} = \left(\frac{1.3.5.....(2r-1)}{2.4.6....2r}\right)^2$.

Let the $2r$ moves consist of e E-moves, w W-moves, n N-moves and s S-moves. Then $e+w+n+s = 2r$, $e = w$ and $n = s$, so

$$P_{2r} = \sum_{e=0}^{r} \frac{(2r)!}{e!e!(r-e)!(r-e)!} (¼)^{2r} = (2r)!\,(¼)^{2r}(1/r!)^2 \sum_{e=0}^{r} \binom{r}{e}^2$$

$$= (¼)^{2r}\binom{2r}{r}\binom{2r}{r}$$

and the alternative expression is easy to obtain from this.

Problem 2 isn't. I have obtained $P_{2r} = Q_{2r} + Q_{2r-2}P_2 + Q_{2r-4}P_4 + ... + Q_2P_{2r-2}$ so that

$$\begin{bmatrix} 1 & & & \cdots & & \\ P_2 & 1 & & \cdots & & \\ P_4 & P_2 & 1 & \cdots & & \\ \cdot & & & & \cdot & \\ \cdot & & & & \cdot & \\ \cdot & & & & \cdot & \\ P_{2r-2} & P_{2r-4} & & & P_2 & 1 \end{bmatrix} \begin{bmatrix} Q_2 \\ Q_4 \\ Q_6 \\ \\ \\ \\ Q_{2r} \end{bmatrix} = \begin{bmatrix} P_2 \\ P_4 \\ P_6 \\ \\ \\ \\ P_{2r} \end{bmatrix}$$

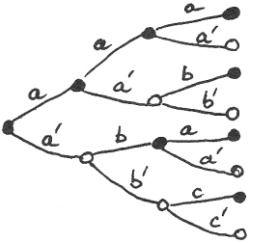
and the $r \times r$ matrix is non-singular, so that in principle we can solve for Q_{2r} in terms of the known P_{2i} (i = 1, 2, ..., r) - but what is the last row of the inverse matrix? Also the expression for Q_{2r} is far from 'neat'.

The following also emerged:
$Q_2 = P_2$; $Q_4 = P_4 - P_2P_2$; $Q_6 = P_6 - 2P_4P_2 + P_2P_2P_2$ ...
$Q_{10} = P_{10} - 2P_8P_2 - 2P_6P_4 + 3P_6P_2P_2 + 3P_4P_4P_2 - 4P_4P_2P_2P_2 + P_2P_2P_2P_2P_2$; ... .
Each term is a partition of $2r$ into sum of even numbers. The numerical coefficients are the

permutations and the sign depends on the 'order' of the particular partition.

I cannot obtain a neater specification of Q_{2r}. Any suggestions?

I also looked at a tree diagram. The solid dots represent arrival at the origin. Clearly $a = P_2 = Q_2$. The meaning of b, c, ... is evident. $a' = 1 - a$, etc.



$Q_2 = a$

$Q_4 = a'b$

$Q_6 = a'b'c$

etc.

It is evident that $a'b'c'... \to 0$ so that $\sum_r Q_{2r}$ converges to 1, i.e. eventually one is certain to return to the origin - so one cannot become 'lost' .

Can anyone help?

NOTICE BOARD

CHEZ ANGELIQUE John Jaworski has written to say that *Chez Angelique* (what do you mean, what's that? It's The Bumper Late-Night Problem Book which also contains, as a bonus, a photo-strip by Richard Ahrens costing £1) has moved. Orders are currently being dealt with from the OU Maths Faculty, c/o John Mason.

LONDON OPEN UNIVERSITY ASSOCIATION: two events

7 May, Saturday, 8pm-1am. LOUA MAY BALL at 'The Cambridge', corner Goodge street and Newman street, Wl. Food. Bar. Nonmembers £l.

20 May, Friday, 8pm-1am. HOW TO GROW TREES by Robin Wilson of the OU Maths Faculty and the Mathematics Institute Oxford. LOUA lecture and social at the Mount Pleasant Hotel (Kings Cross road × Calthorpe street WC1). Refreshments; bar. Nonmembers 50p. If coming to either of these contact Barbara Harrap, Basia Snow or Anne Watts.

EAST ANGLIAN OPEN UNIVERSITY GRADUATE ASSOCIATION Mary Bull is one of the organisers of the association and would like graduates in region 06 to contact her. One aim, she says, is to hold a series of meetings throughout the area. If you write please remember to enclose a stamped addressed envelope.

O MY PEOPLE,..

EDDIE KENT

A remarkable document has just come into my hands: it is a photocopy of an article from an ancient *Eureka*, called "Vipers, logs and all that" by G J S Ross.

Mr Ross mentions (and I have seen this pointed out before) that the arithmetic in *Ezra* is faulty. On this occasion I decided to check the assertion. My results follow. The whole story takes place in *Ezra* ii. In each case the first number in each pair is the verse and the second an increment.

Ezra ii 1: Now these are the children of the province that went up out of the captivity ... . ii.2, 10. 3, 2172. 4, 372. 5, 775. 6, 2182. 7, 1254. 8, 945. 9, 760. 10, 642. 11, 623. 12, 1222. 13, 666. 14, 2056. 15, 454. 16, 98. 17, 323. 18,112. 19, 223. 20, 95. 21, 123. 22, 56. 23, 128. 24, 42. 25, 743. 26, 621. 27, 122. 28, 223. 29, 52. 30,156. 31, 1254. 32, 320. 33, 725. 34, 345. 35, 3630. 36, 973. 37, 1052. 38, 1247. 39, 1017. 40, 74. 41, 128. 42, 139. 43 to 58, 392. 60, 652.

I make the total of all these 29830. I may be wrong on this of course, but it is easily checked. Verses 64 and 65 between them imply that this total ought to be either 49697 (congregation plus servants) or 49897 (with the singing men and women added in). In either case there is a discrepancy of around 20000. I could perhaps make up one of these by reading verse ii.2 as 11 instead of 10 and maybe verse 58 does not imply those preceeding it; perhaps, even, we are meant to include those in 61 to 63 who were "as polluted". But these are hardly likely to make up nearly half the force. I wonder what explanation can be offered.

I don't entirely agree with Mr Ross's offering. He points out that, according to 1 *Kings* vii 29 "were certain additions made of thin work". However, *Kings* comes some way before *Ezra* and in my view would have tended to put the later writer on his guard.

(MS: *Scribe's error above* - v.6 *should be* 2812 *acc. to AV. Sums not checked*.)

CLANK OFF

PHILIP NEWTON

Pencil & paper:- In her article on computers and electronic calculators Sue Davies asks why these should intrude into M500. The brief answer is that they are a new tool and therefore worthy of investigation. People used to write articles on pencil and paper when these first came onto the market (saves time baking your clay tablet ---), and in the case of the computer a vast number of new words is being added to the mathematical vocabulary so that in order to avoid a tower of babel effect there must be discussion in the literature. Again, new insight into mathematics is afforded by the extreme precision required in programming computors.

I quite understand the frustration caused by the appearance of 'problems' which are trivial or

appear so to any particular reader, such as the CoRA algorithm. However it should be born in mind that there are two important facets which M500 caters for in this field. First we have the beginner who requires easy articles and problems to gain confidence and understanding - and most of us were beginners at some time. The second facet is the curious fact of the 'blind spot' - by this I mean the trivial problem which an established student or tutor finds hard for some reason or other. Discussion of this latter type sometimes opens up new realms (in the extreme case) to the victim.

Consider for example Richard Ahrens problem 36.3(c) (which he says is 'hard - i.e. I can't see it immediately') that of showing that in a regular $(2n + 1)$-gon at most two diagonals meet at a point. Krysia Broda in her solution to the part (b) goes around the simple one line solution due to familiarity with set-theoretic principles, and then Eddie Kent in his editorial reckons that the methods for the 7-gon probably won't generalise. Yet anybody who is used to 'practical mathematics' can point to the vital piece $-(2n + 1)/2$ is not an integer.

No, Sue! You cannot expect everything in M500 to be written for you, but fashions change and so do the number of articles on a subject so just be patient, and even try writing the odd solution to help the first year students. It will repay you.

Ed - *Remember, neither the editor nor the Open University necessarily share views appearing in signed articles: in particular those appearing in penultimate lines of penultimate paragraphs.*

Hora aderat briligi. Nunc et Slythaeia Tova
Plurima gyrabant gymbolitare vabo;
Et Borogovorum mimzebant undique formae,
Momiferique omnes exgrabuere Rathi.

"Cave, Gaberbocchum moneo tibi, nate cavendum
(Unguibus ille rapit, Dentibus ille necat.)
Et fuge Jubbubbum, quo non infestior ales,
Et Bandersnatcham, quae fremit usque, cave."

To be continued...

The Times, Thursday March 3rd 1977: *According to* The Scotsman *world prices of butter "are currently more than 100 per cent lower than those in the EEC".*

CONTINUOUS REVERSE ADDITION

PETER MINCH

In M500 38 you asked for someone to continue the CoRA iteration for the number 196 beyond the 43rd reverse addition. I did this on a Wang in BASIC to the 588th iteration without finding a palindrome. The number at this point had 255 digits and it took 1½ hours to process. After an exhaustive search of all the numbers from 100 to 999 I found 13 numbers that will not form palindromes of less than 255 digits at least. Given 196 five of these should have been obvious (they were not but they are now, may Edward de Bono forgive me) namely, 295, 394, 592, 691. As these all give 887 as the first iterate we also have 887 and 788. The sum of 790 and 97 is also 887 so 11 of the 13 numbers result in the same sequence of numbers for the reverse addition.

The remaining two are 879 and 978 which gave a different sequence of numbers at least up to the tenth iterate.

Interestingly there are eight numbers that give palindromes at the 23rd iterate and none that give a palindrome between 23 and 589 iterations.

I suppose the next stage is to check all four figure numbers or to reprogram with a double array to hold the digits so that the process can be continued to 510 digits.

Here is the 588th iterate for 196 if anyone is interested:

196 (588)1314201118083486693914042195971424015477375674164159146355845733279149160099848205537634889297599061591498008016955992287493350064295673289659710800884185161006692888535835602838001060842081237648553542061352476582784400425269591240150839777348982109142 3

PATSY AND PETER

RICHARD AHRENS

My mother and Peter:- cf 41 3. When my mother was at primary school each school was equipped with a set of wooden blocks which could be assembled into a solid with rectangular bottom and sloping sides. (See problem 41.2 *Frustration*). You could then reassemble the blocks to give a rectangular block plus a pyramid and hence obtain the formula for the volume of the first solid. I will bet that Peter was not taught this enchanting result at the age of twelve and I am sure that his development has been greatly hampered as a result. Why does he want Patsy to be put through the same obstacle race that he had to run at her age? (He might have enjoyed four terms of geometry (four terms??!) but I'll bet that nine out of ten of his classmates had no idea what was going on). In any case I don't believe his statement about 36 into 808, and 1502 for 34p versus 2102 for 47p. My eight year old daughter knew exactly which buttons to press on the calculator to get these answers (although she did have some trouble deciding whether 2.666666 was bigger or smaller than 2.2380952).

LETTERS

Nigel Graves: A problem has occurred to me as I grapple with the harsh practical world of M351 after breathing the pure atmosphere of M331.

Let $f: \mathbb{R} \to \mathbb{R}$ be the function such that $f(a)$ is the largest root of the equation $\sin x - ax = 0$. I can deduce sufficient information about the function to answer the particular TMA question which led me to define it, but to satisfy my idle curiosity, can anyone tell me, with proof, whether f is Lebesgue integrable?

John Wllls: I cannot explain how the Romans carried out simple arithmetic (M500 41 5) but their numerical system, complicated by IV and XL and probably CD as substitutes for IIII and XXXX and CCCC continued to be used into the middle ages as the main notation. The mediaeval European method of division was the galley method, so called because you could draw sails on to the sum. Webster's Third tells us that the galley method is equivalent to scratch division "in which the partial products are not set down at all but only the remainders so that it is necessary to scratch out each figure of the dividend from which the subtraction is made and place the remainder each time above it". The children's encyclopædia we had at school gave more detail but the *Encyclopædia Britannica* seems not to have the information. For practical purposes, large numbers may have been rare because of base change, known to us from currency, weights and measures; only now becoming obsolete. Mediæval scientists used base 60 in fractions - or at least postmediæval ones did - for which reason both time & angle are measured in minute parts and second minute parts. I think some people used third and fourth minute parts.

Max Bramer: A recent *Mathematical Spectrum* had a simpler solution to the "prove the product of four consecutive integers cannot be a perfect cube" problem than the one in a recent M500 (and I thought the problems were original!).

Suppose $n(n + 1)(n + 2)(n + 3)$ is a perfect cube, then either $n + 1$ or $n + 2$ is odd and hence relatively prime to the other three. So this term must be a perfect cube and hence the product of the other three must be.

(1) $n(n + 1)(n + 3)$ is a perfect cube. Then either $n(n + 1)(n + 3) = (n + 1)^3$ or $n(n + 1)(n + 3) = (n + 2)^3$. Thus either $n = 1$ (not a solution, by inspection) or $2n^2 + 9n + 8 = 0$ (which has no positive integer solutions).

(2) $n(n + 2)(n + 3)$ is a perfect cube. Then either $n(n + 2)(n + 3) = (n + 2)^3$ or $n(n + 2)(n + 3) = (n + 1)^3$. Thus either $n = -4$ (invalid) or $2n^2 + 3n - 1 = 0$ (no positive integer solutions).

Clive Layton: (to Peter Weir) I am hoping that you are still Membership Secretary - I am using information from the 1976 M500 Special Issue; that is remarkable in itself in that most of my texts, and work room, are buried by dust sheets etc. Because of the builders and sundry other quirks of fate I have been galvanised into joining, at last, THE M500 SOCIETY - in the main due to falling behind on the dreaded MST282.

So please forward the relevant material - especially MOUTHS! or forward this all to your mem. sec.

Michael McAree: One of the odd things about my studies with the OU relates to *The Master Book of Mathematical Recreations* by Fred Schuh (Dover) (from which I will send some examples). When I bought the book some years ago I realised I had made a mistake - I was out of my depth. Now I wouldn't feel quite so adrift reading it but of course I haven't the time.

Since Summer School seems the most likely opportunity of meeting fellow members is there any focal point either at the various universities or even if there wasn't is there some way students could 'advertise' their appearance at a particular Summer School so that members could recognise one another?

One reason I bring this up is that when I went to Stirling last year I can't remember seeing anything about M500. Of course this is no reflection on M500 as I can't really remember looking at notices except where they advertised lectures.

Must go and send deposit for W/E Work-In.

John Hale: Allan Solomon in "M331 - A Course for Everyone" (M500 41) says he wants to make the Lebesgue integral accessible as a 'first introduction' to integration theory. I have recently read in M100 Unit 9 a pretty exact paraphrase of Allan's article, which whilst not actually mentioning Lebesgue, does explain how the difference between the summed smaller rectangles and the summed larger rectangles will enable an average value to be estimated for the area under the curve, and moreover explains that there is the added advantage of being able to estimate an error bound for a given number of rectangles. M100 triumphs again! I notice that the unit mentions only Newton and Riemann however. Perhaps they don't like Mr Legesgue.

Dave Diprose: I wonder if the Maths Faculty could be persuaded to reveal a few details of its plans for the future. The reason I ask is that the large number of prerequisites for third level maths courses requires planning and (more important) commitment to intermediate studies by the student. I know that early maths students had little choice of courses. However the faculty obviously has ideas about the future, and I would like to know what new courses are being considered and which courses are most likely to be withdrawn in the next few years.

Daniel Fox: (To Marion Stubbs) When I wrote asking for a copy of your DST game you enclosed a nice letter (which you have undoubtedly forgotten about now). We have played your game whenever time permits; which is not that frequent. Graduate school has consumed a larger percentage of our free time than was initially estimated. We loves it though, we does.

The M500 you enclosed was above most of us. Abstract math is beautiful and fascinating, but is a time sink we don't need just now so it was sent to a friend in Chicago who is a real mathematician.

The salary of a graduate assistant is less than abundant so I feed myself with a part time job in a toy store near here. One of the girls in the store was in England for a few weeks staying with friends and was most impressed by it all. She said that the people she met in pubs were genuinely friendly - quite a difference from most places here. One sometimes wonders if people here go out

of their way to be obnoxious.Most people seem to think that England is accelerating down the proverbial tube but from what I've seen and heard I think we'll beat you.

Your letter sounds very British - it reads almost like Monty Python speaks. Even normal sentences come out sounding amusing. Back when Monty was new to America we were devotees; now it has become sort of 'in'. They even show the programs on the public broadcasting network: that's the only place where they won't be censored. The commercial networks can be pretty stupid at times. It also gets to be annoying to be interrupted every ten minutes for some douche commercial - 'new natural fragrance'. Are people in England as gullible as people here are?

If you feel at all tempted to write I'd enjoy hearing from you. My address for at least the rest of this year is 6802 Lake Kenilworth Drive Apartment 216 New Orleans, Louisiana 70126 - careful about that permutation between apt # and zip code. The Postal (dis)Service gets very touchy about zip codes. They have these trained morons whose only job is to read the fifth digit in the code and react accordingly by pushing one of ten buttons. Very exciting work, that.

If you really don't have the time to correspond, please see if anyone else might be interested. I really would like to hear about what goes on 'over there'.

Brian Woodgate: M231 exam. In answer to Graham Read I found the exam, taken in 1975, very fair; in fact one of the better ones. It is strange to come out of an OU exam with confidence, this having only occurred twice and M231 was one of those occasions. The things that I liked about it were:

1. Superb preparation - the final unit and the 'exam guide' were very useful.
2. The combination of short questions that allowed some working to be shown and long 'reasoning' questions seemed a good mixture.

Roger Bridgman; My reasons for giving up M500 were:

1. I withdrew from the OU for a year because of domestic problems;
2. I simply couldn't find time to read it.

On reflection, I think the second was the stronger reason - in my opinion M500 is far too densely written on the whole - too many symbols per square inch and not all of them very original. I think Manifold should be your touchstone, with its relaxed style and genuine attempt to provide a different angle on maths. Perhaps you could encourage contributors in this direction.

I might even have a go myself next year!

Jonathan Hamilton: (M100) ...my mathematical interests revolve arounds gravitics - anybody likewise interested?

PRISONERS AND MOUTHS

Last month we printed a letter from Peter Johnson of Broadmoor Hospital. This time there is one from another man in the same place. Then there are some letters on the subject - everything that's printable so far - and next month we shall have a reply from Marion Stubbs.

TIMOTHY WILKINS

Like other people I've worked through the M100 course with units lent by my Tutor-Counsellor but still have to wait for my own units to come in order to do some questions whose answers elude me now I've forgotten what I've learnt. Computing does not seem to bother anyone much but we rarely get everything right first time and have to debug. The SCS postal service with its 24 hour turn round seems very good to me. We used to be able to use a computer in the medical centre here but when the doctor heard about it he sternly forbade us to use it any more because for us to be over there was a security risk.

I have not got parole and a room so I must rely on others kindness when I want to watch television.

I do not find differentiation and integration too hard as I've done A-level maths. I've always found statistics hard and group theory is new to me.

Thank you for the MOUTHS list; most people hold up their hands in horror when one mentions Broadmoor.

Most of M500 eludes me but I suppose I've got plenty of time to improve my maths in the 6 year OU course.

I was most pleased to see Peter Weir's ad in M500 41 as I would like to work in computer programming when I get my degree. I thought a sensible thing to do would be to take a twelve week course leading to City & Guilds 748 (when I've got my degree) to give me some experience in COBOL business programming, then a similar course in FORTRAN as I wish to become a scientific programmer. I hope there are still openings when I qualify.

I was also pleased to see the ad for 'Student Developmental Testers' in the latest copy of *Sesame*. My Tutor pointed this out to me and I've already applied to earn my £5s (and incidentally learn some mathematics at the same time). I'll be pointing the ad out to my fellow student tomorrow when I meet him as a football spectator down the field.

* * * * * * *

MICHAEL GREGORY

There appears to be no justification in the constitution for the selective distribution of the MOUTHS list to members or restricting membership of M500. Any change in the constitution to allow such restrictions would be contrary to the principles of the Open University. Surely M500 should follow this example. If members of Broadmoor are not to have the MOUTHS list, how soon will the Committee decide that someone in prison, or a member of the Communist Party, or someone who criticises M500 should not receive it?

I suggest that after the Committee has consulted the Broadmoor authorities, members should be given the opportunity to withdraw their names from the list, and the MOUTHS list should then be distributed to all members of M500.

* * *

JEREMY HUMPHRIES

It might be said that prisoners should get the MOUTHS list like anyone else. OK in an ideal world I suppose, but then there wouldn't be prisoners anyway.

We must give people the final say about who has their names and addresses, irrespective of our own feelings.

* * * * * * *

ALAN SPEED

Can anyone explain to someone as naive as I must be exactly what the problem is?

If the problem is pure prejudice then I find the thought appalling that we should only be prepared to discuss mathematics with those people we approve of. Surely anyone's thoughts on mathematics are equally relevant whether they have or have not stuck parking tickets on my car, or committed some other hienous crime.

The Constitution in para 2.2 seems quite clear and acceptable. If we were to add anything to exclude one section of the community, namely criminals, we ought also to have another list for me excluding traffic wardens, and no doubt there are those who object to talking to homosexuals, Jews, blacks, etc.

STATUS

THE INSTITUTE OF MATHEMATICS AND ITS APPLICATIONS BRIAN WOODGATE

Have we any members who belong to the above organisation and would be willing to sponsor applicants?

An OU degree is acceptable, but one needs sponsors to sign the form and this can be a problem.

I have written to the faculty but this has so far brought no response.

The probability of a piece of toast and marmalade dropped on the floor landing marmalade-side-down is directly proportional to the cost of the carpet.

Paul Jennings.

MATHEMATICAL TASTE ALAN SLOMSON

There are a number of reasons why mathematics appeals to people. Some are most attracted by the ability of mathematics to provide powerful tools for solving problems, while others are most excited by the logical structure of the subject. This distinction cuts across the usual division that is made into pure and applied mathematics. Powerful techniques turn up in number theory as much as in fluid dynamics, and the logical structure of Newtonian Mechanics appeals to some people more than its ability to predict the behaviour of cannon balls.

Of course, techniques and logical structure cannot be entirely separated from one another. A logical structure is pointless if it is not the structure of something, and often it is the structure of a theory which provides the techniques. On the other hand powerful techniques are useless without some logical structure which ensures that they lead to correct answers.

The distinction I am making appears clearly in Michael Spivak's book *Calculus* and M231 students will be able to judge their tastes by what appeals to them most in this book. Some like calculus because it provides methods for calculating areas and volumes, for locating maxima and minima of functions and so on. In Spivak's book the cannon balls appear in Problem 18 on page 183 but my favourite piece of technique is that in problem 27 on page 329 used to calculate

$$\int_0^\infty e^{-x^2}.$$

Othe people enjoy most from Spivak the careful definition of limits in terms of εs and δs and the construction of the theory of continuous and differentiable functions on the basis of this definition.

Other OU courses can be judged according to how they meet the needs of people with these differing tastes. I shall restrict my attention just to M331 and M332. M332, the Complex Analysis course, contains things for most tastes. The theory of complex functions is developed as carefully as is the theory of real valued functions in Spivak, and there are plenty of examples of powerful techniques which complex variable theory provides, including applications to both number theory and fluid dynamics. The price that has to be paid for all these good things is the rather large amount of course material. It is not necessary to master it all to succeed with the course, but M332 students do need to take some care not to be overwhelmed by the material.

The M331 course. Integration and Normed Spaces, is in sharp contrast. Despite the title it would be wrong to suppose that you will learn new techniques of integration in M331. Indeed the first part of the course is concerned with justifying familiar techniques, and in a somewhat more general context than is to be found in M231. Once the definition and theory of the Lesbegue integral has been developed in the first half of the course, it is used to build up the structure of normed spaces and Hilbert spaces in particular.

These structures do provide the framework for techniques which can be used in quantum mechanics, probability theory and elsewhere, but apart from one unit on Fourier series, little of this emerges from the course. Thus M331 is very much a course for people who like logical structure and justification of mathematical methods rather than those who just like to roll up their sleeves, start calculating and solve problems. The advantage of the course being somewhat one-sided in this way is that its logical structure is clearer and that the amount of material is less than in M332.

Of course these are only my personal opinions; it would be very interesting to hear whether other people agree or disagree with my views of these two courses.

PRONUNCIATION JOHN WILLS

C P Wright (M500 14) is only half (well maybe three-quarters) right. If I am wrong however it is not by living in the North of Germany; I live in Berlin where we regularly and vulgarly call sausage 'voorsht' instead of 'voorst'. When we are in this kind of mood we will call the great physicist Einschtein. When however we try to be cultured we can distinguish from internal 'st'. Is the st in Einstein initial? I should add that I was aware when I wrote my earlier letter that a compound word is spoken as two words (or perhaps some sets of words are written as single words), and I was talking about spoken language. Historically Mr Wright is right: Einstein is fairly typical of the names assigned to nonChristians in central and eastern Europe and means in-stone - whatever that means. But it is very commonly pronounced once-one, the meaning of in-stone being long forgotten, at least by most people. I refuse to ask a German how to pronounce the word because the last time I did I heard only argument about how one *ought* to pronounce it and how it is pronounced wherever each speaker hailed from. But I am quite prepared to make indirect reference to Einstein and hear how the word gets pronounced. Indeed I have done so. On the basis of the results of which, I wrote my last letter. The compound is fossilised. I must try to get an introduction to Mrs Frankenstein and see how she pronounces *her* name. We have met but the meeting was of such a kind that we only smiled pleasantly at each other while those responsible for introducing us discussed something else.

THAT BYWAY: JEREMY HUMPHRIES

I was interested in Bill Midgley's piece on page 41 7. I suppose speaking loosely is forgivable and for most of us unavoidable. What I wonder at is the stuff that gets into print these days. Books and magazines are littered with grammatical errors and just plain 'not saying what is meant'. *New Scientist* and *Amateur Photographer* make your hair stand on end. *Scientific American* is not bad and at that price it shouldn't be.

Mathematicians, who should be very good, seem to be very bad. They can give you pages of maths with never a logical inconsistency or division by zero, but they can't always get from one end of a sentence to the other without dangling a participle or putting a singular verb with a plural subject.

I don't say that I never make mistakes because I do. But *I* don't write for a living. I think anyone who does should try very hard to do it properly.

I notice that the current craze at the BBC newsdesk is to make 'whom' the subject of a sentence; "The Prime Minister, whom, a spokesman said, was at number ten ...".

Please don't print this if there's a mistake in it.

Ed - *Bertrand Russell, in* Human Knowledge, Its Scope and Limits*, uses the form 'none ... were'. I saw this when I bought the book, in the eanly fifties; and more than almost anything else, that gave me strength to carry on.*

FOURTH LEVEL COURSES TONY BROOKS

One would like to ask Brian Woodgate (M500 37 10), why have a fourth level course? This question is not directed at the Maths Faculty but rather at the whole concept of fourth level courses. The point is that the OU does not hand out any extra 'rewards' for doing a fourth level course, they are (as far as I can tell) treated as exactly equivalent to third level courses for honours assessment. I believe there are only two alternatives:

a) You accept that fourth level courses should have some feature which makes them more demanding than third level courses (such as a research element) and give them greater weight when assessing honours.

b) You treat third and fourth level courses identically. But then I do not think one is justified in making fourth level courses more difficult, in which case I see little purpose in calling a course fourth level. If you want to indicate that a course requires other third level courses as prerequisites then this can be done in the normal way in the courses hand book without giving the new course a fourth level label.

I would suggest to anyone who wants a fourth level maths course that they first of all try to get a rational policy on fourth level courses from the OU.

Anyone who cannot cope with mathematics is not fully human. At best he is a tolerable sub-human who has learned to wear shoes, bathe, and not make messes in the house.

Helnleln: *Time Enough for Love* (B Phillips)

PW - *One asks if a converse is true: Does the fact that the membership secretary is an intolerable sub-human who has not learned to wear shoes, bathe, and who makes messes in the house, mean that he can cope with mathematics*?

M500 PROBLEM CORNER, EDITED BY JEREMY HUMPHRIES.

I'm Jeremy Humphries, your new problems editor.

Things are still a bit disorganised after the handing over, and some solutions will appear later than usual. 'Usual' is two issues after the problem appears. Any published item will have reached me before the 20th of the month preceding that of publication, because its a long way to the printers. General rule is the earlier the better.

You can make things easier for me by being neat and brief, sending complete work rather than bits, heading each page with your name and the date, and using only one side.

What I write may not always be correct, and the mistake may be mine, yours or both. If you don't agree with something say so. Debate and argument are two of our aims.

I repeat Eddie's plea. Send something. We have a small band of regulars who keep us going, but there are nearly 400 of you somewhere. M500 is what you make it.

Tell me where you get material from, for copyright reasons.

You can phone me anytime with problems, solutions, advice, abuse, etc. I'm usually up and about until the early hours.

Did you do the crossnumber? Neither did I. There is a mistake in the clues. Assume that the farmer walks $1\frac{1}{8}$ times round Dog's Head and try again. SUE DAVIES put me out of my misery by telling me that. She also says that in New Year Resolutions, 39.1 (see 41 solutions) you can't keep 6 or 7, and, after a bit of thought, I agree.

6 implies two from {1, 2, 3, 4, 5}.

(a) Assume 5 is not one of them. Then you have kept two from {1, 2, 3, 4} so you have kept three.

(b) Assume 5 is one of them. Then you have kept two from {1, 2 ,3, 4} so you have kept three.

A similar argument disposes of 7.

Just as we are going to press STEVE AINLEY writes as follows: 39.1, only the first solution (2, 3, 5, 8, 10, 11) is consistent, surely. 41.4: is this another April Spoof? - It seems insoluble. To which I reply yes and sorry.

Anyway do you want any more crossnumbers?

SOLUTION 31.3 *COMBINATIONS*: Show that $\binom{n}{r} \in \mathbb{Z}^+$, $n > r > 0$. We have another couple of solutions to this, from RON AITKEN and DATTA GUMASTE. Similar ideas, which I outline. (Everything $\in \mathbb{Z}^+$)

1. Equivalently, show that $r!$ divides the product of r consecutive integers since $\binom{n}{r} = n(n-1)\,(n-2)\,...\,(n+r)/r!$

2. Let $(n+1)\,(n+2)\,...\,(n+r)/r! = s/r!$. Divide each $(n+i)$ by r. The remainders are the set $\{0, 1, ..., r-1\}$ $\therefore \exists\,(n+i)$: $r \mid (n+i)$ $\therefore$ $r \mid s$.

3. Similarly $1|s$, $2|s$, ..., $(r-1)|s$.

4. Let $\langle a, b\rangle$ = LCM, (a, b) = GCD of a, b. Then $\langle a, b\rangle = ab/(a, b)$

$$\therefore \langle 1, 2, 3, ..., r\rangle = r!/(1, 2, 3, ..., r) = r!/1 = r!.$$

5. Well-known theorem: $a_1|m, a_2|m, ..., a_n|m \Rightarrow \langle a_1, a_2, ... ,a_n\rangle |m$.

6. $\therefore r!|s; \therefore \binom{n}{r} \in \mathbb{Z}^+$.

We were expecting something like this: (everything $\in \mathbb{Z}^+$.) We can show $\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$. Assume $\binom{n}{r}$ is an integer $\forall n, r: n < k, r < k$ for some k. Then $\binom{k+1}{r} = \binom{k}{r} + \binom{k}{r-1} \in \mathbb{Z}^+ \forall r \le k$ and $\binom{k+1}{k+1} = 1$ by definition of $\binom{n}{r}$. Hence $\binom{n}{r} \in \mathbb{Z}^+ \forall n, r : n \le k+1,\ r \le k+1$. If $k = 1$, $\binom{1}{1} \in \mathbb{Z}^+$. Hence by induction $\binom{n}{r} \in \mathbb{Z}^+ \forall n,r$.

SOLUTION 59.2 *WHAT'S INTERESTING ABOUT* 1977?

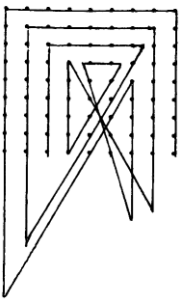
MICHAEL MCAREE says $1977 = 8^1 + 6^3 + 4^5 + 3^6$ and $8 - 6 = 3 - 1$, $6 - 4 = 5 - 3$, $4 - 3 = 6 - 5$. That is, the difference between adjacent numbers, going L to R equals the difference between their exponents going R to L.

My friend BILL COLEMAN produced $1977 = 1^1 (-9^2 - 7^3 + 7^4)$ in about one minute.

SOLUTION 59.5 *THE BLACK ACE* II; what is the expectation for the *r*th of *m* designated cards in a pack of *n* cards?

STEVE MURPHY writes: "I'm convinced that the answer is $\frac{r(n+1)}{m+1}$ but sadly the proof seems to go on and on ..." and it does. Anybody interested can contact me or Steve.

SOLUTION 40.1 *DOTTO*; (a) *Draw* 16 *straight lines through* 81 *points in a square array* (9 × 9) *so that every point is on one and only one line except that two lines may meet* (*but not cross*) *on a point*.



SOLUTION 40.2 *THE NON-ISOMORPHISM THEORY*: Find the smallest finite group G with a quotient group G/N not isomorphic to a subgroup of G.

SUE DAVIES: The quaternion group Q (of M202 group cards) has a normal subgroup $N = \{e, a^2\}$ and $Q/N \approx K$ the Klein 4 group. But $K \not\subset Q$.

There is no smaller group with this property as for o(G) =1, 2, 3, 5 and 7, G has no proper subgroup; for o(G) = 4, G can only have proper subgroups = C_2 and $G/C_2 \approx C_2$, and for o(G) = 6, G has only proper subgroups $\approx C_2$ and C_3, and $G/C_2 \approx C_3$; $G/C_3 \approx C_2$.

SOLUTION 40.5 *EXPLAIN*: *What are* (a) 56.9612412..., (b) 0.692200628...?

ALAN SLOMSON and MICHAEL MCAREE: (a) solution of $x^x = 10^{100}$, (b) minimum value of x^x.

SOLUTION 40.4 *RELATIVE TRUTH*: *Versatility* = 1001(*veracity*). *Value of relativity*? *How many*? *How do you find them*?

The solutions submitted examined VERAC+ACITY = SATIL. MICHAEL MCAREE sent a progress report: "so much-work involved. About 100 cases with V, E, C, Y still to check in each case." Four days later came:

```
R = 8286999980
E = 7767444462
L = 6434113333
A = 1612000018
T = 9358721605
I = 0071132724
V = 3100276151
I = 0071132724
T = 9358721605
Y = 2599668897
```

DAVE DIPROSE explored 115 avenues and found seven of Michael's plus

8652790971.

With DAVID ASCHE's in issue 40 this makes twelve solutions. As Eddie says - golly.

SOLUTION 40.5 *SAFETY*: *Place all the white pieces of a chess set on a board in such a way that no piece threatens another; how many non-trivially different solutions are there.* (Interpolation from Ed - two solutions not sent to Jeremy:

```
. p N p N . . .      p . p . . . . .
. . . . . R . .      p . p . . . . .
. . . . . . R .      p . p . . . . .
p . . . K . . .      . . . . . . . Q
p . . . . . . .      p . . p . . . .
p . p . B . . .      . . . . . . R .
p . p . B . . .      . . . . . R . .
. . . . . . . Q      N B B . N . . .
  M Hodgson            Lucy Poland
```

Each of these seems to give rise to another solution by merely turning the board upside-down. Interpolation now over.)

I solved the first part, one solution, and thought myself no end of a fellow. Then STEVE AINLEY deflated me by writing "I suspect the answer to the second part is too many to be interesting." Steve proposes a modification:

PROBLEM 42.1 SAFETY II: Place all the white pieces, plus the black king and as many black pawns as possible, with no attacks. He's sent a solution with ONE black pawn. Can any of you chess players beat him? Can your positions be reached in actual play?

PROBLEM 42.2 OLYMPIAD II: from KRYSIA BRODA: reprinted by permission of the Mathematical Association from 18 IMO 1976, Lienz.

Let $P_1(x) = x^2 - 2$ and $P_j(x) = P_1(P_{j-1}(x))$ for j = 2, 3, ... Show that for any positive integer n the roots of the equation $P_n(x) = x$ are all real and distinct.

PROBLEM 42.5 REVERSION: p is a non-palindromic 3-digit number $\in \mathbb{Z}^+$. Reverse p, = q. Find $|p - q| = r$. Reverse r, = s. Now $r + s = 1089$. Why?

(For pedants, the numbers are all 3-digit; use a zero if necessary.)

PROBLEM 42.4 INVERSION:

Dialogue from nunerical maths course, Hatfield Polytechnique.

LECTURER: To facilitate inversion we sometimes partition a matrix $X = \left(\frac{A|B}{C|D}\right)$ and seek $X^{-1} = \left(\frac{P|Q}{R|S}\right)$, where A, P are square order m and D, S are square order n. The method involves inverting the simpler matrices A and D.

MATHEMATICALLY OU TRAINED HAWKERSIDDLEYMAN: What if A or D is singular?

LECTURER: O U are awful.

* * * * * * *

Produce a large (i.e. non-trivial) non-singular matrix such that for all partitions of the form described, A, B, C and D are singular.

PROBLEM 42.5 ASSERTION: DATTA GUMASTE

3 divides $n(n-1)(n-2)$ for all n in $\mathbb{Z}^+$; 5 divides $n^2(n^2-1)(n^2-4)$; 7 divides $n^3(n^3-1)(n^3-6)$; ... .

a) Are these true?
b) If so, what is the general assertion?
c) Prove it.

Ed - *I shall finish off this page with what may or may not be a problem but certainly is*

BERTRAND'S PARADOX EDDIE KENT

What is the probability that a chord drawn at random in a circle will be greater in length than the side of the inscribed equilateral triangle?

(i) Each point on the diameter determines one chord, which is perpendicular to the diameter. Hence the probability is ½ . (See figure 1. CD = AB. If the point determining the chord is between B and C the chord will be longer than a side; otherwise not.)

(ii) Each point inside the circle determines one chord, which is perpendicular to the diameter so the probability is ¼. (See figure 2. The smaller circle is of radius $r/2$. It is easily seen that the chord is longer than a side only if its centre point is inside the inner circle.)

(iii) Any two points on the circumference of the circle determine the chord which joins them, which makes the probability $\frac{1}{3}$. (Figure 3: Let A be one of the two points. The chord is only longer than a side of the triangle if the other point is further from A on the circumference than an apex is.)

EDITORIAL

The latest Courses handbook supplement has details of the M101 course which will replace the old faithful, M100. There will be no set book for the course but students will be expected to purchase (that's right, you mustn't buy it or even steal it) a calculator "to investigate mathematical properties". So *that's* why Newton and Gauss failed so lamentably. The 1978 student will, then, be assumed to be not only unable to add but also unable to read. One might well say with the late Mr Shakspear, "O brave new world. That has such people in't."

The correspondence raging about Prisoners and MOUTHS, which will be closed by Marion Stubbs next month, has prompted a couple of thoughts in me. First: do you think it is fair that some members of a voluntary society are exposed willy-nilly while others can hide behind a shroud of anonymity? Secondly, for those who keep bringing up 'the constitution'; have you read it recently? Thoroughly? That's all. Personally I am a pragmatist who thanks the Lord for idealists.

(Perhaps no-one else would think it relevant but I am reminded in all this of the story told by Tristan Tzara about the breakup of the movement-Dada, which foundered on a moral dilemma. The cases are not in any way similar but I think the incident is instructive. Dadaism was founded in 1917 with the stated intention of undermining the fabric of society in order to build it anew, fresh and aware of the things that really matter. One day a waiter at the Café Voltaire went home and left his wallet on a seat. The Dadaists who were present knew they were bound by their manifesto to keep the wallet, but they also knew the waiter had many starving children. In the end they gave the wallet back but were so ashamed of this action that they were unable to face one another and the movement drifted apart.)

S Bull has just written with some suggestions for naming the restricted MOUTHS list: Brackets, Parenthesis or Diffraction Set, (light is diffracted as it goes through a grating!) Closed Mouths seems to me to be perfectly adequate, or why not Inside Mouths (Teeth).

People are still sending me cover designs. This is alright but it would be better if they went direct to Marion as by tacit agreement she is doing all the selecting in that department. To save hassle they should be in jet black on good white paper, not on old scraps of computer and other office waste; please!

Here are the diagrams for Bertrand's Paradox:

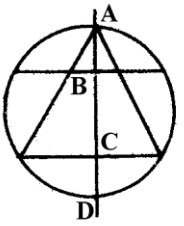


Figure 1

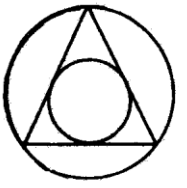


Figure 2

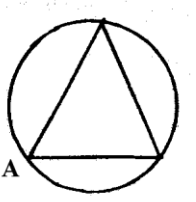


Figure 3

Eddie Kant.