M500 44

M500 is a student operated and owned magazine for Open University students and staff, and friends. It is designed to alleviate student academic isolation by providing a forum for public discussion of the mathematical interests of students.

Articles and solutions are not necessarily correct but invite criticism and comment. Anything submitted for publication of more than about 600 words will probably be split into installments.

MOUTHS is a list of names and addresses, with telephone numbers and past and present courses of voluntary members, by means of which private contacts can be made to share OU and general mathematical interests - or to form self help groups-by telephone or correspondence.

THERE IS ALSO A SPECIAL LIST OF THOSE MOUTHS MEMBERS WHO HAVE EXPLICTLY VOLUNTEERED FOR THEIR MOUTHS DETAILS TO BE DISTRIBUTED TO MEMBERS IN CLOSED INSTITUTIONS, SUCH AS PRISONS.

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PRINTER	Waterside Printing Company 11 Rumbridge Street Totton	

M500 44 published July 1977; subscription £3 for 10 issues. Cheques and postal orders (which should go to Peter Weir) should be made payable to THE M500 SOCIETY and crossed ac payee only not negotiable for safety in the post.

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LINEAR DIFFERENCE RICHARD SHREEVE

THE METHODS OF SOLUTION OF LINEAR DIFFERENCE EQUATIONS

Definition: An ordinary DIFFERENCE EQUATION is a relation between successive values of a discrete variable y_n for $n = 0,1,2,...$

$$F(x, y_n, y_{n+i}, ... ,y_{n+r}) = 0$$

where r is the ORDER of the equation. This equation is linear if it is

$$y_{n+r} + a_i y_{n+r-i} + ... + a_{r-i} y_{n+i} + a_r y_n = \phi(n) \qquad (1)$$

where the coefficients are constants or depend on n.

There is a corresponding HOMOLOGOUS form

$$y_{n+r} + a_i y_{n+r-i} + ... + a_{r-1} y_{n+i} + a_r y_n = 0 \qquad (2)$$

If $y_n = f_1(n)$ and $y_n = f_2(n)$ are two solutions of (2) so is $y_n = A_1 f_1(n) + A_2 f_2(n)$ for any constants A_1 and A_2. Also if $y_n = f(n)$ is a particular solution of (1) and $y_n = f_1(n)$ a solution of (2) then $y_n = f_1(n) + f(n)$ is a solution of (1). Hence the general solution of the linear difference equation (1) is

$$y_n = A_1 f_1(n) + A_2 f_2(n) + ... + A_r f_r(n) + \overline{f(n)}$$

where $f_1(n), f_2(n), f_3(n), ..., f_r(n)$ making up the COMPLEMENTARY FUNCTION are r different solutions of the homogenous form (2) and where $\overline{f(n)}$ is the PARTICULAR INTEGRAL any solution of equation (1) and where $A_1, A_2, ..., A_r$ are arbitrary constants.

The notes which follow suggest methods of solution and explain why those used are applicable.

FIRST ORDER

$$y_{n+1} + ay_n = \phi(n).$$

The complementary function is $y_{n+1} + ay_n = 0$ or $y_{n+1} = -ay_n$. Hence $y_n = (-a)^n A$, A being a constant ($= y_0$). For a particular integral if $\phi(n) = b_n + c$ try $\alpha_n + \beta = y_n$. Complete solution will be

$$y_n = A(-a)_n + \alpha_n + \beta.$$

The function to substitute when obtaining the particular integral can only be found by inspection.

SECOND ORDER

$$y_{n+2} + ay_{n+i} + by_n = \phi(n) .$$

Complementary function $y_{n+2} + ay_n+1 + by_n = 0$. From the type of solution obtained to the first order equation the substitution is made $y_n = \lambda_n$; therefore

$$\lambda^{n+2} + a\lambda^{n+1} + b\lambda^n = 0.$$

If $\lambda = 0$ then nothing can be added to the particular integral therefore take $\lambda \neq 0$ and cancel λ^n to get the AUXILIARY EQUATION $\lambda^2 + a\lambda + b = 0$ with two roots λ_1 and λ_2. The complementary fn is

$$y_n = A_1\lambda_1^n + A_2\lambda_2^n .$$

➔

The nature of this result depends on the discriminant:

(i) a^2 is greater than $4b$, there are two distinct roots.

(ii) a^2 equals $4b$, there are two coincident roots $y_n = \lambda^n$. Substitution of $y_n = n\lambda^n$ shows that this is also a root so the complementary function is $y_n = (A_1 + A_2 n)\,\lambda^n$

(iii) a^2 is less than $4b$, there are two conjugate complex roots λ_1 and λ_2. If these are written in the form $r(\cos\theta \pm i\sin\theta)$ then

$$\begin{aligned} y_n &= r^n(A_1(\cos\theta + i\sin\theta)^n + A_2(\cos\theta - i\sin\theta)^n) \\ &= r^n(A_1(\cos(n\theta) + i\sin(n\theta + A_2(\cos(n\theta) - i\sin(n\theta))) \\ &= r^n(B_1\cos(n\theta) + B_2\sin(n\theta)) \end{aligned}$$

by substituting $A_1 + A_2 = B_1$ and $i(A_1 - A_2) = B_2$

$$\begin{aligned} y_n &= Cr^n\,(\cos(n\theta)\cos\varepsilon + \sin(n\theta)\sin\varepsilon) \text{ where } \cos\varepsilon = B_1/C \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{and } \sin\varepsilon = B_2/C \\ &= Cr^n\cos(n\theta - \varepsilon)\,. \end{aligned}$$

Hence the complementary function. The particular integral may be obtained from inspection and trial and error. Hence the solution.

THIRD ORDER AND HIGHER ORDERS

Since every polynomial function with constant coefficients has as many roots as its order there are as many roots of the auxiliary equation as the order of the equation.

The methods applied to the second order equation may be extended to higher orders of equation.

SQUARE ROOT COMPETITION PETER WEIR

PRIZE WINNERS

As promised here are the prizewinners for the competition first posed in M500 39. The problem set was to come up with a good square root algorithm.

Third prize, a 15p luncheon voucher (to provide food for thought) goes to Cyril Whitehead / Andrew Seeney. Basically their method uses the fact that

$$\sum_{i=1}^{n} i = \frac{n^2 + n}{2}$$

to get approximations from below, digit by digit. Inefficient but stylish. Cyril used it on a hand calculator in the early 50s.

Second prize, a paying in slip to my account, goes to Tony Brooks. His method was bribery and I wish to encourage him. As for the square root routine, it goes something like this: E_0 = initial estimate: $E_{n+1} = (C)(E_n)$ where $C = 2(14k^5 - 5k^4 + 2k^3 - k^2 + k) + 1$; $k = \frac{1}{4}((x/E_n^2) - 1)$. ➔

I trust him that it works! Quite where he gets the formula from I don't know (treat this as a self-assessment question). I wrote to him asking but I may have forgotten to put a stamp on as he didn't reply.

First prize, a lp cheque (a penny for his thoughts) goes to James Chappell for this profound algorithm:

```
1Ø   INPUT N
2Ø   IF N <  Ø OR N >  2ØØØØ THEN 7Ø
3Ø   Q = RND(l)
4Ø   IF ABS   ((Q*Q)–N)>Ø.ØØ2 THEN 3Ø
5Ø   PRINT "ROOT OF"; N;   "IS";  Q;   "GIVE OR TAKE  .ØØ2"
6Ø   GO TO 8Ø
7Ø   PRINT "N OUT OF BOUNDS.    GO TO JAIL.    DO NOT COLLECT £2”
8Ø   END.
```

Note RND(l) denotes a random number, ABS delivers modulus: |x|.

My experience with computer users suggests that the omission of line 4Ø would not be noticed by everyone. A truly magnificent effort.

*   *   *   *   *   *   *   *

THE WHITEHEAD–SEELEY ALGORITHM

```
1Ø     DIM A$ [2Ø] , Y$ [2Ø]
2Ø     A$ = “Ø123456789."
8Ø     PRINT "NUMBER" ;
9Ø     INPUT N
1ØØ      PRINT
11Ø      PRINT  "ROOT OF";  N;   "=";
122      B = Ø
13Ø      S = –1
14Ø      C = Ø
15Ø      C = S +  2
16Ø      N = N –  S
17Ø      C = C + 1
18Ø      IF N  >=  Ø  THEN  15Ø
19Ø      N = N + S
2ØØ      C = C  – 1
21Ø      B = B + 1
22Ø      IF C <  1Ø  THEN 28Ø
23Ø       A = INT  (C/1Ø)
24Ø      D = 1Ø  *  (C/1Ø  –  A)
25Ø      PRINT A$  [A+1,  A+1] ;
26Ø      PRINT A$  [D+1,  D+1J ;
27Ø      GOTO 29Ø
28Ø      PRINT A$   [C+1,  C+l] ;
29Ø      IF B  >  1 THEN  31Ø
3ØØ      PRINT A$   [11,  11] ;
31Ø      IF B = 11 "THEN      35Ø
32Ø      N = N *  1ØØ
33Ø      S =   (S – 1)   *  1Ø –  1
34Ø      GOTO  14Ø
37Ø      PRINT
38Ø      PRINT "ANOTHER NUMBER"
39Ø      INPUT Y$
4ØØ      IF Y$  = [1 , 1]  = "Y"  THEN  6Ø
41Ø      END
```

$\frac{\sin x}{\text{n}} = 6.$ Why? Cancel the n. (Arts student of mathematics; from SD)

Sin 105° = $\frac{1+\sqrt{3}}{2\sqrt{2}}$ since sin 105° = sin 45° + sin 60° = $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}}$ (JH)

LETTERS

Michael McAree: Was Relative Truth an April Fool? I can't say. I only know it kept me amused for some time. For problems like these I always expect someone to come up with an analysis of the question which upon reading and absorbing for ten seconds yields all solutions. (And this would have to be part of the "April Fool", a simple solution.)

Here in Northern Ireland M500 is seriously under represented, having only a handfull of members. Strange to say two of them are doing M201 (same study group); that is myself and one other student. And our course Tutor is a member. I must point out that I joined without any encouragement so this situation seems to be a coincidence. I mention this only in passing; my main point being a request for serious consideration to be given to more simple problems and more simple discussions on general lines. Does no-one write mathematical articles that aim at first level students (who obviously are the more vulnerable) to encourage them to contribute themselves? Or let us have some feedback on courses so that continuing maths students can guage what lies in store if they take a particular course. Indeed let us have some criticism of courses from students on those courses. Does no-one have difficulty with M201 for example? I must confess to being overawed in the differential equations units by the sight of electrical networks. I have heard students complain that this course (so far) has been too compressed, leaving no time for consolidation, or indeed for reflection on the many new concepts introduced. I would like to see comments from students on courses as they actually do them.

Tony Mann: May I add my views on the use and need of electronic calculators? The problem with pocket calculators is that should the operator make an error - and that's just about 100% of errors! - then he or she won't necessarily know that an error has been fed in. This is more so under conditions of stress, ie the housewife in the busy supermarket or the student in the examination room. So I'll argue that unless the machine has a printout - it's hardly 'pocket' then - the value of such a machine is limited.

So I go along with Bill Midgley and continue to use pieces of paper, bits of chalk, etc. As far as MDT241 is concerned I managed without the use of any device other than my well-worn slide rule and the tables supplied with the course material.

So Ray Partridge don't buy a calculator unless you really have to join the conned public. Calculators are today what freezers were in 1970 and Hula Hula Hoops in the 50s - the IN thing; it's the world of commerce's means of parting you from your money by making you think you need something that you really don't. So either, if you have to, get a machine with a printout (and as many buttons - functions - as you care to pay for) or one with as few buttons as possible (ie the lower chance of inputting an error).

Alternatively you might try playing darts. This will give you much practice in adding and subtracting numbers which is all you should require for MDT241.

Brian Woodgate: It might be interesting if a member of the faculty were persuaded to write on the subject of course planning and to explain policy.

eg Why is M231 Analysis to go? Surely this is one of the most popular and successful maths courses. On the other hand MST282, which has had a very poor 'press' in Special Issues is to stay.

One notes that the new course M203 has no set books. Does this mean that Spivak is no more as far as the OU is concerned? If so then I was lucky to come through the system at the right time to meet this work and to profit from it.

Are there any plans for a third or fourth level full credit course? Perhaps something could be designed on the lines of the Technology course T401, which is planned as a project course.

Could I request also, either in this publication or in Course Guides, more background material. eg Consider M331. If Lebesgue was the first to propose the integral bearing his name what got him started? Why was he not satisfied with the Rlemann integral? Did he realise the problem of completeness? This sort of question is bound to come as one becomes interested in a subject.

I have read the note by Allan Solomon in M500 41 but although this explains the difference it does not tell the background story. I suppose that what I am asking for is 'popular' maths history, ie background notes as compared with AM289 which one hears is largely a string of names, dates and facts, and is much too academic.

Tony Brooks: I was very pleased to see in a recent OU mailing that they hope to make all courses available on the associate student programme. If this turns out to be true then I shall be pleased. This may mean that one day I shall be able to take all the maths (and other interesting courses) courses that I have not been able to do as an undergraduate.

I received a piece of very pleasing and surprising news recently. After my election to graduate membership of the Institute of Mathematics I made some further enquiries about becoming an Associate Fellow. As a result I applied to become an Associate Fellow and I received a letter from the Institute a short while ago saying that I had been elected to this higher grade of membership. I must say I did not really expect it less than two months after becoming a graduate member.

I have succeeded in persuading two British employees at Westinghouse to apply to do M101 next year. I told them both to quote their UK addresses at least until they heard whether or not they had been accepted. I don't believe that the OU will normally accept overseas students (even if they are British) although it does allow you to continue your studies if later one moves overseas (as in my case). I have tried to interest them both in M500 and I think one of them will subscribe if he is taken on by the OU.

Jim Sheridan: Many thanks for application form and sample M500, I am very happy to join you.

I am a television engineer and I travel abroad a lot too. I am presently senior engineer for Rediffusion Doric Tv in Ireland and so am based near Clonee, Co Meath. If there is anyone near ➔

me I hope you will let me know. I stay mostly ... within a couple of miles of the Ambassador's Residence.

I gave all the M500 literature to our Counsellor, who is very interested in the Weekend, and she told the other members that were present about it, so I hope you will receive other enquiries. I am due at Stirling University for Summer School 16th to 23rd July. I will fly in to Edinburgh on 15th and I will stay with my cousins there.

Though I am 59 years of age I am young in spirit, ideas and outlook. You will thus understand that modern maths is something new to me all the way. I have always studied in my own field so learning is nothing new. I wish M500 every success and while the situation is so unsettled in the North I would prefer any letters to come to the above address to ensure that I get them.

I hope Marion gets her 200 for the WE Work In. I am looking forward to it now, though her date for full fee is tough - considering Summer School fees plus air fare £50. I just mention it as Foundation students have large expenses at this time. By the time I got the Summer School allotment all boat reservations were completely booked out; air fare is normally only £35 but on weekends in July is £50, plus school fee of £49, plus travel from Edinburgh to Stirling. Its all in a good cause. Thanks for listening to me. Best wishes.

Russell Brass: I'm puzzled over the snatch (Bandersnatch?) of dog latin on page 42 4 (and 43 6 and this issue - Ed). Is the explanation that Lewis Carroll, when not photographing little girls in the Snarkers, took time out to write mathematical books under the ridiculous name of Dodgson?

Or could it be that some of our more erudite contributors have discovered the secret that in the *Snark* Babbage is portrayed as Butcher, de Morgan as Banker and Lady Lovelace (our first programmer) as Beaver?

For more details see Roger Millington in *Computing*, 1 April 1976 and 13 May 1976.

John Hampton: CONTINUOUS ASSESSMENT. I am surprised by the small response to Max Bramer's note (39 6) on this topic. Like some other students (Eric Lamb 39 7 and Ken May 41 5) I am a poor examination candidate. Irrational though it may be I have a nervous disposition and tend to suffer severe pre-examination anxiety and stress. I leave the examination room in a state of nervous exhaustion and my examination performance is well below the standard I could normally achieve. Because of this I put very considerable effort into my continuous assessment work and attempt to do really well at it. I look upon it as general insurance against course failure and a protection against the way I go to pieces in examinations.

I attempt to complete all assessed and non-assessed tutor marked assignment questions. The average time I spend per question is about two hours, but this figure is biased as in several cases it has been doubled or trebled when I have had particular difficulty in getting a solution out. I advocate "excellence is insufficient, only perfection will do", for although ➔

a hard (in fact impossible) precept it does mean I get a great deal out of what I do. In particular I feel elated when a problem I thought I could not do comes out, but suicidal when I cannot do it or I am wrong. I find computer marked assessment questions equally challenging too and my average time spent on them is about three hours per course unit. For M100 (pass), M201 (grade 2) and M202 (in progress) my "running average" continuous assessment marks are TMA assessed 9.60, non-assessed 9.38 and CMA A. So my view is that hard work pays and that the time I spend on continuous assessment is certainly not wasted.

Another virtue of continuous assessment is that it offers a means of keeping up with a course and guaging one's understanding of the material studied. I always try to get my work in by the due date.

Whilst examinations are demanded by society to maintain supposed "academic standards" I nevertheless hope that the OU will place more weight on continuous assessment in the future. In particular I consider that any student who can achieve an overall A or B continuous assessment grade should never be failed on a course due to unsatisfactory examination performance.

John Wills: ISRAELITE: member of a Semitic nation constituted to further the Yahwistic religion; it effectively absorbed the Canaanite nation around 1000 BCE; about 500 BCE it developed Yahwism into Judaism; became Christian 50 CE - 300 CE; known as Palestinian since about 500 CE; 50% Moslem at least since the crusades.

ISRAELI: member of a Turkic nation, originally called Khazar, converted to premessianic Judaism c750 CE under King Bulan, to Talmudic Judaism c800 CE under King Obadiah, migrated in large numbers to Israel 1900-1970 CE, since 1948 CE in constant war with the Israelites.

So don't mix them up! (M500 43 18.)

Godfree Brunton: I'm enjoying M500 - but with only M100, S100 and M201 (withdrew ¾ way through illness) am one of those often baffled by terms I've never heard of. I may do M202 soon just to make my MOUTHS sub pay!

Percy Sillitto: HOW TO BE A LAZY EXAMINER. All you need do to be an examiner and indulge your laziness is to specify that the examination should be CLOSED BOOK and then set questions like "State and prove the - - Theorem." You don't need to know the answer yourself because you mark the paper with the book OPEN, deducting marks for every departure from the text.

I was glad to see several complaints about closed book examinations in the M500 Special Issue 1977. A hint, however. I know one student who got a distinction by tape-recording all the matter to be memorised, and playing the tapes back repeatedly while doing other things like driving to work, taking a bath, etc. Thus absorbing it subliminally. Whether the niggly details were understood or not didn't matter; they were embedded in the memory ready for printout as intelligently as a computer could have done.

LETTER FROM AMERICA *Dan Fox - University of Maryland*

The vast majority of urban schools here do not spend a great deal of effort on science/math. This is partly due to lack of money and partly lack of commitment on the part of the schools. Mainly tho, and unfortunately, the schools in cities seem to be mere "day care" centers - all the way thru highschool. I went to undergrad school in Chicago and while there got involved in several programs to prepare local high school students for college. I was shocked at how little they knew; we had to start at 9th grade general math and barely got into trig functions at the end of the first year. Almost no one could solve an equation like $x = 2 + 3x$; they just didn't see that you could subtract an x from both sides. The questions these high school seniors asked made it clear they hadn't gotten anything out of their last four years of math. Inequalities turned out to be a major project as did graphing simple functions. Very sad. One good point: the people involved in this program did significantly better in their first college year than the average Freshman.

Now that I am 'out' I am not surprised at the lack of mathematical ability displayed by the engineers (mostly electronics engineers) here in our office. Their usual method of solution is to delve into weighty tomes of formulae. Only the physicists and mathematicians seem capable of handling anything unexpected. Meanwhile Society has decided the engineers are more valuable than we are. Bachelors degree engineers straight out of college make as much as masters degree physicists and mathematicians.

At this point I realised our educational system - or at least the names we use - must be different from yours. Thus I'll describe all the words. If you know this already please to forgive.

Around age 6 we start public school. That lasts 12 years. Grades 1 thru 6 are called elementary school; 7 and 8 are junior high school and 9 -12 are senior high school. People in grade 9 are called freshmen, grade 10 = sophomores, grade 11= juniors and grade 12 = seniors.

Most schools from grade 1 - 6 are run as follows - you spend all day in a single classroom with 30 to 50 other kids. In junior and senior high school we have 5 to 7 classes each day and the same schedule is repeated each day Monday thru Friday.

Some 'progressive' high schools vary their schedules each day and allow free time for the student to go off campus or just lie around. The student in this case is free (within certain limits and requirements) to take any courses he wants. This is how I was able to get a programming course as an elective.

After high school we have four years of college at which point you get a Bachelor's degree. Then the next degree is a Master's degree (usually only 1 to 2 years more work depending on the major). Then, finally, a PhD degree (another 2 to 5 years). So we have the sequence

Bachelor of Science	→	BS	→	<u>b</u>ull<u>s</u>hit
Master of Science	→	MS	→	<u>m</u>ore of the <u>s</u>ame
PhD			→	Piled Higher and Deeper

Cute. ➔

Math does exist at Maryland. Supposedly (if you believe school propaganda) it is one of the best in the country. That rating only applies to the graduate school. The average UM student only sees something else. Since there are over 20 000 undergraduates we get more than 5 000 freshmen all taking the first year math courses. This is handled by having huge lecture hall classes of 200 students. Then there are smaller classes run by graduate students, to work problems and generally clarify the lectures. Also the math lectures exist on videotape cassettes that run continuously in the library. You just sit at a desk with a special tv monitor and type in a code number for what program you want to watch. The Audiovisual floor in the undergrad library is really fantastic. You can check out cassettes and watch them in little individual rooms. Alistair Cooke's *America* series is available as is the Nova science program. These are BBC related so maybe you've seen/heard of them. A lot of old movies (*Citizen Kane*.) and avante-garde video stuff is also there. There are about 50 video booths and maybe a few more audio booths with cassette players so you can listen to popular, current music. Anyway, getting back to the math department; they are the only ones that had to resort to ultra-mass education techniques.

Needless to say; with such large numbers of students the registration week is abysmal. Pre-registration the previous semester usually does no good for freshmen or sophomores since so many people are trying to get into the best timed sections (ie something other than 730am or 830pm sections). I would be interested in hearing how your OU works and what it really is. The impression I have is not really clear.

I was just out on a boat 80 miles into the Gulf of Mexico. For 2 weeks. Shell have this policy that new geophysicists should actually see how they get their data. So in our case that means two weeks at sea. Their ships (~180' long; 24 people) tow a 2 mile long cable that has microphones every 100 feet. An airgun is shot under the water near the boat and the reflections from the various layers under the sea bottom are recorded. Using VAST amounts of computer time, a very decent picture of the ground for about 2 miles down is produced. 'VAST' translates as follows: the exploration division (us) has 2 ships soon to be 3) and it takes four Univac 1100/42 computers to handle the processing. The 1100/42 is the largest Univac makes for commercial purposes. They're all in Houston Texas and we connect via a microwave link. Here on our floor there is a 'little' (72 000 word memory) Harris/Datacraft Slash HVMS computer. It handles a lot of little chores but is too small to run any of the processing programs.

PS. How did your postal service come to use such a strange sort of zipcode? Ours just numbers all the post offices, with the first 2 numbers being a state code: 70xxx = Louisiana, 20xxx = Maryland, etc.

Ed - We seem to have Dan because he bought Marion's Dice Star Trek and also because Maryland University use OU material. Has anyone written to him apart from Marion. He seems to enjoy corresponding.

MS - DST is now out of print so no more orders, please, at any price.

THE RERNOUILLIS ALAN URE

The Bernouilli family produced no less than eight distinguished men of science. As mathematicians we probably associate their name with Bernouilli's Theorem and Bernouilli numbers but this family's achievements during the eighteenth century covered many disciplines.

1	James: mathematician.	b Basle 27 December 1654 d Basle 16 August 1705.
	Ars Conjectandi, ie art of guesswork published posthumously in 1713; Bernouilli's theorem and Bernouilli numbers.	
2	John I: chemist and mathematician.	b Basle 7 August 1667 d 1 January 1748.
	Developed the integral calculus and discovered, independently of Leibniz, the exponential calculus; the first to use polar coordinates.	
3	Nicholas: mathematician	b Basle 10 October 1687 d Basle 29 November 1759.
	Posed the problem of the St Petersburg paradox.	
4	Nicholas: mathematician	b 1695 d St Petersburg July 1726.
5	Daniel: mathematician.	b Groningen 9 February 1700 d Basle 1782.
	Systematised hydrodynamics (1738) and made an early effort at kinetic theory; tackled the St Petersburg paradox by developing the idea of the moral value of money.	
6	John II: mathematician,	b Basle 18 May 1710 d 1790.
7	John III; astronomer and mathematician.	b Basle 4 December 1744 d Berlin 10 July 1807.
8	James: mathematician and physicist.	b Basle 17 October 1759 drowned in the Neva July 1789.
	Married a grand-daughter of Euler.	

The family tree shows James and John I as brothers. Nicholas (b 1695), Daniel and John II sons of John I, and Nicholas (b 1687) his nephew. Then John III and James sons of John II.

Ed - The St Petersburg paradox: In the words of Maurice Kraitchik (Mathematical Recreations)

A and B play under the following conditions: A is to toss a coin until it falls heads, at which time the game is to stop and A is to pay B 2^{n-1} *dollars, where n is the number of throws. How much should B stake on the game*?

Let us calculate the value of B's expectation. He has probability ½ *of winning* $1, *probability* ¼ *of winning* $2, $1/8$ *of winning* $4, *and in general he has probability* $1/2^n$ *of winning* 2^{n-1} *dollars. Hence the value of his expectation* is $\frac{1}{2} . 1 + \frac{1}{4} . 2 + \dots$, *which is a limitless infinite sum. Thus no matter how much B pays he buys a bargain.*

There are 5½ *pages of analysis of this paradox in Kraitchik, including the rather spurious formula developed by Daniel. Read it..*

HOW LONG IS A STRING? TONY FORBES

When I go to work in the mornina I have to get through the front door by inserting a plastic card into a lock and typing out a certain 4-digit number on a keyboard. On the rare occasions when I am fully awake I get the number correct and the door opens. However usually I don't and it doesn't; so I continue typing in digits and the door opens as soon as it recognises the correct sequence. This suggests a problem;. What is the minimum length of a string of digits which contains every 4-digit number? The answer is in fact 10003 and in what follows you are invited to prove this.

More generally, suppose you have *n* different symbols.
Consider strings of these symbols and call a string (*n,m*)-complete if it contains every one of the *nm* sequences of length *m*. For example

abbbaaabab

is (2,3)-complete since it contains the eight sequences *aaa, aab, ..., bbb*.

It is easy to show that an (*n,m*)-complete string must be at least *nm* + *m* – 1 long. The problem is for which (*n,m*) are there (*n,m*)-complete strings of length *nm* + *m* – 1. Let us call such strings minimal.

If we also insist (as in the example above) on making the last *m*–1 symbols the same as the first *m*–1 then the problem can be stated thus:

For which (*n,m*) is there a permutation *f* of the group J_{n^m} = ({0, 1, ..., n^m – 1} with addition modulo *nm*) such that

$$0 \le f(p) - nf(p-1) < n \quad \forall p \in J_{n^m}.$$

(Think of bending the string into a circle using *m*–1 symbols to join the ends together and treat sequences of *m* symbols as *m*-digit *n*-ary numbers.)

I suspect that minimal (*n,m*)-complete strings exist for all (*n,m*) and here (in the form of a list of problems) is a brief account of my attempt to prove this.

1 Prove by induction that minimal (*n*,2)-complete strings exist.

2 Assume you have a minimal (*n*,3)-complete string and you want to construct a minimal (*n*+1,3)-complete string with *x* as the extra symbol.

3 Construct four strings:

i. The minimal (*n*,3)-complete string you already have. Its length is n^3+2.

ii. A string of length 3n^3+2 containing each of the 3n^3 sequences of length 3 which contain exactly one *x*.

iii. A string of length 3*n*+2 containing each of the 3*n* sequences of length 3 with two *x*'s

iv. The sequence *xxx*.

4 Somehow splice these four strings together to give an (*n*+1,3)-complete string of length $(n+1)^3$ +8. If you are ➔

clever you can do this in such a way that you can also discard six symbols and still retain (n+1,3)-completeness.

5 2, 3 and 4 thus form the induction step of a proof of the existence of a minimal (n,3)-complete string.

6 Now try a similar method for (n,4). The only extra complication is finding a string of length $6n^2+3$ containing all length 4 sequences with two x's.

7 For (n,5), (n,6), ... it looks is if the same method will work but things start to become very messy.

This is about as far as I have got. I believe that the induction step from (n,m) to (n+1,m) will always work and that the minimal (n,m)-complete strings exist for all (n,m). Maybe there is a completely different and much easier way of proving this rather than trying to generalise the induction step. Or perhaps it's not true anyway .

I

$$\int_a^b g \times (Df) = [g \times f]_a^b - \int_a^b f \times (Dg)$$

D

INTEGRATION BY PARTS

Ray Morland, OU Summer School Reading, 74

...

Dum requiescebat meditans uffishia, monstrum
 Praesens ecce! oculis cui fera flamma micat.
Ipse Gaberbocchus dumeta per horrida sifflans
 Ibat, et horrendum burbuliabat iens!

...

PROBLEM SECTION EDITED BY JEREMY HUMPHRIES

I seem to have a lot of material this month, so I'll get to the solutions quickly and chat as I go.

There is one thing. Some of you are calling me 'Mr Humphries' and even 'Problem Editor - Dear Sir'. Consequently I am putting on airs and am about to become insufferable. My friends wish you would all call me Jeremy.

SOLUTION 41.4 CROSSNUMBER. SUE DAVIES and various non-members.

Across. 1:38720; 5:32; 6:44; 7:352; 8:1610; 10:72; 11:1913; 14:792; 15:27; 16:16.
Down. 1:355; 2:7396; 3:22; 4:142; 6:45; 7:30976; 8:12; 9:11; 10:792; 12:19; 13:325.
Nobody but Sue sent a solution. No need I suppose. You know if you've got it right. However, many have said that they enjoyed it, so, for your further delight, there's another one this month.

SOLUTION 41,5 DISC COVERING. (*How many discs of unit radius are required to cover a disc of radius $n \in \mathbb{N}$?*)

CHRIS PILE writes: As I posed the question I feel that I should offer some contribution to the solution, though I doubt if my strategy is anything close to optimum.

I propose an attack which is based on finding the smallest number of unit discs which will fit round the circumference of the disc of radius n. Clearly this number is approximately πn.

For n=2, six discs just suffice, and a centrally placed disc completes the covering. For n=3 retain the solution for n = 2 and find the additional number of discs required; $= \pi 3$. Nine discs are not quite enough as the inscribed nonagon has a side of 2.05. Ten suffice and these overlap the existing discs to complete cover. Total 17.

Continuing, we find that n=4 requires 13 outer discs and these overlap those for n = 3, completing cover with 30 discs.

It may happen that the discs around the circumference do not overlap existing discs. Then more discs must be added until the outer ring can be moved inwards far enough to cover. For very large n, the number of discs in the outer ring is approximately $2\pi n/\sqrt{3}$, which is the number of unit discs required to cover a strip of size $(2\pi n \times 1)$. The table shows the efficiency.

Large disc		Unit disc		
Radius	Area	Number	Area	Area ratio
1	π	1	π	1
2	4π	7	7π	0.571
3	9π	17	17π	0.529
4	16π	30	30π	0.533

For large n the ratio of areas in the outer ring is approximately $\sqrt{3}/\pi = 0.551$. ➔

Chris goes on: "If anyone submits a better solution throw this in the wpb." Nobody has however, nor a worse one.

I think that for large n it would be better to go back to the regular covering used for $n-2$, ie the centres of the discs form a net of equilateral triangles of side $\sqrt{3}$. The density of this packing, D, is 1.209... and the area ratio, $1/D$, is 0.828... . For some n Chris's strategy will yield to this one. I think. Investigate, anyone?

Two books I have which treat coverings and packings are: *Regular Figures* by L Fejes Toth, mentioned by RICHARD AHRENS last month, and *Unsolved and Unsolvable Problems in Geometry* by Herbert Meschkowski, Oliver and Boyd 1966. Martin Gardner's 1966 book *New Mathematical Diversions* has a chapter on sphere packings, but that is now out of date. The problem of packing hyperspheres in n-space had stuck at $n = 8$. Last year I went to a lecture on this by John Conway at Cambridge. He says that n is now up in the mid twenties and many marvellous things have come out of the studies. Does anyone know of an up to date non-technical treatment?

By the way, if you get a chance to see Dr Conway do so. He's excellent. See him create all numbers out of nothing by means of the Dedekindest cut of all.

SOLUTION 42.1 SAFETY II. (*Place all the white pieces, the black King and as many black pawns as possible on the chessboard, with no attacks*.)

STEVE AINLEY proposed this one, and sent a solution with one black pawn, and STEVE MURPHY has managed to use four black pawns:

```
.  R  .  .  .  .  .  .
N  .  .  bK .  bp .  Q
B  .  .  .  .  p  .  p
B  .  .  .  .  p  .  p
N  .  .  K  .  p  .  p
p  .  .  .  .  .  .  .
.  .  .  .  .  .  Q  .
.  .  R  .  .  .  .  .
```

STEVE AINLEY

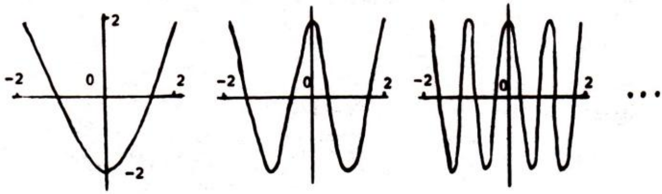
```
N  B  B  N  .  .  .  K
.  .  .  .  R  .  .  .
.  .  .  .  .  R  .  .
p  p  .  bK .  .  .  p
bp .  .  .  .  .  .  p
p  .  p  p  .  .  .  .
p  bp bp bp .  .  .  .
.  .  .  .  .  .  Q  .
```

STEVE MURPHY

Steve Ainley also asked: can the position be reached in actual play? I think his position can. Steve Murphy's on the other hand can't. Look at his six white pawns on the Queen's side. None of then can have started off any further 'east' than King 2. This requires a starting position with six pawns on five squares. Dirichlet's pigeonhole principle says that this implies that at least one square contains at least two pawns - and that's against the rules.

SOLUTION 42.2 OLYMPIAD II. (*Let* $P_1(x) = x^2$ and $P_j(x) = P_1(P_{j-1}(x))$ for $j = 2, 3, \ldots$. *Show that* $\forall n \in \mathbb{Z}^+$ *the roots of* $P_n(x) = x$ *are real and distinct*.} ➔

RICHARD AHRENS: If $P_j(x) = 0$ then $P_{j+1}(x) = 0^2 - 2 = -2$ and if $P_j(x) = \pm 2$ then $P_{j+1}(x) = (\pm 2)^2 - 2 = 2$. This is enough information for a rough sketch.



To get from one graph to the next: All points where P_n is ± 2 become points where P_{n+1} is 2. All points where P_n is 0 become points where P_{n+1} is -2. If we now draw the line $y = x$ on the graph of $P_n(x)$ we can see that $P_n(x) = x$ has 2^n real distinct roots.

STEVE MURPHY: Problem 42.2 doesn't seem too difficult and it's not too hard to construct a direct proof. I thought this proof by induction interesting.

P_j has degree 2^j and so is continuous. $P_j(x) = 0$ cannot have more than 2^j distinct real roots. Assume that the following three propositions are true for some j (easily verified for $j = 1$).

I. $P_j(x) = 2$ has at least $2^{j-1}+1$ distinct real roots which can be written in increasing order $a_1, a_2, a_3, \ldots$.

II. $P_j(x) = -2$ has at least 2^{j-1} distinct real roots $b_1, b_2, \ldots$.

III. $a_1 < b_1 < a_2 < b_2 < a_3 < b_3 < \ldots$.

I, II, III and the Mean Value Theorem imply that $P_j(x) = 0$ has 2^j distinct real roots (ie all roots are real and distinct). Now if $|P_j(x)| = 2$ then $P_{j+1}(x) = 2$ and if $P_j(x) = 0$ then $P_{j+1}(x) = -2$. From these results if I and II are true for j they are true for $j+1$. If $c_1, c_2, c_3, \ldots$ are the roots of $P_j(x) = 0$ then $a_1 < c_1 < b_1 < c_2 < a_2 < \ldots$ and it follows that III holds for $j+1$. We now use induction to prove that $P_j(x) = 0$ has 2^j distinct real roots $\forall j \in \mathbb{Z}^+$

SOLUTION 42.3 REVERSION. (*p is a non-palindromic three digit positive integer. Reverse p = q. Find |p−q| = r. Reverse r = s. Then r + s = 1049. Why?*

Answers came from MAX BRAMER, SUE DAVIES, THURSTON HEATON, MICHAEL MCAREE and STEVE MURPHY.

$p = 100p_1 + 10p_2 + p_3$ then $q = 100p_3 + 10p_2 + p_1$. $r = |p-q| = |99p_1 - 99p_2| = 99n$ where $1 \leq n \leq 9$. Therefore $r = 100(n-1) + 90 + (10-n)$ then $s = 100(10-n) + 90 + (n-1) = 1089 - 99n$ so that $r + s = 1089$.

All the solutions were very similar to this one, which is ➔

Sue's. 1089 is $10^3 + 10^2 - 10 - 1$. For any base B the procedure produces $B^3 + B^2 - B - 1$. I hoped to see some M100 weapons used on this - equivalence classes, partitions, ring homomorphisms, etc, but I suppose simplest is best.

SOLUTION 42.4 INVERSION. (*Produce a large nonsingular matrix X such that for all partitions*

$$X = \left(\frac{A|B}{C|D}\right)$$

where A and D are square, A, B, C and D are singular.)

When I said singular I meant not invertible, and I apologise for any confusion. For most partitions *B* and *C* are not square and so not invertible. I included them with *A* and *D* in the problem just to add a bit of interest if any of your partitions made them square. I understand that in some texts 'singular' and 'not invertible' are not synonyms; singular matrices are required to be square.

STEVE MURPHY and MICHAEL MCAREE sent examples. Steve gave

$$A_n = (a_{ij})$$

where

$$a_{ij} = \begin{cases} 1 & (i,j) = (1,n) \text{or } j = i+1 \\ 0 & \text{otherwise.} \end{cases}$$

Michael's was (a), below. I didn't ask for it but this *can* be done so that every partition into four gives non-invertible submatricesi (b).

$$\begin{pmatrix} 0&0&0&0&1 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 1&0&0&0&0 \\ 0&0&0&1&0 \end{pmatrix} \qquad \begin{pmatrix} 0&0&1&0&0 \\ 1&0&0&0&0 \\ 0&0&0&0&1 \\ 0&0&0&1&0 \\ 0&1&0&0&0 \end{pmatrix}$$

(a) (b)

SOLUTION 42.5 ASSERTION. ($3|n(n-1)(n-2)$; $5|n^2(n^2-1)(n^2-4)$; $7|n^3(n^3-1)(n^3-6)$, *for all poiitlve integers n. Ttue? Generalise; prove.*)

Datta's problem attracted a group of our regulars: MAX BRAMER, SUE DAVIES, THURSTON HEATON, MICHAEL MCAREE and STEVE MURPHY. This is Max.

Yes it is true. In general $(2k+1)\,|a^k(a^k-1)(a^k-2k)$ $\forall a \geq 0$, $k \geq 1$, $(2k+1)$ prime and $a,k \in \mathbb{Z}_0^+$

Proof: If *p* is prime and *a* is a strictly positive integer, $a^{p-1} \equiv 1 \bmod p$ (Fermat's Theorem). Taking *p* an odd prime $= 2k+1$ we have $(a^k-1)(a^k+1) = a^{2k}-1 \equiv 0 \bmod (2k+1)$; ie $a^k \equiv \pm 1 \bmod (2k+1)$. This can be written in terms of positive values as

$$a^k \equiv 1 \text{ or } 2k \bmod (2k+1)$$

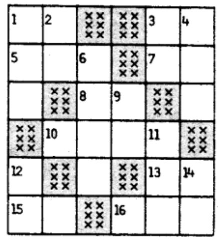
thence

$$(2k+1)\,|a^k(a^k-1)(a^k-2k)$$

(*Note: The a^k term can be replaced by a, but this seems to be the expected form of answer.*)

PROBLEM 44.1 ST SWITHIN'S SCHOOL. 1950. (*Heinemann Educational Books Ltd have very kindly given me permission to use some Crossnumbers from Fun With Figures by L H Clarke. This is the first one.*)

St Swithin's is a small school catering for young girls and presided over by that distinguished headmistress, Miss Murgatroyd. Miss Murgatroyd is very reticent about her age but if you are clever enough you will be able to discover it.



Across

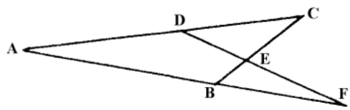
1 Most of tne girls leave at this age.
3 The number of girls in the school.
5 The headmistress hopes to increase the number of girls in the school by 100. The school will then hold this number of girls.
7 The age of Miss Murgatroyd when the school was founded.
8 The age Miss Murgatroyd will be (if she lives) in 1979.
10 The year (AD) when the school was founded.
13 Five times the number of staff.
15 Half of 9 Down.
16 same as 5 Across.

Down

1 Double the number of staff and girls added together
2 The number of girls (reversed).
3 Miss murgatroyd's age.
4 Twice the headmistress's age.
6 The year (AD) Miss Murgatroyd was born.
9. Seven of the girls are absent at the moment through sickness. How many are present?
11 The fourth power of the number of staff
12 Half of 8 Across.
14 The full number of pupils.

PROBLEM 44.2 GEOMETRY: RICHARD AHRENS

AB + BE = AD + DE
show that
AC + CE = AF + FE.



PROBLEM 44.3 JOBS: ANDY MCGOWAN

Three men have two jobs each:

1. The chauffer offended the musician by laughing at his long hair.
2. The musician and the gardener used to fish with John.
3. The painter bought a quart of gin from the consultant.
4. The chauffer courted the painters sister.
5. Jack owed the gardener £5.
6. Joe beat Jack and the painter at quoits.
7. One of them is a hairdresser and no two have the same job.

Who does What?

PROBLEM 44.4 OLYMPIAD IV (based on problem 4 of the 18th International Mathematical Olympiad, 1976 at Lienz Austria. It was the first problem of the second day, July 13 and ought to take approximately one third of four hours. For new readers this was the event where Britain came second to the Soviet Union. Sent in by KRYSIA BRODA.)

Determine, with proof, the largest number which is the product of positive integers whose sum is 1977.

PROBLEM 44.5 QUICKIES AND TRICKIES.

I Find the first and last number:

–, 121, 144, 202, 244, 400, 1210, 10201, – . (TONY BROOKS)

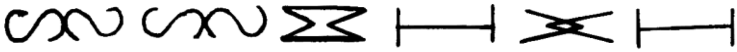
II Next two terms :

1, 2, 4, 11, 24, 112, 1000, ...

III Which salary scheme would you rather have: £200 increase per year or £50 increase per half-year?

IV What did Mark Twain say when asked "Why do you wear a white suit?" (1959 *Time*) Top marks for the right answer but anything printed.

V What is missing?



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By the way May 26 is very significant in the History of Mankind (see 43 editorial). John Wayne was born, the Venerable Bede died, Frankie Laine began his all time record dance marathon at Atlantic City (145 consecutive days), and John and Yoko began their bed-in to save the World.

And, finally, there is a book I've found recently which is really gripping. It is *Complex Approaches to Mathematical Problems* by Nievergelt, Farrar and Reingold, Prentice Hall. Praise for a computing type book from me is praise indeed. I find most of them a slog. But this one stands alone. Rave reviews - a fascinating work - it glows as a gem. Full of items of interest, eg Machin, π, games, random processes and so on.

EDITORIAL

This is all the space Jeremy has left me this month, so I must be brief. In fact all I'd better say is, sorry we're a bit late and please get some contributions in; or their might not be a 45.

Eddie Kent.