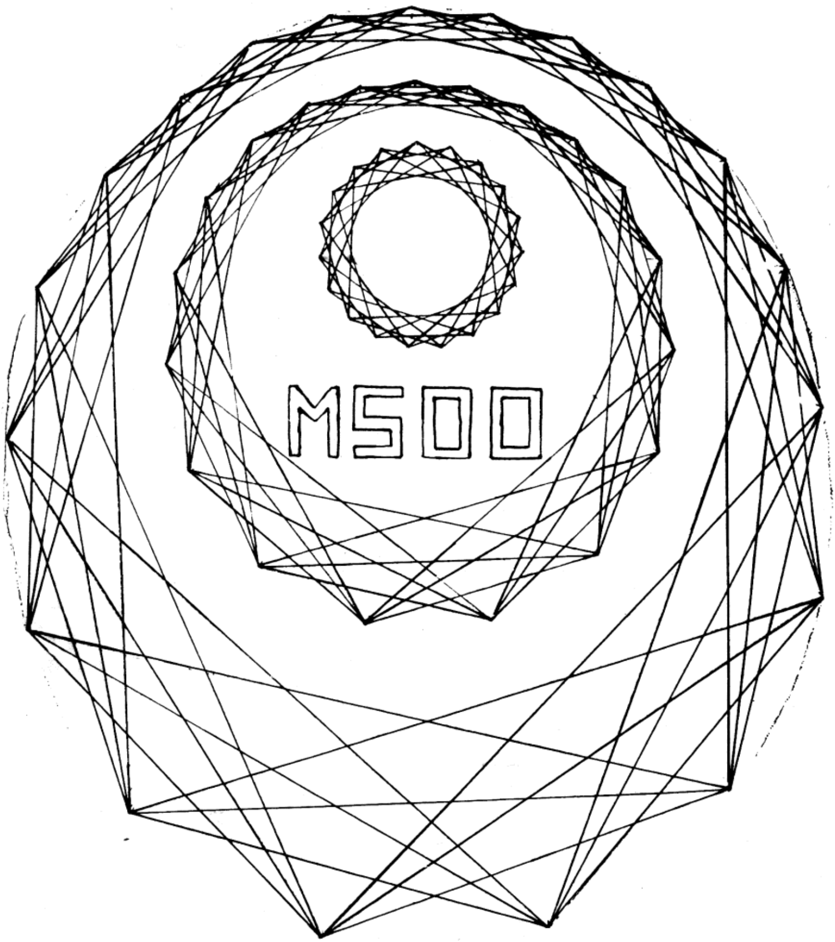

M500 49

M500 is a student-owned, student-operated magazine for Open University mathematics students, staff and friends. It is designed to alleviate student academic isolation by providing a forum for public discussion of the mathematical interests of members.

Articles and solutions are not necessarily correct but invite criticism and comment. Anything submitted for publication of more than about 600 words will probably be split into instalments.

MOUTHS is a list of names and addresses, with telephone numbers and details of past and present courses of voluntary members, by means of which private contacts can be made to share OU and general mathematical interests - or to form self-help groups by telephone or correspondence.

MATES is a special list of MOUTHS members who have explicitly volunteered for their MOUTHS details to be distributed to members in closed institutions such as prisons and special hospitals.

The views and mathematical abilities expressed in M500 are those of the authors and may not represent those of either the Editor or the Open University.

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EDITOR Eddie Kent articles and tetters for publication

PROBLEMS EDITOR Jeremy Humphries problems and solutions for publication

MEMBERSHIP SECRETARY Joyce Moor subscriptions - changes of address - MOUTHS/MATES data - application forms for membership and sample copies (for which please enclose 30p in loose, unused, small value stamps, and state your current courses)

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WEEKEND ORGANISER 1978 Sidney Silverstone

PUBLICITY Nick Eraser

Printer Waterside Printing Company, 11 Rumbridge St, Totton, Southampton.

M500 49 published February 1978. Subscription £4 for 10 issues. Cheques and postal orders should be payable to THE M500 SOCIETY, crossed "ACCOUNT PAYEE ONLY. NOT NEGOTIABLE" for safety In the post.

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ANYTHING SENT TO ANY OFFICER OF THE SOCIETY WILL BE CONSIDERED FOR POSSIBLE PUBLICATION IN THE MAGAZINE UNLESS OTHERWISE SPECIFIED.

The cover design is by Tony Brooks who writes about it in this issue.

COLOURING A CUBE DAVID ASCHE

How many different ways can we colour the faces of a cube using at most three colours?

Each of the six faces has to be painted uniformly in one colour. No mixing of colours is allowed and the colouring two cubes are regarded as equivalent if we can rotate one cube so as to match the other. The 36 possible colourings partition into classes of equivalent colourings and we are going to calculate the number of equivalence classes.

Notation

C = the set of 3^6 colourings.
$C_1, C_2, \dots, C_t$ = the set of classes of equivalent colourings.
t = the number of 'different' colourings.
R = the set of rotations of the cube.
$R(c \to c')$ = the set of rotations which take colouring c to colouring c'.
$R(c) = R(c \to c)$ = the set of rotations which fix the colouring c.
$C(r)$ = the set of colourings fixed by the rotation r.
$C_i(r)$ = the set of colourings in C_i fixed by r.
$S_i = \{ (r,c) \mid r \in R(c), c \in C_i \}$.

Result 1

$$\sum_{c\in C_i}|R(c)| = |S_i| = \sum_{r\in R}|C_i(r)|.$$

Count the set S_i in two ways. For a fixed $c\in C_i$, there are $|R(c)|$ rotations fixing c. Then, for a fixed $r\in R$, there are $|C_i(r)|$ colourings in C_i fixed by r. The result follows.

Result 2 For $c, c' \in C_i$, we have

$$|R(c)| = |R(c \to c')| = |R(c')| = \frac{1}{|C_i|}\cdot|R|.$$

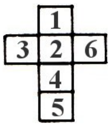
Following $r_1 \in R(c)$ by $r\in R(c \to c')$ gives another rotation in $R(c \to c')$. Also, following $r\in R(c \to c')$ by $r_2\in R(c')$ gives another rotation in $R(c \to c')$. In this way we can set up one–one correspondences between $R(c)$, $R(c \to c')$ and $R(c')$. So all these sets have the same number of elements. For each $c\in C_i$, every element of R belongs to exactly one of the sets $R(c \to c')$ and there are $|C_i|$ such sets. The result follows.

Result 3

$$t = \frac{1}{|R|}\sum_{r\in R}|C(r)|.$$

From results 1 and 2 we get $\sum_{r\in R}|C_i(r)| = \sum_{c\in C_i}\frac{1}{|C_i|}\cdot|R| = |R|$. So $\sum_{r\in R}|C(r)| = \sum_{i=1}^{t}\left(\sum_{r\in R}|C_i(r)|\right) = \sum_{i=1}^{t}|R| = t\cdot|R|$ and the result follows. ➔

What we need now is $|R|$ and each $|C(r)|$. Label the faces of a cube 1, 2, 3, 4, 5, 6. (See illustration below.) The identity rotation leaves all faces fixed while the other rotations have axes which are lines which join



1. midpoints of opposite faces;
2. midpoints of opposite edges;
3. opposite vertices.

We can write them down as permutations in cycle notation

(1) (2) (3) (4) (5) (6) = identity,

(1245) (3) (6) + five others like it,

(14) (25) (3) (6) + two others like it,

(14) (26) (35) + five others like it,

(123) (456) + seven others like it.

Counting these up we get $|R| = 24$.

<u>*Result* 4</u> If $m(r)$ is the number of cycles in the permutation corresponding to the rotation r, then

$$|C(r)| = 3^{m(r)}.$$

The rotation r fixes colouring c if and only if all the faces in a cycle are coloured the same. The result follows.

<u>*Result* 5</u> $t = 57$.

From results 3 and 4 and our list of permutations we get

$t = \frac{1}{24}(3^6 + 6\cdot3^3 + 3\cdot3^4 + 6\cdot3^3 + 8\cdot3^2)$. The result follows!

<u>*Generalisation.*</u> Replace 3 by n and get

$$t = \frac{n^2(n+1)}{24}\cdot(n^3 - n^2 + 4n + 8).$$

CONFERENCE

The Open University will hold a one-day conference at Durham University, Elvet Riverside, on SATURDAY 22 APRIL 1978. Present plans for the conference include a review of developments since the March 12 conference (where the idea of a course, "Mathematics Across the Curriculum" was discussed. See M500 39 13), ...

HIPY PAPY BTHUTHDTH THUTHDA BTHUTHDY* MARION STUBBS

M500/MOUTHS (before it became a SOCIETY) was 'born' at about 2300 hrs, 16 February 1973, In Southampton, England, (latitude 51° north, longitude 1°25' west). Happy 5th birthday, M500 .

According to Roger Elliot's book: *Astrology for Everyone* (Hodder Causton, 1976; £6.95) it has the following basic characteristics.

SUN SIGN: Aquarius.

Character: Independent, observant, out-of-the-ordinary. Uses reason to work out problems and seems truthful, sensible, ... and a bit cool. Humane and progressive in many ways, but likes to follow its own road.

Motivations: Strongly motivated by the search for truth, respect for commonsense, hatred of prejudice. Believes in the Good Society and wants to help others.

ASCENDANT SIGN: Scorpio.

Outer temperament: intense, strong-willed, somewhat magnetic in character. Takes life seriously and can beam in on ideas, people and activities with great concentration. Basically introspective. Subtle, secretive, a loner in life if need be. Full of devotion to the right person or cause.

WITH VIRGO MIDHEAVEN; great attention to detail and determination to complete a task perfectly. An overzealous critic of itself and others. Tends to become more fidgety and finicky with age, but will seek to lead a pure life.

MOON IN LEO

Disposition: warm-hearted and enthusiastic. Vain, likes to be noticed, needs an aura of affection. Reacts to life in an honourable, sometimes pompous way. Quite traditionally-minded, especially where authority is concerned. Likes to feel that life is for enjoyment and pleasure, not duty and obligations. Emotionally proud and can easily be hurt by a thoughtless word or gesture.

MERCURY IN PISCES

Intelligence: absent-minded and dreamy in some ways, intuitive rather than rational, sometimes missing the detail but grasping the overall point. The mind coloured by the imagination and fantasy. Likes the security of 'safe' ideas.

Words and gestures: can be silent through shyness, but voluble once given the chance! Fast talker with plenty of expressive gestures.

VENUS IN AQUARIUS

Love: looks for mental companionship as well as sexual compatability. Can be friendly with all kinds of people. Most unsnobbish. Likes to maintain personal independence, so never quite falls ➔

for someone hook, line and sinker!

Beauty: likes the cool, modern look. Likes things to be well-made and doesn't mind furniture, art etc that looks like machinery.

Ideal day: meeting many people.

MARS IN CAPRICORN: Life-force is controlled. Doesn't waste energy and takes pride in its ability to be efficient. Excellent at lengthy tasks and can pace itself to last the distance. Desires are carefully thought out and based on realism.

Anger: Can lose temper deliberately. Can also be coldly furious. Only attacks when provoked. Can put a plan of action into effect with disregard for human feelings.

JUPITER IN CAPRICORN: Not blindly optimistic ... simply realistic. Believes that success must be earned and can be obtained if it is crafty, persistent and occasionally hard on itself and others. Feels free once it is on the road to success.

SATURN IN GEMINI; Has a weakened sense of responsibility and tries to evade and wriggle out of duties. Has an inferiority complex about education and often tries to catch up in adult years what was missed in teenage schooling. Takes games seriously and sticks to the rules. May experience difficulties through its brothers and sisters.

(**Pooh looked on admiringly*.)

ΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛΛ

INFINITY COLIN DAVIES

There have been two queries recently about infinity, (Sid Finch 46 5 and Brian Woodgate 48 12). Brian mentions tanθ. I put the proposition that tanθ is a continuous function that joins up round the back somewhere to Ray Zahar at an M2O2 Summer School. From what I recall of his reaction I think we can forget that idea.

George Gamow wrote a book called *One two three ... infinity*. (My copy cost 16/- in 1963.) He discusses three levels of infinity, which he calls:

Aleph-nought	The number of all integers and fractional numbers,
Aleph-one	The number of all geometrical points on a line in a square or in a cube,
Aleph-two	The number of all geometrical curves.

Gamow describes Georg Cantor as being "the creator of the arithmetics of infinity". He discusses the one–one correspondence between the set of natural numbers and the set of positive rationals ($\mathbb{N}$ and $\mathbb{Q}^+$). This is also discussed in M100 34 5 2. M100 and Gamow use the same argument to show that a mapping of $\mathbb{N}$ to $\mathbb{Q}^+$ is ➔

one–one. This can be extended to show that a mapping of $\mathbb{Z}$ to $\mathbb{Q}$ is one–one. So the sets $\mathbb{N}$, $\mathbb{Q}^+$, $\mathbb{Z}$ and $\mathbb{Q}$ all have aleph-nought members.

M100 34 5 3 discusses the set of reals on the interval]1, 0[and shows that the number of real numbers in this interval exceeds the number of rational numbers. M100 calls the number of all reals C (for continuum). This must be the same as Gamow's aleph-one. He does not use words unlikely to be understood by the general reader, so he uses (what amounts to) a one–one mapping to map the points on one line to the points on another of a different length. By extending his mapping to the sets of all ordered pairs and all ordered triples (he does not go to n-tuples but one could) he shows that all lines of whatever length contain the same number pf points and that this equals the number of points in a plane or in a solid-body. They all contain aleph-one points and this is larger than the number of all integers. ie

$$\forall a,b \in \mathbb{Z}; \text{ rationals in }]0,1[\,=\, \text{rational in }]a,b[\,=\, \aleph_0$$
$$< \text{reals in }]0,1[\,=\, \text{reals in }]a,b[\,=\, \aleph_1 \text{ or } C$$

If f is the mapping (M100 34 5 2)

$$f : \mathbb{Q} \to \mathbb{Z}$$

f is one–one and so is its inverse; but for any mapping g,

$$g : \mathbb{R} \to \mathbb{Z}$$

g is many–one so g^{-1} is one–many.

After this M100 and Gamow differ. M100 discusses the possibility of transfinite numbers between $\aleph_0$ and C, but comes to no conclusion. Gamow states that it has been found that "the variety of all possible curves, including those of most unusual shapes, has a larger membership than the collection of all geometrical points and thus has to be described by the third number of the infinite sequence $\aleph_2$".

Gamow does not explain why this is. Does anyone have any ideas? Perhaps one can invoke Topological Connectedness in some way. Other ideas that occur to me are that a curve is an ordering of points, so that the number of ways of ordering the points in a space is something like (the set of all permutations of factorial$\aleph_1$}. But, first, $\aleph_1 \times \aleph_1 = \aleph_1 = \aleph_1^n$ for all n, I see no reason why $\aleph_1! > \aleph_1$. Secondly, is $\aleph_1!$ well-defined?

(I think $\aleph_0!$ is well-defined, and equal to $\aleph_0$, and while $\frac{22}{7}!$, say, is not normally given a meaning one could define it here as

$$f^{-1}((f\tfrac{22}{7}))!)$$

since f^{-1} is one–one; but $\pi!$ can never have a meaning as g^{-1} is one–many.)

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A science is said to be useful if its development tends to accentuate the existing inequalities in the distribution of wealth, or more directly promotes the destruction of human life.

G H Hardy, 1915. (from JH)

SRETTEL

From ROSEMARY BAILEY What a devil the man Jaworsky is. (*See* M500 48 18: *insubstantial prize offered for the largest number using* ≤ 10 *symbols*.) Are we supposed to send answers to you, him, Jeremy or Beelzebub? My first attempt at an answer consisted entirely of those well-known acceptable symbols F and *, and soon resolved itself as

F*** J J.

However, a moment's sober reflection convinced me that the winning answer must have the form

$\sum$ the rest + 1

so long as not more than one entrant submits this type of answer, otherwise all such will be undefined. This makes the competition isomorphic to "Finchley Central" (*vide* Chez Angelique) and therefore harder than it would appear at first sight by a factor of at least 10 x 10. Therefore before I expend enormous amounts of brain power on it I should like to know the exact nature of the "insubstantial prize" so that I can judge whether it's worth it.

From MIKE PURTON I thought I would let you know that I miss the general correspondence sections of M500 and trust it will pick up again after the exams.

Don't worry too much about M500 . Apparently we all have just a year to go. According to Marion's latest circular the entire membership is going to expire in December 1978 except for the lucky '49' members who are going to expire in January 1979. You can expect mass resignations next November.

From AUSTEN JONES I do not know if you received a questionnaire from the OU concerning our thoughts about the broadcast arrangements but one question was a gem:

'Did you miss the radio broadcasts for this course because: ...

Previous radio broadcasts were not worth watching'.

From DERYK JENKINS I enclose 1978 renewal form and accompany it with an open cheque. This is because even after scrabbling around in my dustbin for quite a few freezing minutes to search for the envelope with its mysterious number I was unable to find it, I have no idea how much I owe you for subs.

This year I am doing Engineering Mechanics (T231). I suppose its nearest on your list is MST282. The last M500 W/E was unable to cover MST282 - in marking the "perhaps" space I assume the same difficulty will occur this next year which will guarantee my non-attendance .

PS. You can round up to the nearest pound as a donation to the equipment fund.

From ALAN GURR I am not planning to continue to study for Honours in 1978 but I would like to continue to subscribe to M500 as an ex-student, possibly just for curiosity's sake.

You may be interested to know why I have dropped out having come so near - I have seven credits (no exemptions) and had I not

[*continued on page* 11

GAUSS IV

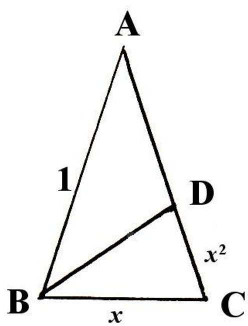
JEREMY GRAY

In this episode I shall state some mathematical problems connected with Gauss's work and describe how you might attempt to solve them.

First, some problems in classical geometry:

Gl. Using only a straight-edge and a compass show how to construct a regular 5-sided figure.

Hints: There are two classical methods. The construction in Euclid (Book IV, Prop II) rests on the observation that the interior angle at any vertex must be 108°, and therefore the exterior angle 72°. In an isosceles triangle ABC with $\widehat{B} = 72° = \widehat{C}$ and therefore $\widehat{A} = 36°$ (see figure) if you bisect $\widehat{B}$ by BD two more isosceles triangles are obtained (because $\widehat{A} = \frac{1}{2}\widehat{B}$). If BC = x and AB = AC = 1 then, from the similarity of triangles ABC and DBC



$$\frac{AB}{BC} = \frac{BC}{CD} \Rightarrow DC = x^2.$$

But BD = DA = BC (why?) = x. Therefore

$$AC = 1 = AD + DC = x + x^2.$$

So to construct an angle of 72° is equivalent to solving $x^2 + x - 1 = 0$ or drawing a segment $x = \frac{-1+\sqrt{5}}{2}$; the ratio 1 : x is called the golden mean and related to the Fibonacci numbers. ➔

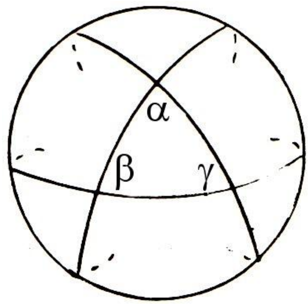
The second method is due to Pacioli (c 1494) and first constructs a regular decagon.

In a circle of unit radius, draw a pair of perpendicular diameters say AOB and COD where O is the centre of the circle.

Bisect AO at E, and draw a circle centre E radius EC = ED to meet AOB in F between O and B. OF has the length of one side of a regular decagon inscribable in the unit circle - why?

G2. How can a regular 15-gon be constructed?

G3. In spherical geometry triangles are formed from the arcs of great circles. Show that the area of a triangle of angles α, β and γ is $\alpha + \beta + \gamma$.



Hint: Consider the double segments in the above figure.

Next, some very hard problems in number theory.

A *quadratic form* is a modified sum of squares, typically $x^2 + y^2$, or $x^2 + 5y^2$, or ... .

N1. By taking small integral values of x and y show that some numbers can be written as a sum of two squares, but others cannot.

N2. All primes other than 2 are odd and are either of the form $4n + 1$ or $4n - 1$. Formulate a conjecture concerning which primes are a sum of two squares by trying several examples.

N3. Find a reformulation of $(a^2 + b^2)(c^2 + d^2)$ which shows that if two numbers are each a sum of two squares so is their product. ie $(a^2+b^2)(c^2+d^2) = x^2+y^2$.

Fermat, the originator of modern number theory, investigated several questions of this kind and proposed several in challenges to his contemporaries. They are reminiscent of pythagorean triples: integer triples (a,b,c) such that $a^2+b^2=c^2$. In 1657 he proposed (I paraphrase slightly): "if A is not a square, show there are infinitely many square integers x^2 such that Ax^2+1 is again a square". In other words show $Ax^2+1=y^2$ has infinitely many solutions in integers. I propose

N4. Show by examples that $Ax^2+1=y^2$ has any solutions at all, take $A = 2,3,5,6$, say.

As examples, when A=3: $3 \times 1^2 + 1 = 2^2$; $3 \times 4^2 + 1 = 7^2$. This problem is very hard, even to find solutions in rationals is quite good so I propose

N5. Set $y = 1 + \frac{mx}{n}$ and substitute in $Ax^2+1=y^2$, thereby finding a parametric representation of rational solutions to the equation $Ax^2+1=y^2$.

Alas, this is no help with Fermat's problem. Brahmagupta, an Indian mathematician of the 6th century AD, defined a mathematician as anyone who could solve $92x^2+1=y^2$.

N6. Consider what numbers can be written in the form x^2+3y^2. In particular, what primes are of this form (guess, don't prove!)?

If we write i for $\sqrt{-1}$ we can consider the so-called Gaussian integers $p+qi$, where p and q are integers.

N7. Show that $p^2+q^2 = (p+qi)(p-qi)$.

N8 Formulate and prove a similar result for numbers of the form $p + q\sqrt{-3}$.

Numbers of the form $p+qi$ are said to be prime if they cannot be factorized $p+qi = (p' + q'i)(p'' + q''i)$ non-trivially. That is $p'' = 0$, $q'' = \pm 1$; or $p'' = \pm 1$, $q'' = i$.

N9. Prove that every Gaussian integer can be factorized uniquely into primes by considering the norm $N(p + qi)$;

but

N10. Not every ordinary prime is a Gaussian prime. 5 for instance.

N11. Which Gaussian integers are prime?

N12. Compare your answer to N8 with your answer to N10. Notice anything?

What about factorization of numbers of the form $p + q\sqrt{-n}$? To show what can happen try

N13. Show that factorization is not unique for numbers of the form $p+q\sqrt{-5}$ by factorizing 21 in two different ways.

In fact unique factorization is seldom the case. Gauss conjectured but could not prove that $p + q\sqrt{-163}$ is the last class for which factorization is unique. In fact this result was only proved recently (by Alan Baker and Harold Stark). May I end with a malicious problem for calculator freaks?

N14. Is $e^{\pi\sqrt{163}}$ an integer?

You might care to practise on $e^{\pi\sqrt{43}}$.

Answers will be given in a future episode.

continued from page 6)

abandoned M334 and M341 last year I would be BA Hon by now.

The pressure of OU work on top of family and (managerial) job pressures gradually grinds one down to the point where studying is only just possible. If, on top of this, you have little interest in the particular course then the threshold is crossed and heaven is no more TMAs!

I believe myself to be a victim of the lack of choice of courses for genuine Honours students. To get Maths Honours you have no real choice: you have to take every M course in sight. If Analysis is your preference then you are not too badly off but if, like me, you put the differential operator in the same class as toilet paper (essential but not something you wish to write a thesis on!) then it becomes an up-hill struggle.

Some day I may finish my Honours; but only when there is an M<u>3</u>02.

From RICHARD SHREEVE (*editor of the* 1979 *Special Issue*)

NOTA BENE

MY HEARTFELT THANKS TO THOSE WHO HAVE PUT PEN TO PAPER OR FINGERS TO KEYS TO PROVIDE THE SPECIAL ISSUE 1978. WE STILL NEED MUCH MORE QUOTES, QUIPS AND ESSAYS,

IF YOU WERE PRIVILEGED TO TAKE M251, PM951, SM321 OR M321 IN 1978 THEN I HAVE A SPECIAL PLEA: YOU ARE A MINORITY AMONG THE M500/MOUTHS MEMBERSHIP AND A HIGHER PROPORTION OF YOU MUST RESPOND IF WE ARE TO GET A MEANINGFUL CONTRIBUTION FOR THESE COURSES,

From ERIC LAMBE On page 19 of the 1978 Degree Handbook it is stated that M231 is an excluded combination with the new half credit course M211, *An Introduction to Algebra and Geometry*. I queried this and received a letter from Walton Hall in which it is stated that this is a mistake. The letter goes on to say that M231 is an excluded combination with M212 and M203, and that M211 is only available in 1979.

From JOHN WILLS On names of big numbers; n-llion = million^n where n on the left is 'mi' for 1, Latin distributive root for 2 and 3, Latin cardinal root for $n > 3$.

decillion*million = undecillion; decillion*decillion = vig(es)illion; decillion↑10 = centillion; million↑1000 should = mil(l)iliion, messy. Should we switch to Greek and have kilillion, megillion, ... , exillion?

For smaller numbers English does a little better than Carl E Lindholm knows: 10↑0 = one/unit(y); 10↑1 = ten; 10↑2 = hundred; 10↑3 = thousand; 10↑4 = myriad; 10↑5 = lakh; 10↑6 = million; 10↑7 = crore. Does anyone know any short names for 10↑*n*, $n > 7$?

From NICK FRASER Enclosed is a clip from the *Radio Times* . I though it quite amusing:

"Sir George Porter ... is particularly anxious about young mathematicians: 'A mathematician is finished by the age of 30 to 35' ."

Are you finished yet?

M500 AND OUSA SIDNEY SILVERSTONE

On Sunday 6th November I attended a meeting of OUSA Societies' Standing Committee to discuss affiliation. They had invited representatives of all OU societies who had made enquiries about affiliation. Individuals there included Rex O'Hare, OUSA General Secretary; Robin Fennell, OUSA Vice President Finance; and Alec Pendle, OUSA Vice President Constitutions. There were representatives from OU Liberal Society, Open Science, OUGS, OU Sociological Society, OU Jewish Society, Open Medicine Trust, OU Psychological Society and even an organisation called the OU Adventurers Association (I think). It's amazing how many of these societies there are; and everybody was there at OU's expense.

OUSA are all for us affiliating and to encourage us the National Executive Committee have approved a payment to each affiliated society of £50 per society plus £15 for each 100 members (or part thereof) of the society who are currently studying student members. That's for the year 1977. The grant for 1918 will probably be the same. I asked why they wanted us and Rex O'Hare said that part, of OUSA's function was to assist any group of students who wished to join together for some particular activity compatible with OUSA and societies such as M500 were in that category. I also asked what they expected of us. Not very much it seems. In order for us to be affiliated they have to approve our constitution and any amendments thereto; but this is something of a formality because they are only concerned to ensure that we are a democratic organisation whose aims are not inimical to those of the OUSA. By democratic they mean that we have some mechanism for electing officers and taking decisions so that those in power can't become a self-perpetuating oligarchy (down with all despots and tyrants). OUSA will want us to submit copies of our annual audited accounts and also audited membership figures. They didn't say so, but I assume they will also want us to submit names and student numbers of members so that they can check on the validity of any claims we might make.

Affiliated societies have the right to send one or two delegates to OUSA national conference each year, to submit motions to conference and for the delegates to speak for those motions. They can also send representatives to the Societies' Standing Committee. All of this representation is at OUSA expense.

My personal view is that M500 should apply to affiliate, I do not believe that any student should be indifferent to OUSA as presently constituted. They are being given by the University £120000 to spend each year (See *Sesame* September/October, page 1). If you are against them then you ought to be campaigning to have them abolished and the money utilised somewhere else. If you are not against them then you ought to be in there having your say as to how the money is spent and how they should behave.

Let us have some discussion in the next couple of issues of M500 and then put it to the vote.

OBITUARY - KURT GŐDEL

Professor Kurt Gődel died on January 14 in Princeton, NJ, at the age of 71.

He was born in Brünn, Czechoslovakia on 28 April 1906 and became Privat-Dozent at the University of Vienna from 1933-1938. He settled in Princeton in 1938 and became a permanent member of the Institute of Advanced Studies in 1946 and Proffessor in 1953. He had the Einstein Award and the National Medal of Science, and in one form or another was in the US National Academy of Sciences, the American Academy of Arts and Sciences, the Royal Society, the Institut de France, the British Academy and the London Mathematical Society.

In 1930 he published his doctoral dissertation. This was his completeness proof for the first order functional calculus.

His most celebrated result came out in the following year: the incompleteness theorem for axiomatic systems. In particular he showed that the structure of Whitehead and Russell's *Principia Mathematica* was inadequate even for proving its own formal consistency, let alone deciding all mathematical questions. It is an inherent inadequacy of a sufficiently strong axiomatic system - and it changed the philosophical conception of the foundations of mathematics.

In 1938 Gődel showed that if the system of *PM*, or a stronger system, is consistent then it remains so on the addition of the Axiom of Choice; and also the Generalised Continuum Hypothesis.

In 1938 he married and from then on stayed put. He took no students and worried about his health.

xxxxxxxxxxxxxxxxxxxxxxxxxm

COVER DESIGN TONY BROOKS

I had been intending to do a new design for some time, your note spurred me into action. The basic idea behind the design on the cover is quite simple. You divide the circumference of a circle with a prime number of points (19 in the present case) and then project these points onto surrounding or enclosed circles, ellipses, or convenient shapes according to your taste. Then the points on each circle, ellipse, etc, are joined. First of all join successive points until you come back to your starting point, then every other point until you again return to your initial point. This process is repeated with every third, fourth point, etc until the desired effect is obtained. The use of a prime number of points ensures that you pass only once through each point before returning to your starting point.

I noticed in the Mathematical Games section of the December *Scientific American* a mention of a number much larger than Skews number. As far as I know Skews number ($10^{10^{34}}$ I believe) has always been claimed to be the largest number used in a mathematical proof. The article mentions a number used in a proof in graph theory which is too big to be written in exponential notation (or even in my star notation, M500 46).

PROBLEMS SECTION - JEREMY HUMPHRIES

Not a lot has come in this month. I'm getting rather short of problems and you seem to have stopped sending them. Anybody got any good ones?

Some of this issue's problems are quite easy so I expect a deluge of answers.

C S Evans writes to say that his conscience is troubling him because of the continuing correspondence on RELATIVE TRUTH. The original problem was his and he had no idea it would still be going eight issues later. He maintains that he is 'almost innocent', since the 'relativity' part was not in the original problem and was introduced by EK.

Don't worry, CS. I think the correspondence is finished now and anyway all letters are welcome - as I frequently say.

CHRIS LYONS has sent 'two silly solutions' to BILL MIDGLEY'S umbrella problem (*There were ten men sharing an umbrella and they didn't get wet*) .

1. It was not raining.
2. Each man sat on the shoulders of another with the top man holding the umbrella.

Well the first one is right, of course. And if the bottom man of the ten can find an eleventh man to sit on then the second one is right too. Well done.

That reminds me that there is a world record for the number of people sitting down without using a seat. I think it is more than a hundred. The people stand in a circle and then each one sits down on the knees of the person behind. It must take some doing - I think we should try it at this year's Weekend.

BOB BERTUELLO says that Turing's F notation (JOHN JAWORSKI, 48 18: $F_1(x) = 2x + 1$; $F_{n+1}(x) = F_n\,(F_n\,(F_n \ldots (F_n(x)) \ldots)\,)$ where the composition is taken $Fn(x)$ times) generalises, so that

$$F_{n+1}\,(x) = 2^{F_n(x)}(x+1) - 1$$

whence

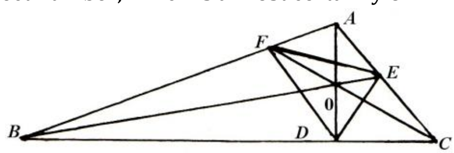
$$F_3\,(2) = 2^{95} \cdot 3 - 1 = 1 \cdot 188 \times 10^{29}.$$

In the *Scientific American* for November 1977 Martin Gardner describes a notation which has been invented to cater for large numbers. It is used to represent a number which cropped up in a bit of maths, and is the largest number ever found in a proof.

I don't have the issue to hand but I think that the work is a piece of graph theory and the large number is an upper bound for the correct number, which is almost certainly 6.

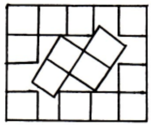
SOLUTION 45.4 BISECTOR *continued*

Yet another solution to this has arrived. JOHN READE says: There is a 'one-line' answer to this problem. ➔



DEF is the diagonal point triangle of the quadrangle ABC0. Therefore the pencil from D is harmonic so, since AD is perpendicular to BC, AD must bisect ∠FDE.

SOLUTION 47.1 OLYMPIAD III continued



STEVE AINLEY is still the only person to get fourteen cubes of volume 2 in the 2×5×6 box. This is how he does it.

SOLUTION 47.2 ARRAY *A sequence is formed by the mapping*

$$(x,y) \to (x+\Delta x, y+\Delta y) \bmod 5.$$

		X	1	2	3	4	5
Y	ΔY	ΔX	3	1	4	4	2
1	3						
2	1						
3	2						
4	3						
5	4						

The sequence stops when an element appears which has already been used. Which start positions give max and min numbers o£ elements? Are there other sets ΔX, ΔY *which allow all the elements of the array to be included in the sequence*?

Nobody, I fear, has done anything on this except MICHAEL GREGORY, whose problem it is. He writes:

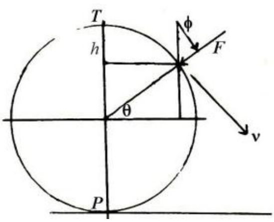
1. (1,1), (1,2), (1,3), (1,4) and (1,5) all give the maximum number of elements - six. The minimum number, four, is given by (2,2), (2,3), (2,4), (2,5); (3,2), (3,3), (3,4), (3,5).

2. I do not think there are any sets of ΔX, ΔY which generate all elements of the array. I think the maximum number is twenty one, obtained when the *x*-loop includes all five elements and the *y*-loop has four elements. eg:

$$x\ \ 12345 \qquad\qquad y\ \ 12345$$
$$\Delta x\ \ 11111 \qquad\qquad \Delta y\ \ 11112.$$

There are many other values of Δx, Δy which give this result. If anyone gets more than twenty one I shall be interested to hear.

SOLUTION 47.4 SPHERE AND PARTICLE *A frictionless particle slides from the top of a sphere radius r under gravity* γ, *and strikes the plane on which the sphere stands. How long is its path on the sphere*? *where does it strike the plane*?



THURSTON HEATON and TIMOTHY WILKINS sent solutions to this. Timothy says that it is really an A-level problem but he's glad I included it because, unlike all the other problems in M500, he can do it.

Velocity of particle = $\sqrt{2\gamma h}$ = ν.

Force $F = m\gamma\sin\theta$, where m is the mass of the particle.

Centrifugal force = $m\nu^2/r = C$.

When $F = C$ particle leaves sphere, ie $m\gamma\sin\theta = 2m\gamma h/r$. Now ➔

$h = r(1 - \sin\theta)$. $\therefore\ \theta = \sin^{-1}\frac{2}{3}$. Hence length of path on sphere:

$r \cos^{-1} ⅔$.

Total time of fall, t_1, is $\sqrt{4r/\gamma}$ and time to separation, t_2 is $\sqrt{2h/\gamma} = \sqrt{2r/3\gamma}$. Time of free fall $= t_1 - t_2$. Horizontal velocity $= v \sin\theta = 2\sqrt{2\gamma r/3}\ /3 = v_1$. Free horizontal distance $= v_1(t_1 - t_2)$. Horizontal distance on sphere $= \sqrt{5}/3\ r$; whence total horizontal distance, or distance of landing from $P = 1.39r$, independent, as Thurston notes, of gravity.

SOLUTION 17.5 POTTON *V* BARFORD

The ten wickets fell at the following scores: 48, 123, 128, 161, 181, 183, 194, 211, 212, 213. FRANK ARGENT, CHRIS LYONS and SIDNEY SILVERSTONE solved this. They point out that the solution to 18 down (*Three partnerships scored more than this*) can be 21 or 31.

I no longer have the Clarke book, and I didn't copy the answers but the question certainly said three. Regular readers will know that, by tradition, M500 crossnumbers have misprints or omissions. Perhaps this time it is the original that is wrong, and 18 down should be 'Two partnerships ... ' . That would give the unique answer, 41 .

PROBLEM 19.1 POETRY

This is an example of a particular function which maps word sequences to letter sequences:

f: JACK AND JILL WENT UP THE HILL $\rightarrow$
JAJWU THANI EPHIC DLNEL KLTL.

It is fairly easy to see the rule and it is very easy to go in the direction of the arrow, but it is not so easy to come back.

x is a rhyming couplet from a well-known English poem;

f:$x \rightarrow$ TVADH MHKTC HCWAC THILE OUENW EEORN IOELL CWCEA RUIDP.
OLLHW STLTH AAADE EGRIR EEND.

Find *x*; and who wrote the poem?

Note. *x* is not the first two lines of the poem. I tried one like that on my mother and she solved it in one minute by the sneaky method of looking through the index of first lines in the *Oxford Book of Verse*.

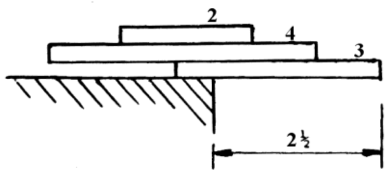
PROBLEM 19,2 LOCK AND KEY MICHAEL MCAREE

A company has four directors on the board. They decide to provide their safe with a certain number of lock's, all different, and to provide each board member with keys to some of the locks, so that any three members can open the safe, but no two members can open it.

How many locks should be put on the safe, and how many keys should each member receive?

PROBLEM 49.3 PLANKS

This is taken from an old issue of *Eureka*, the journal of the Cambridge University Mathematical Society. I don't know if the magazine still exists. I have written to the published address in vain. Does anybody know?



There are three uniform thin planks of lengths (and weights) 4, 3 and 2. In this diagram they are arranged on the edge of a surface to project a distance of 2½.

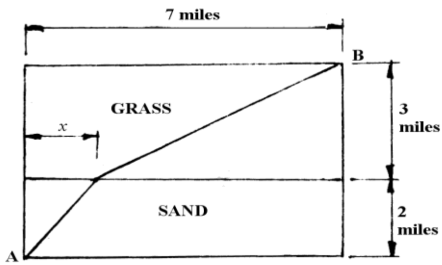
How can they be arranged to project the maximum possible distance?

PROBLEM 49.4   BILLIARDS   C S EVANS

Two men, Pot and Cannon, are playing billiards.
Pot: We will play five games.
Cannon: How do my chances of winning compare with yours?
Pot: Your chance of winning three games is the same as your chance of winning four games.

What is the chance that Cannon will win all five games?

PROBLEM 49.5 HORSE RIDING   PETER MINCH



A rider on horseback sets out from A to see his lover who lives at B. Naturally he is in a great hurry. He can ride twice as fast on the grass as he can on the sand. What is the distance *x* so that he gets there in the minimum possible time?

*   *   *   *   *   *   *   *

What baffles me is how people find things like this:

$194979 = 1^5 + 9^5 + 4^5 + 9^5 + 7^5 + 9^5$

or this:

$24678051 = 2^8 + 4^8 + 6^8 + 7^8 + 8^8 + 0^8 + 5^8 + 1^8$.

EDITORIAL

The other morning my feet were cold and the pubs weren't open and I had some waiting to do so I went into a bookshop. The section nearest the hot air outlet was labelled 'Reference'; there I saw this book:

> The Universal Encyclopedia of Mathematics, Pan Books, 1976. Translated and adapted from *Meyers Rechendaden*, Bibliographisches Institute, Mannheim, 1960.

I grabbed it for a look, of course. Only £1.5, not bad. But no author, editor or translator. There was a Foreword by James R Newman (who's he? Editor of *The World of Mathematics* for his sins! It says so on the title page) who says it is clearly written, sensibly arranged and reliable.

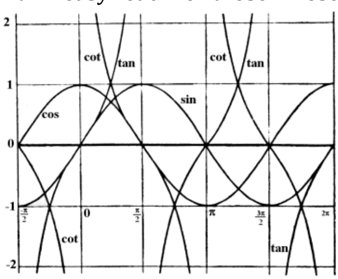
So I pulled up a stool and sat down, thus barring the way into the main body of the shop to the elderly ladies. Flicking through I saw various headings: Area, calculation of; Affine Transformations; (try again) Logarithm, Hyperbola, Determinants (pages of this). Right at the back pages of tables - but x is in radians. I've never seen that before in a book of tables. So I had a closer look under some of the headings. Power series produced the usual boring few then this

$$\sum_{n=0}^{\infty} n!\, x^n = 1 + x + 2!x^2 + 3!x^3 + \ldots \qquad \text{converges only for } x = 0.$$

Beautiful.

There are many entries under most of the usual headings - for instance scalar product but not dot product. Some of the entries are very brief; others go into a subject reasonably thoroughly (though not at more than foundation level). The first entry is Absolute Value, the last is Zero and the one about the middle is 'Imaginary Numbers (so called)'. None of these three rates more than a few lines.

You certainly couldn't learn mathematics from this book and it would be difficult to use without prior knowledge. But it is useful for putting quite a lot of basic information within easy reach for those whose memory cells are dying exponentially. By the way there's no index. But then, one wouldn't expect one from Pan.



There is also a jolly nice picture on page - oh they don't have page numbers - anyway towards the end of the book. It is called "Trigonometrical functions" and here is a piece of it.

So I bought it.

Eddie Kent.