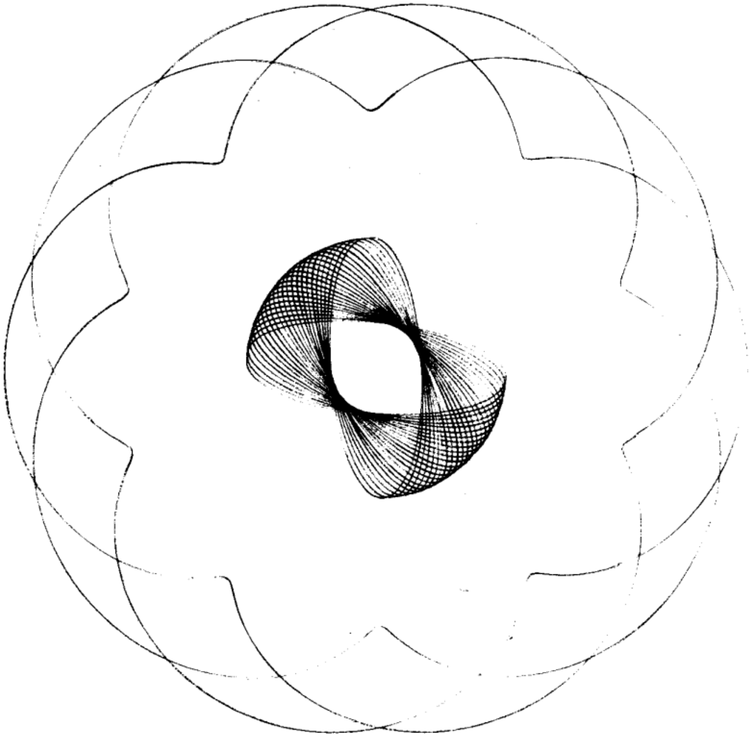

M500 53

M500 is a student-owned, student-operated magazine for Open University mathematics students, staff and friends. It is designed to alleviate student academic isolation by providing a forum for public discussion of the mathematical interests of members.

Articles and solutions are not necessarily correct but invite criticism and comment. Anything submitted for publication of more than about 600 words will probably be split into instalments.

MOUTHS is a list of names and addresses, with telephone numbers and details of past and present courses of voluntary members, by means of which private contacts can be made to share OU and general mathematical interests - or to form self-help groups by telephone or correspondence.

MATES is a special list of MOUTHS members who have explicitly volunteered for their MOUTHS details to be distributed to members in closed institutions such as prisons and special hospitals.

The views and mathematical abilities expressed in M500 are those of the authors and may not represent those of either the Editor or the Open University.

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THE NUMBER OF EQUIVALENCE RELATIONS ON A SET

DAVID ASCHE

Consider the problem of counting the number $m(k)$ of equivalence relations on a set of k elements. Now, as most of you will know, this amounts to counting the number of possible partitions of a set of k elements. Let us take some very small sets and look at their partitions.

$k = 1$ 

$k = 2$ 

$k = 3$ 

From these we conclude that $m(1) = 1$, $m(2) = 2$ and $m(3) = 5$. If you are energetic you can check that $m(4) = 15$, but it would soon become rather tiresome to have to draw these diagrams to verify results such as $m(5) = 52$, $m(6) = 203$, $m(7) = 877$, $m(8) = 4140$.

What we need is a systematic procedure for stepping up from a set of size k to one of size $k+1$. To do this let us introduce the number $m(k,r)$ which is to be the number of partitions of a set of k elements into exactly r classes. Thus, for example, $m(3,1) =1$, $m(3,2) = 3$ and $m(3,3) =1$. We see that we must have

$$m(k) = m(k,1) + m(k,2) + ... + m(k,k) .$$

Now, to get a partition of $k+1$ things into r classes we can either take a partition of k things into r classes and pop the extra element into any one of the r classes or else take a partition of k things into $r-1$ classes and make another class to contain just the one extra element. From these considerations we get the formula

$$m(k+1,r) = rm(k,r) + \mathrm{m}(k,r-1).$$

This is exactly what we need to compute a table of values of $m(k,r)$ and hence work out each $m(k)$.

k \ r	1	2	3	4	5	6	$m(k)$
1	1	0	0	0	0	0	1
2	1	1	0	0	0	0	2
3	1	3	1	0	0	0	5
4	1	7	6	1	0	0	15
5	1	15	25	10	1	0	52
6	1	31	90	65	15	1	203

For example $m(6,4) = 4m(5,4) + m(5,3) = 4 \times 10 + 25 = 65$.

Readers will no doubt want a nice compact formula for the nth term of the series 1,2,5,15,52,203; so we will pursue this a little further.

We will regard the rows of our table as the coefficients of polynomials and put

$f_1(x) = x$

$f_2(x) = x + x^2$

$f_3(x) = x + 3x^2 + x^3$

$f_4(x) = x + 7x^2 + 6x^3 + x^4$

etc. In general we have

$f_k(x) = m(k,1)\,x + m(k,2)\,x^2 + \ldots + m(k,k)x^k.$

Using our formula we can check that

$f_{k+1}(x) = x(f_k'(x) + f_k(x)).$

Now this looks like a nasty mixture of a differential equation and a difference equation, but with a little fiddling around we can find a way of simplifying it. If we write

$F(x) = e^{e^{e}-1},$

then it turns out that $F_k(x) = F^{(k)}(x)$, where $F^{(k)}$ denotes the kth derivative of F, and moreover $F_k(0) = f_k(1) = m(k)$. This means that we have the Taylor series

$e^{e^{e}-1} = 1 + \sum_{r=1}^{\infty} \frac{m(r)}{r!}\, x^r.$

So, to get $m(k)$, all you need to do is to differentiate $e^{e^{e}-1}$ k times and put $x = 0$. Alternatively you might like to work out the coefficient of x^k in the expression

$1 + \left(x + \frac{x^2}{2!} + \cdots\right) + \left(x + \frac{x^2}{2!} + \cdots\right)^2 + (x + \ldots)^3 + \ldots .$

To end up with here is another method of getting $m(k)$. Take the polynomial equation

$x(x-1)(x-2)(x-3)\ldots(x-k+1) = 1$

and write it in the form

$x_k = 1 + \alpha_1 x + \alpha_2 x^2 + \ldots + \alpha_{k-1} x^{k-1}.$

Then we have

$m(k) = 1 + \alpha_1 m(1) + \alpha_2 m(2) + \ldots + \alpha_{k-1} m(k-1)$.

For example, taking $x(x-1)(x-2) = 1$, we can write it as

$x^3 = 1 - 2x + 3x^2,$

so

$m(3) = 1 - 2m(1) + 3m(2) = 1 - 2 \times 1 + 3 \times 2 = 5.$

Why should this work?

M331 ALAN SLOMSON

I am tutoring M331 for the fourth year and agree with almost everything said about it in the Special Issue of M500. It is a course for people for whom the main attraction of mathematics is its logical structure; if on the other hand you like mathematics mainly because it provides powerful techniques for solving problems M331 is not for you. Thus it appeals to a minority taste, and although it is a good course for people who want to see the logical construction of the Lebesgue integral I have always thought that, given the rather small number of courses that the OU is able to offer, it is rather a strange choice for a third level course.

On the question of whether M231 is a necessary prerequisite for M331 the enclosed table showing what happened to students taking M331 in the Yorkshire region in 1976 and 1977 may be of interest. The table only includes students who submitted the first assignment.

M331 \ M231	1	2	3	4	5	Q	W	-
1	4	1						1
2	1	3					1	
3	1	4						
4								
5			2	1		1		
W		1	4	2			1	2

The message is very clear. Of the fifteen students who had previously passed M231 with grades 1 or 2, 14 passed M331 and one withdrew. Of the 15 students who had previously passed M231 with lower grades or who had withdrawn from the course, or who had not taken M231, only two passed M331. Of the rest, four failed and nine withdrew (of course some students withdrew for non-academic reasons). The student who got a grade 1 in M331 without having taken M231 is someone who was already familiar with the sort of mathematics in M331 from his professional work.

It would be interesting to know if the pattern is similar in other regions.

INFINITY LUCY POLAND

Following on from Colin Davies in 49 4 on infinity, according to AM289 Unit 10 it has been shown by Gődel among others that, whether or not there exists a transfinite number l such that

Aleph-nought < $l < C$,

is an independent axiom of set theory. Consistent theories can be described with or without l.

Cantor proved that if Aleph is a transfinite number, ie representing a set of Aleph objects, then more than Aleph subsets of that set are possible. Thus there are infinitely many transfinite numbers:

$\mathcal{C}$ is the number of possible subsets of the natural numbers, Aleph-two is the number of possible subsets of the reals, etc.

But how many transfinite numbers are there?!

---

x^x BRIAN STEWART

My head of department handed me this function one day, and cheerfully asked me if I could integrate it (presumably he meant find a primitive). I couldn't. Neither could he; and neither could the person who had cheerfully handed it to him.

$f : x \longmapsto x^x, x > 0$

(the ultimate in exponential functions?)

In fact $\lim_{x\to 0} x^x=1$ so we can define $f(0) = 1$. and obviously it is meaningless for negative x.

Having just completed M231 I decided to pit my entire knowledge against this function; I didn't get very far.

$f\,'(x) = x^x(\ln x + 1)$

which means $f\,'(1/e) = 0$ and we can graph f, figure 1. (For number freaks $f\,(56) = 7.9 \times 1097$ which is the largest integer I can get f of on my calculator.) $f\,'$ looked so reasonable that I had an attempt to differentiate further in an effort to get a Taylor series. $f\,''(x)$ wasn't too bad -

$f\,''(x) = x^x((\ln x + 1)^2 + 1/x)$

and clearly $f\,''(x)$ has a minimum value of $ef(1/e)$ at $1/e$, so we can sketch $f'(x)$, figure 2.

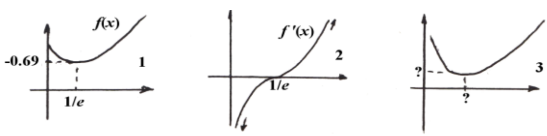
After this it all exploded:

$f\,'''(x) = x^x((\ln x + 1)^2 + 3(\ln x + 1)/x - 1/x^2)$

so the graph of $f\,''(x)$ is something like figure 3. Can anyone solve $f\,'''(x) = 0$ to find the numbers missing from the diagram?

And can anyone find a general formula for $f(n)(x)$? Is there a simple (?) function g such that $g' = f$, or can anyone prove that no primitive exists?

And if there is any practical situation where this function comes up, let me know; otherwise I've wasted a lot of time.



RATIONAL SEQUENCES JOHN HULBERT

I was interested to read in a popular mathematics book of the following 'remarkable' relationship which is derived from the Fibonacci sequence:

$$
\begin{array}{ll}
\frac{1}{89} & = 0.0112358 \\
& \quad\quad\quad\quad 13 \\
& \quad\quad\quad\quad\;\; 21 \\
& \quad\quad\quad\quad\;\;\;\; 34 \\
& \quad\quad\quad\quad\;\;\;\;\;\; \ldots \\
\hline
& 0.011235955056179\ldots\ .
\end{array}
$$

We can restate this by denoting the terms of the Fibonacci sequence as F_1, F_2 etc. Then

$$\frac{1}{0.89} = F_1 + \frac{1}{10}F_2 + \frac{1}{100}F_3 + \ldots = \sum_{i=0}^{\infty} 10^{-ai} P_{i+1} \quad \text{as} \quad \sum P\langle 10^a \rangle$$

where P is some sequence.

Now, if we consider $\sum F\langle 10^2 \rangle = 1.010203050813\ldots$ we find that it is equal to 1/0.9899. Similarly $\sum F\langle 10^3 \rangle = 1/0.998999$, and so on.

However the Fibonacci sequence is not unique in this regard. Simply taking the sequence $\mathbb{I}$ of integers we have

$$\sum \mathbb{I}\langle 10 \rangle = 1.2345679012\ldots = 1/0.81.$$

The triangular numbers $\mathbb{T}$ (1,3,6,10,15,...) have

$$\sum \mathbb{T}\langle 10 \rangle = 1.37174211248285\ldots = 1/0.729.$$

And the 'once-removed' Fibonacci sequence G (1,1,1,2,3,4,6,9,13,19, ...) gives

$$\sum G\langle 10 \rangle = 1/0.899$$

$$\sum G\langle 10^2 \rangle = 1/0.989999$$

$$\sum G\langle 10^3 \rangle = 1/0.998999999.$$

The sequence $\mathbb{S}$ of perfect squares gives $\sum \mathbb{S}\langle 10 \rangle = 1/0.66272727\ldots$ which is interesting, but not as satisfying as the other sequences.*

I offer no proof for the above, nor have I more than a reasonable confidence in their correctness. However, the topic seems to be fruitful, so perhaps somebody else would care to take it up from here.

(* On further consideration, perhaps it is surprising that $\sum \mathbb{S}\langle 10 \rangle = 0.\dot{1}5089163\ldots66941\dot{0}$.)

PREREQUISITES FOR S354 ALAN COOPER

S354 is called *Understanding Space and Time*, and will be available in 1979.

In Science there are certain terms which everyone is expected to know. These include: atom, pressure, density, temperature, telescope, magnet, gravity, electric charge, friction, radio waves, gas, ellipse, orbit, simple pendulum. Consequently they will not be defined in S354. The Penguin *Dictionary of Science* can give any help that might be needed.

All other terms will be explained on introduction; either briefly as in the case of: frequency, neutron, chemical element, etc; or in considerable depth as with, for instance, velocity, energy, coordinate system, etc. It is as well to have looked at Units 6 and 28 of S100.

The concepts from mathematics will be as appear in MST281, MST282 and MS283, although the chances are that if you have taken any maths course you will have acquired the necessary general background and practice.

Maths needed for S354

Simple algebraic manipulation.
Idea of a function - exp, log, sin, cos, tan.
Meaning of a derivative as a slope or limit of $\frac{\Delta y}{\Delta x}$.
Derivatives of simple functions, eg $\frac{d}{dx}(\sin x) = \cos x$.
Idea of a differential equation and constants of integration but not methods of solution.
Coordinates - cartesian and polar.
The idea of a vector and its components. Definition of scalar and vector products.

(There will be a short resume of this at the end of Block I.)

A Few Books - among many others

College Calculus with Analytic Geometry, Protter and Morrey (Addison Wesley), chapters 1-6, 11-14 (omitting hyperbolic functions).
Maths for Engineers and Scientists, Jeffrey (Nelson), chapters 5 and 6.
Calculus and Analytic Geometry, Thomas (Addison Wesley), chapters 1-4, 6, 11-13.

Prince, I approach your throne in all humility,
Nor do I want (or dare) to be abusive
But, speaking just in terms of Probability,
You can't be Independent and Exclusive.

STEVE AINLEY

GAUSS VII JEREMY GRAY

It must seem, with his work on function theory and algebraic number theory so dramatically begun that Gauss was well on the way to becoming a successful pure mathematician, yet it is with the *Tagebuch* entries on astronomy that we first encounter Gauss's lifelong vocation. He was employed for many years as an astronomer, and most of his friends and colleagues were other professional astronomers, Bessie and Olbers among them. The bulk of his mathematical correspondence is with such men and not the leading mathematicians of his day with whom, it seems, Gauss had little to do. In part this should not surprise us. There was not, in the early 1800s, the successful academic business there is today. Universities were only just beginning to establish reputations as centres of research rather than as teaching institutions, and they did not offer the safely glamorous careers the professions have arrogated to themselves in our more democratic times. German universities, Göttingen and somewhat later Berlin excepted, were poor in this regard - worse than the new French Grandes Écoles, although much better than the moribund English institutions. So it was not obvious that Gauss should seek, or want, a professorship. Furthermore, astronomy has always called forth from some of the best mathematicians some of their best work - Gauss was following Newton, Euler and Laplace, and was to be followed in his turn by Poincaré. As we shall see, planetary astronomy was entering one of its most exciting periods before the modern exploration by space vehicles. It was probable that to significant new mathematics would be added the spice of new discoveries. Nor should one forget that Gauss's mastery of calculation would render the huge computational element in astronomical work almost congenial to him; he did in fact write of the satisfaction the number-crunching gave him.

And yet there is an arrogance in Gauss that is wholly unappealing. Mathematicians of his calibre, or nearly his calibre, did write to him and their letters sometimes went unanswered. He never collaborated with any of the French who were almost his equals; even when strictly correct his judgements on his colleagues have a dismissive tone to them which is strange in one so magnificently gifted. Thus of Abel's work in elliptic functions, which so excited mathematicians in the late 1820s, Gauss merely said: Abel has gone about one third of the way. It seems that Gauss would admit that astronomy was a joint enterprise but would share his mathematics with almost no-one.

We can be quite precise about the final decisive stimulus that brought Gauss to astronomy. An accident of nature had ordained that if the Earth be considered to lie ten units from the sun then all the visible planets lie, in order, $4 + 3 \times 2^n$ units away, where n is $-\infty$ for Mercury, 0 for Venus, 1 for the Earth, up to 5 for Saturn. Except, and it is a big exception, there is no planet corresponding to $n = 3$. This exception seems to have consigned this strange law, called the Titius–Bode law after Titius who borrowed it and Bode who stole it from him, to oblivion. Then in 1781 the lonely and obscure Englishman William Herschel made the moving and exciting discovery of a new planet: Uranus, a distance of approximately $196 = 4 + 3 \times 2^6$ units from the sun. The Titius–Bode law was suddenly more intriguing. Perhaps a diligent search with the new generation of telescopes would reveal a planet between Mars, for which $n = 2$, and Jupiter ($n = 4$). On New Year's Eve 1800-1801 the distinguished astronomer Piazzi caught sight of a planet in the right place and followed it for forty days, which was about 9° of its orbit. Then, tantalizingly, the planet went to near the sun and could no longer be seen. The problem facing astronomers was to predict, from Piazzi's limited data, where and when this elusive but important new planet would reappear.

The chief problem in the determination of an orbit is the over-determination of the data, that is, the number of observations taken vastly exceeds the three necessary to determine the orbital curve (which must be a conic section) precisely. Different sets of three give, accordingly, conflicting orbits and taken together the observations are inconsistent. The source of this incompatibility is, of course, the inescapable errors made in each observation. A second, considerable, problem is the sheer computational drudgery necessary in any astronomical calculation and the attendant delicate estimations of valid approximations of the data. It is important to ensure, for instance, that no error-term blows up in the course of a calculation into a significant amount.

Gauss took up the challenge of determining the orbit of the new planet in September 1801, the year his *Disquisitiones Arithmeticæ* was published, and published his solution but not his method in Zach's *Journal* in December of the same year. He had looked for an elliptical orbit whereas the distinguished astronomer Olbers had assumed only a circular orbit, and his estimate differed by several degrees from that of his professional rival. Looking for a faint object at night is difficult indeed, but one may easily enough check an estimate. On the 7th of December Zach found the new planet where Gauss had said it would be; Olbers found it, again confirming Gauss's estimate, on January 1 1802. In one year Gauss not only became the Prince of Mathematicians but the prince of astronomers!

What was it that so distinguished Gauss's method from that of his rivals? It was, quite simply, the method of least squares, which he later claimed to have discovered in 1795 and is the first satisfactory technique which allowed all observations to be used. Gauss even derived a way of recursively up-dating the observations so that later information could be used without wholesale recalculation of the results. So efficient was this method that it has been adapted with little change for computer work and employed to monitor Apollo space-flights. Previous error estimates had attempted to eliminate the worst pieces of data on a fairly rough and ready basis, although Laplace had sought curves which minimised the sum of the absolute errors (ie the absolute deviations of each observation from the calculated 'true' one). In comparing the two methods Gauss noted in the *Tagebuch* for June 17 1798 (88): The calculus of probability defended against Laplace. It is important to observe that what is involved is not simply a way of calculating a 'total error' but a whole theory of errors connected with ideas of maximum likelihood and precision through the mechanics of the ‘Gaussian’ normal distribution. I hope to show in a later episode how great an improvement upon prevailing common sense ideas of error Gauss's theory was.

The new planet, called Ceres, turned out to be the first of many and Gauss became fascinated by the mathematical problems they raised. Olbers found Pallas in 1802 and Vesta in 1807; Harding found Juno in 1804, and Olbers produced the famous theory that they were the result of a gigantic catastrophe in which somehow a planet had been shattered into the tiny asteroids of the belt. As Gauss studied their orbits he found that they were not true ellipses. Instead the orbits are deformed by the gravitational influences of the other bodies in the solar system, chiefly Jupiter. Gauss, in the years 1805 onwards, took up the famous perturbation problem, previously studied by Newton, Euler and Laplace on the curious case of the moon. It is no longer a two body problem (eg sun and planet) but a three or even *n*-body one (eg sun, Jupiter, Ceres), and is still unsolved; Newton said it was the only problem which had ever given him a headache and he wound up having to fudge it in his *Principia*. Gauss published his findings, now publicly linked with the method of least squares, in his justly celebrated *Theoria motus corporum colestrum* in 1809. At once he was embroiled in controversy, which he so much disliked, for Legendre had by then presented the method, shorn of any reference to a theory of probability, in his 1805 treatment of the orbit of comets. Gauss knew of Legendre's work, but he mentioned it only slightingly, perhaps confident of his own priority - and superiority - in the matter. Since he had done little better when discussing Legendre's work on the law of quadratic reciprocity in the *Disquisitiones*, the Frenchman was enraged and pursued Gauss in print for an apology and due acknowledgement. None was forthcoming, and not for the first time Laplace had to intervene and seek to restore the peace.

Finally priority of discovery was assigned to Gauss, and priority of publication to Legendre, both men working independently.

BREAKTHRU EK

Towards the end of last year Max Bramer sent a cutting from the *Times Higher Education Supplement* of the 25th of November 1977 in to M500. There was a tiny article called "Priming the Numbers Game" which gave publicity to some recent work in number theory.

It seems that Dr James Jones and three fellow researchers have been awarded something unspecified from the Mathematics Association of America for producing a polynomial generator for the primes.

The article points out that in the past many eminent mathematicians have claimed that there could be no such thing. However in 1971 a Russian mathematician (who unfortunately has no name but was 21 at the time) gave an existence proof for such a formula. Dr Jones took heart from this and, bingo! as they say.

In an interview Dr Jones said (and note this carefully, you aspiring mathematicians!) "The prime numbers are a very difficult set to describe mathematically because their occurrence is very irregular." See that 'very'? It takes an astute man to spot something like that where most of us had thought they were only regularly irregular.

Here it is then. THEOREM 1: The set of prime numbers is identical with the set of positive values taken on by the following polynomial as the variables range over non-negative integers

$$\begin{aligned}
&(k+2)(1-(wz+h+j-q)^2-((gk+2g+k+1)(h+j)+h-z)^2-(2n+p+q+z-e)^2\\
&-(16(k+1)^2(*)(k+2)(n+1)^2+1-f^2)^2-(e^3(e+2)(a+1)^2+1-o^2)^2\\
&-((a^2-1)y^2+1-x^2)^2-(16r^2y^4(a^2-1)+1-u^2)^2-(((a+u^2(u^2-a))^2-1)\cdot\\
&(n+4dy)^2+1-(x+cu)^2)^2-n+\ell+v-y)^2-((a^2-1)\ell^2+1-m^2)^2-(ai+k+1-\ell-i)^2\\
&-(p+\ell(a-n-1)+b(2an+2a-n^2-2n-2)-m)^2-(q+y(a-p-1)+s(2ap+2a-p^2\\
&-2p-2)-x)^2-(z+p\ell(a-p)+t(2ap-p^2-1)-pm)^2).
\end{aligned}$$

(*) means I don't guarantee that particular index; it doesn't mean that I guarantee anything else, owing to the exigencies of newsprint.

Put everything equal to 0 and I think you will get the result –2186, which is not one of the standard primes. But never mind; how could one have any prime that is the product of k+2 and something else? (Actually I know the answer so don't write and tell me. Just agree that it is of little help here.)

On December 21 I wrote to the *Times Higher Education Supplement* pointing out my reservations and asking what was the source of the information. I have had no reply to my letter and no further mention had appeared in the paper by February, when I stopped looking. Can the *THES* have been the victim of a hoax?

Do you like me imagine Mrs Sylow -
That Nordic groupie - hastening to peel off
And join the Primal Power of Mr Sylow
Upon some sun-drenched Scandinavian Lilo?

STEVE AINLEY

NOTES AND QUERIES

MICHAEL GREGORY writes: My article has appeared - they seem to have turned the photograph upside-down since they sent me the proof! The details are: "Complete Threading of Surface Models" M Gregory, J Recreational Mathematics, vol 10(2) 1977-78, pages 87-95.

The paper is based on "Constructions" in M500 6 and 28: 'To thread up a set of holes around perimeter of a disc according to a set of rules; a complete threading is one which uses all the holes and the thread forms a continuous loop'.

The two types of threading described in M500 are dealt with. An algorithm is described for investigating the characteristics of any particular threading and deciding whether it will be complete.

The previous publication of parts in M500 and discussion with OU colleagues are acknowledged.

MRS P J COLEY, secretary to OLIVER PENROSE writes: A student has just written to us to ask for a copy of the University Postgraduate Degrees by Course and Research in Applied Mathematics in the UK 1978-78, as mentioned in M500 magazine'.

In fact the quickest way to obtain these booklets is by writing direct to Professor J Heading Department of Applied Mathematics University College of Wales

From STEVE AINLEY: How nice to have M500 again - a chance to do some maths is a nice antidote to M202.

A Mathematician's Miscellany by J E Littlewood: some devil having pinched mine and it being out of print, I will offer a handsome prize to anyone who can bear to dispose of his.

Morphism: a concept (Gk Morpheus, god of hypnos/sleep) so hypnotic as soon to bore its own proponents.

Does anyone know of a good book on Groups to follow up Herstein? A really meaty one?

GRAHAM FLEGG: Following the first presentation of AM289 there have been a number of applications for post-graduate study by research in various aspects of the history of mathematics, and I hope that an increasing number of graduates who have successfully completed AM289 will continue to come forward for history of mathematics research.

The University is now exploring the question of post-graduate courses, and I would be particularly interested to know if there is any likely demand for a post-graduate course in the general area of history and/or philosophy of mathematics.

If you think you might seriously be interested in taking such a course would you please write to me at the Faculty of Mathematics preferably giving some indication of your own special interests.

JEREMY HUMPHRIES: Did I really send you problem 51.2? And then 52.3? If 51.2 has a solution we can say it was intentional, although 52.3 must be easier. (They should have been in reverse order.)

RESULTS OF VOTING WINTER 1977/78 MARION STUBBS

This last winter at renewal time there was only one item for a Constitutional Vote. This was "Amend Rule 2 to allow for three major publications. Add new sub-paragraph

3 MATES

MATES shall be a directory of voluntary members who are also subscribers to M500. Members of MATES shall undertake to correspond with fellow members in closed institutions, to assist them mathematically or to share mathematical interests and problems. The MATES directory shall be circulated only to MATES.

Amend Rule 3 by deleting 'directory' and substituting 'and/or MATES directories'."

Results:	YES	231
	NO	3
	ABSTAIN	23
		257

The second 'vote' was merely an opinion poll, not a Constitutional alteration. The results were intended to reveal the general opinions, in principle, of our renewing members about affiliation of THE M500 SOCIETY with OUSA. At the time of the poll no specific offers had been made by OUSA but since then some concrete offers have been made and were reported by Sidney Silverstone, our Liaison Officer with OUSA, in M500 49 qv. Results: "Do you approve in *principle* of affiliation with OUSA...?

Results:	YES	93 =	36%
	NO	62	24
	DON'T CARE	89	35
	ABSTAIN	13	5

which comes to 257 votes, 100% of Renewal Subscriptions received by 26/2/78.

The actual real Constitutional vote ref OUSA affiliation will be taken next winter 1978/78, according to the current Constitution. Please express your views fluently in M500 and even do some pressurisation pro or con by telephone via MOUTHS if you wish. It will do no good if you plaintively say that you do not have any information facts or whatever on which to make up your mind. You have the opportunities to make yourself fully acquainted with what is involved - ring Sidney Silverstone if in doubt. You have three months in which to prepare!

From PETER BLOOD: *Thanks for the copies of* M500 51 & 52 *which I greatly enjoy reading though I don't understand most of it. It gets clearer each year as I study the OU maths courses. I note on the envelope of my last copies the number* (48) *after my name which suggests my membership fee has run out. However I note from my cheque book I paid on the* 20*th November to* THE M500 SOCIETY *for this year. I hope I won't be getting a demand note.*

Pub - The expiry no. after your name is altered on the master set of address labels. It usually gets altered by hand only once a year, when your sub is due to expire, on the actual sheets being cut up. If M500 *comes your sub is OK. If it doesn't arrive please enquire. MS.*

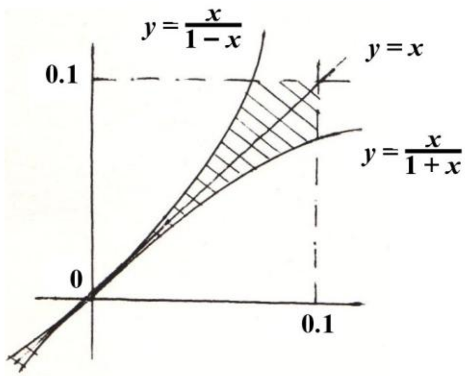
PRBLMS CRNR - JRMY HMPHRS

On the Delimiter Game MICHAEL MASTERS (M500 51 1) writes: I suggest that an answer would be to select as the delimiter a sequence of two characters which would not naturally occur; ie . , . This of course assumes that such characters are in the allowable set. To specify the delimiter as part of the message it would be necessary to have

., the delimiter is . followed by , ., .

SOLUTION 48.2 MODULUS (*If* $|x|< 0.1$ *and* $|y|< 0.1$ *what is the probably, p, that* $|x–y| < |xy|$?)

I've had some more letters about this problem. John Reade says that the solution set of the inequality $|x–y| < |xy|$ is the area between the hyperbolas $x–y = \pm xy$ or $y = x/(1\pm x)$. On the square $|x|<0.1,|y|<0.1$, the ratio of this area to the square is Michael Gregory's result. The diagram will clarify.



Michael used the fact that the curves are symmetric about the lines $y = \pm x$. He found the shaded area below the line $y = x$ and multiplied by four for the whole area. Hence his expression:

$$p = 4\left(\int_0^{0.1} x\,dx - \int_0^{0.1} \frac{x}{1+x}\,dx\right)/(0.2)^2$$

I received other answers for calculating the shaded area but I think these are wrong as they bring in areas which are outside the open square.

SOLUTION 48.3 SCRAMBLE *Write down a four digit number* (=*p*). *Scramble the figures to make another four digit number* (*q*). *Tell me any three of the digits in* $|p–q| = r$. *I can nearly always tell what the fourth digit is. What is the probability that I get it right*?

STEVE AINLEY points out that I made a rash assumption at the end of my solution (52 14).

I said "If all the numbers 0,1,... ,9 are equally to be the missing one ... " .

He has found that when two-figure numbers are used for 'scramble' the probabilities for the missing digit are P(0) = 35/180; P(9) = 17/180; P(1-8) = 16/180, and he suggests that a similar thing will happen with four figures.

I'm sure he's right. I've looked at the number of occurencies of each digit with three-figure scramble and they are

0 - 3864 1 - 1140 2 - 1104 3 - 1068 4 - 1032 5-996 6-960 7 - 924 8 - 888 9 - 1704.

That took over four hours to run on the HP67. I haven't yet looked at four figure scramble, which produces a total of 828 000 but if I do get round to it I'll let you know the correct answer.

SOLUTION 49.5 HORSE RIDING

I have requests for a less literary solution to this problem (see 52 15).

Let t = time of journey; v = speed over sand; $2v$ = speed over grass.

By Pythagoras $t = \sqrt{(4+x^2)}/v + \sqrt{(9+(7-x)^2)}/2v$. But

$$0 = \frac{dt}{dx} = \frac{x(4+x^2)^{-½}}{v} - \frac{(7-x)(9+(7-x)^2)^{-½}}{2v}.$$

This simplifies to

$$(x-1)(3x^3 - 39x^2 + 140x + 196) = 0$$

which has two complex conjugate solutions, one negative solution and

$$x = 1.$$

SOLUTION 51.1 POWERFUL DIGITS *Find the two consecutive four digit numbers which are equal to the sum of the fifth powers of their digits.*

ANGUS MACDONALD says that since the numbers, x and y, are consecutive clearly x ends in zero and y ends in one*. Also fifth powers of digits greater than five are too big so trial and error quickly finds

$$x = 4150; \qquad y = 4151.$$

(* The Hulbert MacDonald Theorem: $(a)^b \wedge 10|a \Rightarrow (a+1)^b$.)

SOLUTION 51.2 AGES *When my sister is four times as old as she was when I was twice as old as she was when I was twice as old as my brother, my brother will be two-thirds as old as I will be. My brother and I are teenagers; how old was my sister on her last birthday*?

This question mysteriously grew. The intended version appeared as 52.3. However, STEVE AINLEY says that there is an answer and it is

11 or 12

and who shall say him nay?

SOLUTION 51.4 DIVISION BY SIX *Show that if j is an integer* >1 *then* $t = (2j-1)j(j-1)$ *is divisible by* 6.

This is ANGUS MACDONALD'S solution:

$$t_j = 2j^3 - 3j^2 + j. \quad t_2 = 6. \quad \therefore 6 \mid t_2.$$

Assume $6 \mid t_k$, (ie $6 \mid (2k^3-3k^2+k)$). Now $t_{k+1} = 2k^3+3k^2+k = t_k + 6k^2$. $\therefore$ $6 \mid t_k \Rightarrow 6 \mid t_{k+1}$. $6 \mid t_2$. $\therefore$ $6 \mid t_j$ $\forall$ integers $j > 1$ by induction.

Not many solutions came for the 51 *problems. Nobody has yet sent impressive examples of* $(a)^b$ *but I'm sure machines are now buzzing away to get results in time for our next issue*.

PROBLEM 53.1 CIRCLES

Draw a circle *A*. Mark any five points on its circumference, labelled cyclically 1 to 5. On each of these five points as centre draw a circle of arbitrary size, but such that

1. The circle on point *i* does not contain either of the points *i*+1, *i*–1.
2. The circle on *i* intersects the circle on *i*+1 in two points and one of these points lies on the circumference of *A*.

(Note: 1+1 = 5; 5+1 =1.)

Draw the pentagon defined by the five intersection points which do not lie on *A*, and produce its sides until you have a five-pointed star.

Isn't that pretty? Why does it work?

PROBLEM 53.2 AN INANITY

S is a string of letters such that

1. Every other letter is a vowel.
2. If a letter is not a vowel it is 'n'.

Who can write a reasonably sensible sentence containing the longest S?

PROBLEM 53.3 POWER

What is the last digit of 3333377777 – 7777733333 ? (I don’t mean the 3.)

PROBLEM 53.4 SQUARE EDDIE

Given a length of 10, construct with straight edge and compasses a square of area 300.

PROBLEM 53.5 MULTIPLICATION

In the multiplication sum

```
  A B C D
        E
_________
F G H I J
```

the letters represent different digits in the scale of 10. If E = 4 what is A + B + C + D?

* * * * * * * * * *

Following BRIAN WOODGATE'S 'Define a function' (M500 52 2); I have a nice book called *Set Theory* by Charles Pinter (Addison Wesley) 1971. His definition is:

A function from A to B is a triple of objects $\langle f, A, B\rangle$ where A and B are classes and f is a subclass of $A \times B$ with the following properties

F 1. $\forall\, x \in A, \exists\, y \in B$ such that $(x,y) \in f$.

F 2. If $(x,y_1) \in f$ and $(x,y_2) \in f$ then $y_1 = y_2$.

By 'class' he means any collection of objects. All sets are classes but some classes do not qualify as sets since they are badly behaved and lead to paradoxes in intuitive set theory (eg Russell's).

Another book I have is *Differentiable Manifolds* by Brickell and Clark (van Nostrand) 1970, and part of the third sentence in it says

... but it is important to notice that we do not suppose that the domain of a function

$$f\colon A \to B$$

is necessarily the whole of the set A.

Don't, however, let your M101 tutor catch you saying that. I remember that my tutor used two lines to write a function, eg

$f\colon \mathbb{R} \to \mathbb{R}$

$f\colon x \longmapsto x^2$.

He didn't really approve of the OU way: $f\colon x \longmapsto x^2$, ($x \in \mathbb{R}$.) which does not define the codomain. For instance the function

$g\colon \mathbb{R} \to \mathbb{R}^+$

$g\colon x \longmapsto x^2$

is not the same as f but the one line method doesn't show the difference. He was fond of functions such as

Blend: {Juice,Fruit} $\to$ {Juice}

which demonstrates that many–one functions have no inverse. If you take the juice from the codomain and put it through your anti-blender (Blend^{-1}) it doesn't know whether to stay as juice or turn back into fruit.

* * * * * * * *

I saw this when it appeared some months ago, thought it must be a joke and forgot about it. I have now seen it reproduced as a filler in the *Mathematical Gazette*, so perhaps he meant it.

"I would wager that very few maths teachers can solve the following problem: can it be proved that 9.9 recurring is 10?

"The trick is that you have to know something of astronomy to see the answer. If the universe is eternal, if infinite time can be allowed for the calculation, then 10 will sometime be reached. But if space and time are going to end, thousands of millions of years hence, as some scientists predict, then 9.9 recurring will never be seen to reach 10."

Adrian Berry, *Daily Telegraph*.

Jerry

EDITORIAL

In pursuance of my policy of keeping members of THE M500 SOCIETY up to date with happenings in the alternative worlds I will pass on the result of a couple of reports in *Astrophysical Journal*, May 78.

From observations at Mount Palomar and at Kitt Peak, using apparatus developed by Dr Alec Bocksenberg of UCL, there seems to be a Black Hole in galaxy M87, Virgo constellation. The hole is about 300 light years across (0.003% of the diameter of M87), is in the centre of the galaxy, and has a mass about 5 000 000 000 times that of the sun.

Since noting the above I see *Scientific American* has picked up the report. Even so I believe the first Black Hole deserves to be commemorated in M500.

They can't scoop me on this one though: Do you remember Kaprekar's Constant: 6174? (Put those four digits in descending order, reverse them, take one from the other, and get 7641 – 1467 = 6174. Do the same thing often enough with any four digit number and end up with 6174. See M500 35 1.)

Well, how much of a coincidence is it that rearranging the digits gives 4671 which multiplied by 59 equals 9 123 046 875. Shift the digits one place and 1467 × 221 = 3 076 521 984. Both these long numbers are PANDIGITAL (contain 0-9 exactly once each)? There are two shifts to go. Any volunteers?

Since becoming editor of M500 I have had three books sent to me to review. The first was called *Appropriate Technology*: *problems and promises*, edited by Nicolas Jequier and published by the Development Centre of the Organisation for Economic Co-operation and Development, Paris 1976 at £5.60. I imagine we got it because M500 was for a time on the directory of 'Periodicals that progressive Scientists should know about'. Somehow I never seem to have got around to reading it. It has a silver cover and contains 344 pages, packed with information about the uses and abuses of technology, particularly in the underdeveloped parts of the world. It is in two bits, the first by the editor and called 'The major policy issues', the second by various authors with interesting looking names, under the general heading 'The practitioners' point of view'. Chapter XV of part two is 'Small Scale Distillation of Potable Spirits From Palm Wine'; it contains the unnerving sentence: 'The final product is usually water-white in colour, sometimes straw yellow. Solid contamination is usually visible. The flavour is characteristically strong.' This chapter is followed immediately by 'Appropriate Technology in Ethiopian Footwear Production' so you see there is a good range. I find the style somewhat turgid; a little like certain OU texts, but that is no doubt to be expected from a committee of authors and no doubt translators. If anybody would like the book in exchange for a review we can publish I will send it along; ease my conscience a little.

The latest book to arrive is *Proof in Mathematics* by P R Baxandall (who has lectured at OU Summer Schools), W S Brown, G St C Rose and F R Watson published at the University of Keele by the Institute of Education, from whom it is available at 75p including postage. This will be reviewed later. Finally there is, of course, *Chez Angelique*. which if you haven't got you must immediately send £1 for to

John Mason, Maths Faculty, Walton Hall, &c.