M500 55

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Articles and solutions are not necessarily correct but invite criticism and comment. Anything submitted for publication of more than about 600 words will probably be split into instalments.

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M500 55 published October 1978. Subscription £4 for 10 issues. Cheques and Postal Orders should be made payable to "The M500 Society" and crossed "Account payee only. Not negotiable".

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The cover is a drawing by Roger Stangar, a friend of Jeremy Humphries, of Prof. P.R. Halmos, author of "Naive Set Theory". With this drawing, we bid a sad farewell to M202 and the authors of its set books, of which Halmos was one. Prof. Halmos has given his happy consent to the publication of this sketch, in a letter to Jeremy Humphries.

A POLYNOMIAL GENERATING ALL THE PRIME NUMBERS

ALAN SLOMSON

This note is prompted by Eddie Kent's Breakthru in M500 53. It has been known since 1970 that there exist polynomials which generate all the prime numbers. I want to sketch the history of this problem and give some references to places where the details may be found.

The study of *Diophantine equations* is very ancient. The name derives from that of Diophantus who flourished in Alexandria about 250AD. A Diophantine equation is one like

$$x^2y - 5xz + 3y^2 - 5 = 0. \qquad (1)$$

or

$$x^2 - 2y^2 = 0 \qquad (2)$$

that is, it is an equation in which on the left hand side we have a polynomial in a finite number of unknowns, and whose coefficients are positive or negative integers. The problem in each case is to try to find *postive integer solutions*.

In some cases a Diophantine equation does have positive integer solutions. For example, one solution of (1) is

$$x=7,\ y=2,\ z=3.$$

In other cases it is possible to show that there are no solutions. For example, it is well known that equation (2) has no positive integer solution. It is easy to see that this is just another way of saying that the square root of 2 is irrational.

However in general it is difficult to decide whether or not a Diophantine equation has a solution. In 1900 in a famous address to the International Congress of Mathematicians in Paris, the great German mathematician David Hilbert listed twenty three problems "from the discussion of which an advancement of science may be expected". The tenth of these problems was "Given a Diophantine equation with any number of unknown quantities: TO DEVISE A PROCESS ACCORDING TO WHICH IT CAN BE DETERMINED BY A FINITE NUMBER OF OPERATIONS WHETHER THE EQUATION IS SOLVABLE IN RATIONAL INTEGERS." Although the wording of this problem suggests that Hilbert thought it would be possible to discover such a process, Hilbert was almost certainly aware of the possibility that it might not exist.

The proof that no such process exists depends on having a general theory of processes involving "a finite number of operations". Today we call such processes *algorithms*,. A general theory of algorithms did not begin to develop until the 1930s when a number of mathematical logicians put forward ostensibly different precise characterizations of the notion of an algorithm. In fact these characterizations all turned out to be equivalent. M202 students will be familiar with some of them, especially that in terms of Turing machines, which was put forward by the English mathematician Alan Turing in 1936.

However for the purposes of this note it is adequate to think of an algorithm as a *calculation which can be carried out by an ideal computer*. By an *ideal computer* I simply mean a digital computer where we are not restricted by the practical problem of storage space, so we assume that the store of our computer can be increased indefinitely as required. Now for a few technical definitions.

A *recursive function* is a function f such that there is an algorithm which enables us for each natural number n to calculate the value of $f(n)$. A set X of natural numbers is *recursive* if there is an algorithm which enables us for each natural number n to decide whether or not it is in the set X. For example, the set of prime numbers is recursive; the algorithm for deciding whether or not n is prime is to test all the natural numbers up to the square root of n to see whether they devide into n.

A set X of natural numbers is *recursiely enumerable* if (either X is empty or) there is a recursive function f such that

$$X = \{f(0),\ f(1),\ f(2), ...,\ f(n), ...\}.$$

It is important to appreciate that a recursively enumerable set need not be recursive. Given the enumeration of X as above we would, in general, have to check the whole infinite list to see whether or not a particular number is in X and this is not a process involving only a finite number of operations. It is a basic fact of the theory that

FACT 1. THERE ARE RECURSIVELY ENUMERABLE SETS OF NATURAL NUMBERS WHICH ARE NOT RECURSIVE. (HOWEVER ALL RECURSIVE SETS ARE RECURSIVELY ENUMERABLE.)

Now for the connection with Diophantine equations. Let $P(k,x,y,z, ...)$ be a polynomial which involves the numerical parameter k and the unknowns $x,y,z,...$. Then the Diophantine equation

$$P(k,x,y,z, ...) = 0$$

will have a solution for some values of k and not for others. The set X of those k's for which a solution does exist is called the Diophantine set defined by the equation. In symbols

$$X = \{k: \exists x \exists y \exists z \ ...\ P(k, x, y, z, ...) = 0\} \qquad (3)$$

where $\exists x$ means "there exists a positive integer x" etc. A set of natural numbers which has a definition of the form of (3) is called a *Diophantine set*.

For example consider the set

$$S = \{k: \exists x \exists y\ (x^2 - ky^2 = 0)\}.$$

It can be seen that S is the set of perfect squares which is thus a Diophantine set. Clearly we have

FACT 2. IF THERE IS AN ALGORITHM TO DETERMINE WHETHER AN ARBITRARY DIOPHANTINE EQUATION HAS A SOLUTION, THEN EVERY DIOPHANTINE SET IS RECURSIVE.

The solution of Hilbert's Tenth Problem is completed by establishing the following

FACT 3. EVERY RECURSIVELY ENUMERABLE SET IS DIOPHANTINE.

It can be seen that Facts 1, 2 and 3 together imply

THE DAVIS–MATIJASEVIČ–PUTNAM–ROBINSON THEOREM:

> THERE IS NO ALGORITHM TO DETERMINE WHETHER AN ARBITRARY DIOPHANTINE EQUATION HAS A SOLUTION.

Thus Hilbert's Tenth Problem has a negative solution. I must now explain why there are four names attached to this theorem. Fact 1 was proved in the early days of the theory of recursively enumerable sets. The hard work was proving Fact 3. Martin Davis, Hilary Putnam and Julia Robinson are three American mathematicians who, working at first independently and then together, made a major advance towards establishing the truth of Fact 3. They almost succeeded in proving it, but their argument left one gap. That was the problem of whether the exponential function has a Diophantine definition. In other words, whether there is a polynomial $P(m,n,x,y,z,...)$ with two numerical parameters m,n and unknowns $x,y,z,...$ such that for all m,n

$$m = 2^n \Leftrightarrow \exists x \exists y \exists z (P(m,n,x,y,z,...) = 0).$$

In a series of papers published in the 1950s and 1960s Davis, Putnam and Robinson showed that if such a polynomial exists then Fact 3 is true. In 1970 a young Russian mathematician completed the proof by showing how to give a Diophantine definition of the exponential function. He is Yuri Matijasevič, and thus his is the fourth name attached to the theorem above.

What has all this to do with a polynomial which generates all the primes? Read next month and find out.

SAVE YOUR SPIVAK   MARION STUBBS

There are sundry complaints that SPIVAK (and indeed many OU set books) literally falls apart with much usage. Mine has not - neither has any other of my OU set books. But then, I am a librarian by trade! Here are my counsels on how to make your books last longer. The methods are tried and tested.

SPIVAK is an example of contemporary 'Perfect Binding' - which is, of course, anything but perfect. It means simply that it is unsewn: the backs of the pages have been chopped off, strong glue forced about 2mm deep between the pages, and this glue is all that holds the book together. M500 is an example of the contrary state - pages are not chopped and 'stitching' in the form of a metal staple is applied to the folds. Consequently M500 cannot fall apart unless the staple falls out or goes rusty, which is why bookbinders use thread for stitching, not metal.

1. *Never*, under any circumstances, place anything thicker than a paper bookmark between the pages of *any* book, let alone a 'perfect' book. Do *not* place your latest Assignment booklet inside SPIVAK or inside a Unit or inside anything else. The extra thickness forces the glued pages apart and the book will disintegrate very rapidly. For the same reason never turn such a book over face downwards to keep your place while you go off to do some other job.
2. Try, especially with SPIVAK, applying invisible sellotape down the joints at front and back where the endpapers are helping to hold the book togecher. Do not use ordinary sellotape since this goes gooey with age. Invisible tape, also known as ‘ghost tape’, disappears on

contact with paper and is very special, rather expensive. It does not go gooey. Use full-length strips for the job and trim to be level with the top and bottom of the book. This looks neater and more professional and in any case is stronger than a few inches of tape stuck somewhere amateurishly in the middle.

3. Cover your book(s) with transparent plastic film, obtainable in small quantities from WHSmith and other stationers. The transparent film helps to hold the book together and keeps it clean too, Transparent film may be sticky or plain; the sticky kind works like Fablon or Contact and is best for the job - you could even use Fablon or Contact if you like. Method is to start by peeling away 1" of the backing material, apply the sticky side to the book, carry on peeling with one hand while the other hand holds a ruler or straight edge and keep pressing the sticky side flat onto the book. With luck you get no air bubbles. The plain sort is simpler and is applied according to the directions on the pack. The you mitre all the corners at 45 degrees, turn in, and if necessary use sellotape to hold the turned in edges to each other. You need to trim with scissors, preferably in a curve to accomodate the spine of the book. The trimmings are thrown away, not turned in.

4. When all else fails and a book falls apart you could try forcing some new adhesive (eg Cow Gum) along the backs of the pages, after knocking the book into correct shape.

INFINITE QUALITY CONTROL G MARRIOTT

With regard to recent articles of M500 48 and 49 on infinities, I would say that the number of infinities that exist probably depends on how many infinities we wish to bring into existence; ie how many infinities we wish to define. This being so, the main criterion would presumably be the pay-off, in terms of results, to be obtained from defining any particular number of infinities.

I would agree with Brian Woodgate (issue 48) that, intuitively, it seems natural to define just one infinity. I would think that to define more than one infinity could lead to possible inconsistencies, perhaps to the idea of infinity not being 'well-defined'.

The idea of differing infinities tends to arise through notions of 'countability', which depends on existence proofs for 'counting' functions.

Taken in isolation this might not be too solid a basis; if something is said not to exist may it not be that the object in question merely has not yet been defined?

If we accept that the Cartesian Product of an infinite number of sets is non-empty (Axiom of Choice) we are then able to 'select' a 'real number'. There is no restriction on the use of the Axiom of Choice so we may apply it again to select another ‘real number’.

If we repeat this process indefinitely and add the restriction that for all real numbers r in the sequence, $r_k \neq r_n$ for any $n < k$ could we not use that to define a counting function f for the real numbers? eg

$$\forall k, n \in \mathbb{N},\ k \neq n \Rightarrow r_k \neq r_n$$

by definition. So f is one–one. Also the only condition r must satisfy to be in the image of $\mathbb{N}$ under f is that r be a real number. As $\nexists$ a real number which cannot be 'selected' using the Axiom of Choice, $\therefore$ f is onto.

As f: $\mathbb{N} \rightarrow \mathbb{R}$ is one–one and onto we may conclude $\#\mathbb{N} = \#\mathbb{R}$, where # is the cardinality mapping.

I believe it may be more helpful to categorize sets of 'varying infinities' in a qualitative sense, rather than by quantity. To do this one would probably need some naive notion of an 'element' of a set. eg if a set could be a flock of pigeons then an element could be a pigeon. Given this naive notion one could categorize types of elements as we do with sets. eg we may have a set of sets so we may have an element of any number of elements.

The idea of an element is implicitly important to the idea of a one–one mapping as it is the elements which are in one–one correspondence. Until some idea of an element is made explicit one cannot be certain that any one–one correspondence is matching elements with elements. eg an element of one set may be a 'single' element, an element of another set may be an unlimited number of elements; without some notion of element we may find ourselves matching all the elements of one set to only one element of another set.

If we catagorize sets according to the type of element they contained, we could say that a set has quality $\aleph_0$ if its elements are single elements; a set has quality $\aleph_1$ if its elements are sets of quality $\aleph_0$; a set has quality $\aleph_2$ if its elements are sets of quality $\aleph_1$; ... ; and so on.

I think that such a qualitative approach would still be able to absorb and reflect, in a natural way, the ideas of 'varying infinities' while establishing the logical necessity for only one 'quantitative' infinity. eg if $\#\mathbb{Z} = \#\mathbb{Q} = \aleph_0$ then $\mathbb{Z}$ and $\mathbb{Q}$ have quality $\aleph_0$ as they contain single elements only. If $\#\mathbb{R} = \#\mathbb{R}^2 = \#\mathbb{R}^n = \aleph_1$ then $\mathbb{R}$, ..., $\mathbb{R}^n$ have quality $\aleph_{1;}$ eg a real number is an infinite set of quality $\aleph_0$.

If $\#G$ = where G is the set of all geometric curves, then G has quality $\aleph_2$; this is because G is an infinite set of real lines, and each real line has quality $\aleph_1$.

Anyone have any further comments on this?

We ... have dreamt the world. We have dreamt it as firm, mysterious visible, ubiquitous in space and durable in time; but in its architecture we have allowed tenuous and eternal crevices of unreason which tells us it is false.

J L Borges: *Avatars of the Tortoise.*

"THE WORLD OF MATHEMATICS" CONTINUED

TOM DALE

As I mentioned last month, one item in *The World of Mathematics* (edited by J R Newman) can be referred to Krysia Broda's article in M500 47 1 . This is from *The Great Mathematicians* by HWTurnbull. Writing on the Greek mathematicians, Turnbull says "to

1	1
2	3
5	7
12	17
29	41
70	99

approximate to √2 they built a ladder of whole numbers. A brief scrutiny of the ladder shows how the rungs are devised: 1 + 1 = 2; 2 + 3 = 5; 2 + 5 = 7; 5 + 7 = 12 and so on. Each rung of the ladder consists of two numbers, x and y, whose ratio approaches nearer and nearer to the ratio 1: √2 the further down the ladder it is situated. Again, these numbers x and y, at each rung, satisfy the equation $y^2 - 2x^2 = \pm 1$. The positive and negative signs are taken at alternate rungs, starting with a negative."

He goes on to say that'"such a ladder could be constructed for any irrational but the only other

1	1
1	2
2	3
3	5
5	8

example is one which he admits has not been found in Greek writing. As shown, it is connected with the Fibonacci series. The right member of each rung is the sum of the pair on the preceeding rung. The ratios approximate (by a little more and a little less) to the limit √5+1 : 2, the Golden Section.

Later in the same article, discussing Archimedes, Turnbull states that "he casually states approximations to √3 in the form

$$\frac{265}{353} < \sqrt{3} < \frac{1351}{780},$$

which is an example of the ladder arithmetic of the Pythagoreans. As these two fractions are respectively equal to

$$\frac{1}{3}\left(5 + \frac{1}{5+1/10}\right) \qquad \text{and} \qquad \frac{1}{3}\left(5 + \frac{1}{5+\frac{1}{10+1/5}}\right)$$

(⅓(5;5,10) and ⅓(5;5,10,5) in Krysia's notation) it is natural to suppose that Archimedes was familiar'with continued fractions, or else some virtually equivalent device."

1	5
5	26
51	265
260	1351

The continued fractions above are expansions of $\sqrt{27}$; a ladder could be constructed by adding alternatively five times or ten times each rung to the previous rung; this seems to depend on knowing the continued fraction. The ladders could be convenient ways, however, of finding successive convergents (in effect equivalent to (3) of Krysia's article) and the device seems to have been a clever method of approximation to a square root in the days before decimals (and Newton-Raphson!)

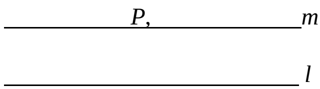
My old algebra book which is my only source of information on continued fractions gives 1;3,2,3,2, ... = √(5/3) and 1;2,3,1,2 3 , ... = $^{1}/_{7}(4 + \sqrt{37})$. I wonder if anyone can see how to do them. I can't.

GAUSS IX

JEREMY GRAY

Gauss was always interested in geometry and characteristically proceeded from the first with a fresh point of view. In 1792, if his letter to Schumaker written forty years later is to be believed he struck new ground in the investigations concerning non-Euclidean geometry.

At that time researches into the foundations of geometry were acknowledged to be in a scandalous state. First the Greeks, then the Arabs, and then Western mathematicians had looked at Euclid's geometry and wondered: is the parallel postulate necessarily true? is it really necessary to assume it? The postulate asserts in a version equivalent to Euclid's statement that given any line *l* and any point *P* not on *l* there is one and only one line, *m*, say, through *P* which never meets *l*



The line *m* is called the parallel to *l* through *P*. It irked mathematicians that it should seem necessary to assume such a thing when it seemed evident that it was true of space. It had rather the character of a theorem and many attempts were made to prove it. These attempts were published, and they fall into three classes:

1. Those which sought to show that there cannot be a geometry in which the line *m* does not exist. These attempts were generally successful (Saccheri, Lambert, Legendre).

2. Those which sought to show that there cannot be a qeometry in which there are many lines *m* through *P* which do not meet *l*. These attempts were inevitably unsuccessful, and were either abandoned (Lambert) or published with errors in them (Saccheri, Legendre).

3. Those which tacitly or explicitly made use of a postulate effectively equivalent to the parallel postulate. Thus: that similar figures (ones having the same shape but

different size) to a given figure exist; or that the curve which is everywhere equidistant from a straight line is itself straight; or that three distinct points either be on a line or on a circle.

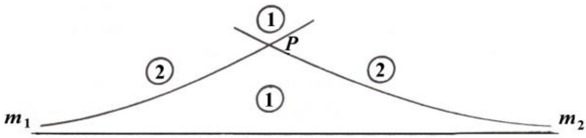
In the eighteenth century the problem had been treated at length by Saccheri (1733) and later by J H Lambert (1766, published 1786). They had shown that any geometry which differed from Euclid's only in respect of the parallel postulate could only be either

a. one in which no parallels exist, the angle sums of trianglesalways exceeded it; and in fact that no such geometry could exist; or

b. one in which many parallels (m) existed and the angle sum of triangles always fell short of π. The amount by which the sum $\hat{A} + \hat{B} + \hat{C}$, say, differs from π, Lambert showed, was proportional to the area of the triangle:
Area ($\triangle$ *ABC*) $= \pi - (\hat{A} + \hat{B} + \hat{C})$.

But there the matter rested. The French mathematicians, with the exception of Legendre, dismissed it with irritation and it became the happy hunting ground of amateurs eager to rush into print with their invariably fallacious defences of Euclid.

Gauss may or may not have read Saccheri, I do not know. Perhaps guided by Kästner, a mathematics professor at Göttingen and the man who had first suggested that Lambert work on the problem, Gauss did read Lambert's work on arriving at the university in 1795. But he claims to have begun work even before then with what is in fact an elegant reinterpretation of a figure of Saccheri's. Saccheri had shown that, on the assumption that the angle sum of any triangle is less than π, the lines through P divide into three families (see figure on opposite page). Those in region (1) will always meet l. Those in region (2) will never meet l, and diverge from it in *both* directions. Then there are m_1 and m_2; $m_1$approaches l asymptotically to the right and diverges to the left, m_2 approaches l asymptotically to the left and diverges to the right. Whilst m_1 and m_2 might well be called parallels to l they are by no means the only lines through P which do not meet l. Gauss took this figure and interpreted it to involve



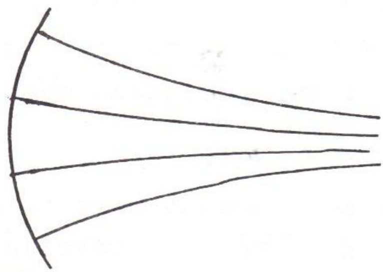
directed lines. Now only one of m_1 and m_2 can be parallel to l and in the same direction, and in a little memoir he showed that parallelism of directed lines is an equivalence relation:

l is parallel to itself

if *l* is parallel to *m*, *m* is parallel to *l*

if *l* is parallel to *m* and *m* parallel to *n* then *l* is parallel to *n*.

In a second memoir he showed that given a family of parallel directed lines a locus could be obtained meeting each line of the family at right angles in a curious curve later called the horo-cycle.



But what of the validity of the parallel postulate? Was there any sense in investigating other geometries? And if geometry (Euclid's geometry) is about space what could a new geometry be about? Gauss hedged. In 1799, in a letter to Bolyai Farkas, a former fellow student, he wrote that his researches made him doubt the truth of geometry. In 1817, writing to the astronomer Olbers he declared that the truth of the matter was not for human understanding. In 1818 he endorsed Schweikart's researches into what Schweikart had called an Astral Geometry (which was indeed to be a non-Euclidean geometry). In 1829 he wrote to Bessel that he feared "the howl of the Boetians" if he published his views - the Boetians were regarded as particularly barbaric by the chauvinistic Greeks. In 1831 he did write down his earliest researches for fear they otherwise perish with him but only in 1832 did he unreservedly accept the existence of a non-Euclidean geometry, for by then the matter had been taken out of his hands. Not one but two mathematicians had done what Gauss had always refused to do: publish an account of a geometry different from Euclid's.

This geometry, independently discovered by the Hungarian Janos Bolyai (son of Farkas) and the Russian Nicolai Lobachevskii, was first described in terms of its trigonometry. That is, formulae relating the sides and angles of triangles were given, somewhat like the formulæ of Euclidean plane geometry but more like those occuring in spherical trigonometry. The familiar functions sin and cos are replaced by new functions sinh and cosh which, oddly enough, had been introduced and studied for a different purpose by Lambert in 1761. In terms of these formulæ the relationship between the distance PA (the perpendicular from P to l) and the angle between PA and m_1 can be derived and all of non-Euclidean geometry deduced

This remarkable discovery, the first decisive separation of the mathematically describable and the real worlds, was to have an immense effect on all of mathematics. Yet the reactions of even the mathematical community for a generation was a mixture of hostility and neglect. Bolyai, Lobachevskii and Gauss were all to die before the discovery was to begin to be appreciated at its true worth. The key ingrediant then, paradoxically enough, was to be another of Gauss's ideas as the next episode will discuss.

"PROOF IN MATHEMATICS" GRAHAM FLEGG

Proof in Mathematics - (Ed.) F R Watson - Keele University - is essentially a collection of interesting problems of various degrees of sophistication illustrating ideas related to proof in mathematics The material is held together and given shape by useful introductory explanations of specific topics, eg 'the role of proof', 'the idea of implication', 'language and notation', etc. Neither the form of presentation nor the examples are new, being largely extracted from sources quoted at the end of the book, especially Courant and Robbins, Moses, Polya, M100 17, etc. Inevitably there are some barely satisfactory aspects of such a work, and some places where greater care and attention to detail would have paid considerable dividends. The discussions of implication and equivalence, for example, lack the necessary clarity; and the authors (as do so many others) seem unable to present clearly and unambiguously the different uses of the word 'or' in everyday speech in relation to their logical representations. Their own use of 'or' is not always strictly logical; The relationship between implication and causality needs much better clarification. There is some curious splitting up of related material, eg why postpone discussion of $\sim Q \Rightarrow \sim P$; why not discuss Peano's Axioms with mathematical induction, etc? The forward references could have been avoided by a more logical grouping of the subject matter. More space could with profit have been devoted to discussions of necessity and sufficiency and of the universal and existential quantifiers. The density of presentation is very varied, and it is not clear just what level of mathematical sophistication on the part of the reader is being assumed. There are a few typing errors and one diagram (paae 81) is not in accord with the corresponding text. Historical reference have been taken from secondary sources - sometimes a dangerous practice, as the Authors are no doubt well aware! The book is at its best in the discussion of the pitfalls of hidden assumptions, naive geometrical 'proofs' etc. and overall the Authors are to be commended for assembling such a useful collection of material from a number of sources under one cover. (I have not gone in detail through all the exercises.)

Ed - The book came to me from Professor Pengelly in July with the following covering letter: ... I am sending you a copy of a booklet on "Proof in Mathematics" which has been published by the Institute of Education at the University of Keele. One of the authors - P R Baxandall - based his own contributions to the book largely on the lecture on "Problems and Proofs" which he has given at several M201 Summer Schools. Consequently he thought that M500 might be interested in publishing a review of the book. ...

The first thing I noticed was that it used the same production method we do on M500*: the whole thing typed out on* A4*, reduced by the printer and stapled together. Unfortunately the typist has used too small a type-face and too great spacing between the lines, making the book irritattng to read. Also most of the symbols are put in by hand as are many ccrrections and afterthoughts; these, in some cases, are almost unreadable. The other main irritation is mentioned by Graham above: the forward references. You turn forward to look, find you have to read a good deal to understtand what is going on then forget where you were. There are uncorrected errors but I had no trouble spotting them. I found the book really interesting reading; 1 was able to read most of it in bed. And it is extraordinary value at 75p. I recommend you buy a copy immediately and hope that by the second edition they can afford a typewriter like this.*

OBITUARY - PROFESSOR HERBERT DINGLE

Herbert Dingle died on September 4th at the age of 88. He was born in London but moved early to Plymouth on the death of his father. There he attended various schools till he left at 14 and spent the next 11 years working as a clerk and studying for his scholarship to Imperial College London where he read Physics. He stayed at London University in one position or another till he eventually became Professor Emeritous on retirement.

His early work was in spectroscopy and it was always his ambition to analyse the sun during an eclipse. There were three chances in his working life. ln 1927 he missed out at Colwyn Bay because of clouds. The same thing happened at Montreal in 1932 then in 1940 his expedition was called off because of the war.

Another of his interests was in relativity. In 1922 he wrote *Relativity for all* and got himself the reputation of being one of the six people in the world to understand the subject. (Presumably this reputation was among people who had read neither Einstein nor Dingle.) In 1932 he wrote of the mathematical aspects of the so-called expanding universe theory and on EA Milne's alternative cosmogony.

He wrote heaps on scientific philosophy. His central themes were, first, that science is firmly rooted in experience and secondly that it is a rational and coherent scheme of thought, though not necessarily the only such scheme. Eddington and Jeans seemed to him to present science to the lay mind as a set of mysterious paradoxes, at times almost as something irrational.

This all led to his contraversy which started in about 1969 and lasted for years, culminating finally in a book. It began in *Nature* when he attacked that aspect of the Special Theory known as 'time dilation' where it is asserted that the time registered by a clock depends on the speed it is moving at. He was of course attacked bitterly by most of the scientific community and eventually the editor of *Nature* called a halt to the correspondence. So he transferred to the *Listener*, Even an account by John Taylor of an experimant in which two clocks were sent off round the world at different speeds and which then told different times was dismissed by Dingle. He said that of course they showed different times: they were given different accelerations! Nothing to do with their speed. Several times the editor tried to stop the correspondence but each time it blew up again. I remember Friday mornings became an exciting event in my life for a while.

His basic contention was that so much of present day technology is based on nuclear effects and thus ultimately on Special Relativity that if there is a flaw in the theory so basic and so unnoticed, one day we are all likely to vanish when that flaw reaches a critical point.

One of the examples he used was an equation relating workmen to holes in roads. Since it was a quadratic it had two solutions, 3 and –7. He said that we know that only the 3-solution is useful because from our experience of the world we know there are no minus-men. Therefore in Special Relativity we should reject any solution calling for time dilation because that also obviously does not happen. I was doing M100 at the time and thought "Fool; has no one ever told that no function is defined until its domain is specified."

Sorry; I hadn't meant to go on so long. But if you ever have a minute to spare you might call in the local library and look up those old Listeners. The correspondence makes fascinating reading and justifies all our faith in professors.

EK

PROBLEMS EDITED BY JEREMY HUMPHRIES

In the editorial for 53 Eddie wrote of his discovery concerning Kaprekar's constant 6174. If the figures are rearranged, $4671 \times 5^9 = 9123046875$; shifted one place - $1467 \times 2^{21} = 3076521984$. The large numbers are pandigital. BOB BERTUELLO has found an example with the next shift

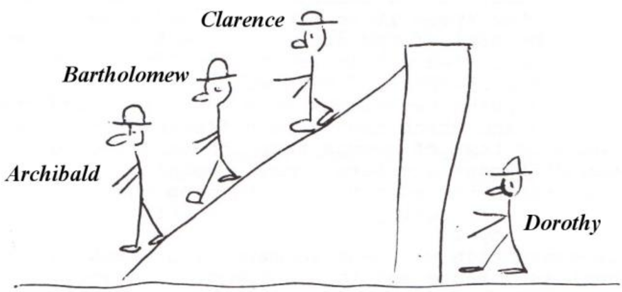
$$7146 \times 5192 = 1\ 924\ 853\ 706\ .$$

He writes: I also tried the third shift $6714 \times a^n$ and I believe I have exhausted all $n > 1$ (and myself) without finding a solution. Any volunteer for $n = 1$?

My friend CHRIS PANAYI showed me a variation of 'naming a chosen card' with which you can baffle your victims. Notice that many playing cards are not symmetric about the horizontal bisector - one half has more symbols than the other, or more symbols point one way than point the other way. Select these and arrange them all to 'point' the same way. Discard the rest. Fan the pack, face down, and ask the victim to select two, say, and commit them to memory. While he is doing this close the pack turn it round and refan it so that he can replace his cards. He won't see you turning it round. Now shuffle the pack, let him shuffle it then examine it and simply select the two cards which are the wrong way round.

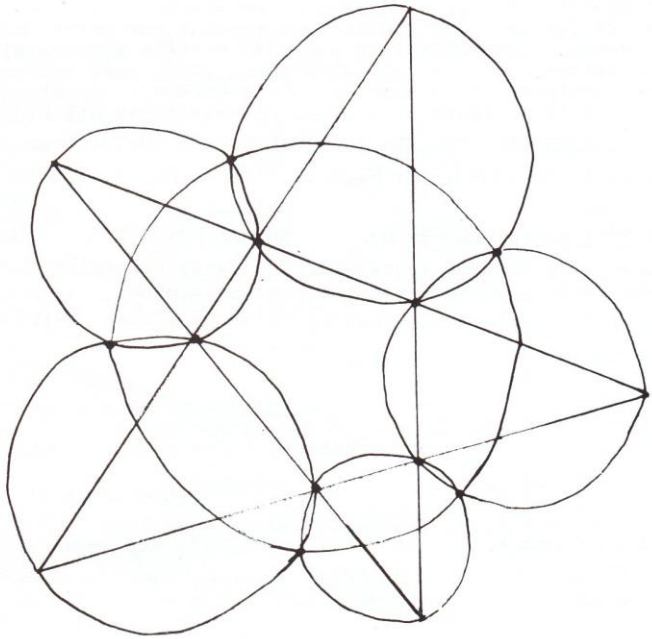
Here is a quicky from my friend LIBBY DRAKE.

Four men, Archibald, Bartholomew, Clarence and Dorothy, are each wearing a hat. They all know that two hats are black and two are white. No one can see his own hat.



Archibald, Bartholomew and Clarence stand on a ladder so that Bartholomew can see Archibald,and Clarence can see Bartholomew and Archibald but Dorothy stands behind a wall. He can't see anyone and no one can see him.

Whan a man knows the colour of his hat he says so. What can happen?



SOLUTION 53.1 CIRCLES

I gave instructions for drawing this. The circles are of any size but must intersect as shown. I asked "Why does it work?" meaning "Why do the star's points lie on the circles?" The only response was "Why does what work?" And I must admit that when you don't know what is supposed to happen it's quite easy to get the points nowhere near the circles. Accurate drawing is essential.

Now you do know, how about a pretty proof to go with it?

SOLUTION 53.2 INANITY *S is a string of letters such that every other letter is a vowel and any non-vowel is an 'n'. Write a sentence containing a long S.*

We began to produce such sentences one afternoon in the office and the fact that it got into M500 is an indicator of the number of maths problems I have in reserve. Still, it's a bit of fun. I got three answers.

ANGUS MACDONALD was the only one who actually produced a sentence. He said that the nurse who came on the ambulance from the Catholic hospital didn't have any bandages:

that's INANE, NONE, NO NUN ON A NINE NINE NINE NEglects things like that.

That uses 31 letters. I didn't require that the string *S* be the whole sentence. BOB BERTUELLO and JOHN METCALF sent verbless strings thinking that I did. John's has 37 letters and could easily be incorporated in a sentence and extended thereby to 39 letters. It is a remark by a hotel proprietor to his old nurse.

O, NO, NANA NAN, ONE NUN IN A NINON IN ONE-NINE-NINE

to which could be added, for example, 'NEeds her tea'.

SOLUTION 53.3 POWER *What is the last digit of* $33333^{77777} - 77777^{33333}$?

The answer is 6, sent by BOB BERTUELLO, ANGUS MACDONALD, JOHN METCALF and GORDON ROBSON, who all had the same reasoning:

The last digit of any number ending in 3 and raised to the power n is
3 if $n \equiv 1 \bmod 4$
9 if $n \equiv 2 \bmod 4$
7 if $n \equiv 3 \bmod 4$
1 if $n \equiv 0 \bmod 4$.
The last digit of any number ending in 7 and raised to the power m is
7 if $m \equiv 1 \bmod 4$
9 if $m = 2 \bmod 4$
3 if $m \equiv 3 \bmod 4$
1 if $m \equiv 0 \bmod 4$.

So 3333377777 ends in 3 and 7777733333 ends in 7 so the answer is 6.

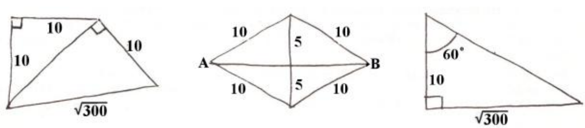
SOLUTION 53.5 MULTIPLICATION $E(ABCD) = FGHIJ$; *the letters represent different digits in the scale of* 10. *If* $E = 4$ *what is* $A+B+C+D$?

Many people gave me answers to this and both of them got it right. A+B+C+D = 19. SIDNEY SILVERSTONE says: $A = 7$, $B = 0$, $C = 3$, $D = 9$, $E = 4$, $F = 2$, $G = 8$, $H = 1$, $I = 5$, $J = 6$ but I don ' t know if it is unique.

Somebody at the recent Weekend also gave me 19 but I'm afraid I've forgotten who it was. Many apologies.

SOLUTION 53.4 SQUARE Given a length of 10 construct a square of area 300 units using straight edge and compasses only.

EDDIE KENT, ANGUS MACDONALD, GORDON ROBSON, SIDNEY SILVERSTONE and BRIAN WOODGATE sent correct constructions. The problem is obviously solved when we have a length of √300. Simplest were:



Eddie   Gordon. AB = √300   Brian.

PROBLEM 55.1 BLACK AND WHITE

Remove two black and two white squares from a $(2n)^2$ chessboard. Show that if at least one black and white square is interior the remaining board can be tiled with 2×1 dominoes.

PROBLEM 55.2 COINS

A tosses two coins, hidden from B. "Is there at least one tail?" B asks. A says "Yes". (a)

A then drops one coin accidentally. (b)

When A and B find the coin they see it is a tail. (c)

"That's OK," A says. "It was a tail to start with." (d)

At each point, a,b,c,d, B calculates to the best of his knowledge the probability that both coins are tails. What are the probabilities?

PROBLEM 55.3 CUBE

I have a wire model consisting of the edges of a cube. How many different structures can I produce by removing three edges?

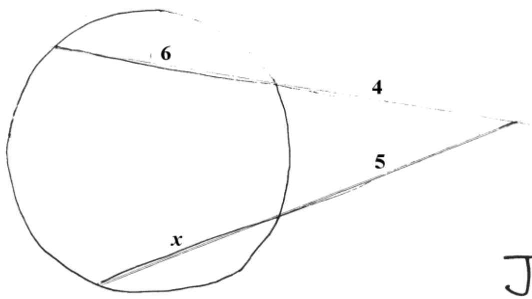
PROBLEM 55.4 PYRAMIDS

You know the problem of making four triangles with six matches, where you place the six matches to form the edges of a regular tetrahedron.

Now imagine that the mid-points of the matches are joined by lines. How many tetrahedra are there in the resulting structure?

PROBLEM 55.5 CHORDS

A group of us looked at this at the recent Weekend and were puzzled for a while even though some of us were nearly sober. The round part is a circle; find x.



Jerry.

EDITORIAL

Wherein lies originality? Jeremy Humphries talks about "Eddie's discovery concerning Kaprekar's Constant 6174". What I did was notice one of the factors in each of two examples from a list of Pandigital numbers I saw somewhere; I think in *Games and Puzzles*. I assumed at the time they were straight shifts from KC and had the article written before discovering my mistake, at which time it seemed simpler just to alter a number rather than scrap half a page wrought with anguish from my fevered brow. At the recent Weekend Nick Fraser sought me out and gave me the same answer as that reported from Bob Bertuello, except that I think he said he had checked *all* powers of n in $6174 \times a^n$. If I am wrong I am certain he will write and say so.

Conversation at that Weekend:
Weekender: Why are you wearing a yellow badge? Self: I'm editor of the magazine. Weekender: What magazine?

Two words about books, before I forget; both relating to items in this issue. First *Proof in mathematics* (page 11) As I said in my note, I received the book in June and read it in the next few days and thoroughly enjoyed it but between then and now I have got married and moved to London so it has become temporarily misplaced. Therefore I cannot print the names of the four authors. I will attempt to do the whole thing in the form of an advertisment soon. The other book is the one written by Herbert Dingle which I mentioned on page 13. It was called *Science at the Crossroads,*1972. (After that, in 1974 , he wrote *The Mind of Emily Brontë*.)

I suppose everybody has heard of the British chess master who wagered £1000 that no computer could beat him in ten years. Well in Toronto on September 5 the last match was played. In fact the man was David Levy, the British international chess master.

In 1968 he bet $1250 (£650) that a computer could not be programmed to beat him before the end of August 1978. He defeated the computer after 42 moves in the fifth game of a best-of-six match at the Canadian National Exhibition, winning the series by 3½ points to 1½. (My source is *The Times*, 6ix78.)

I met Robin Wilson at the Weekend where he gave his celebrated lecture on the four colour theorem. I asked him if he would like to write something for us on the subject but he couldn't, being off to America on a lecture tour shortly. But he very kindly let us have the text of a paper he is preparing; so that will go in just as soon as Jeremy Gray has finished off Gauss. Robin also promised to send me copies of the Maths Faculty Newssheet he edits so we can keep in touch with what goes on at WH. Here is a straight quote from No. 15 (June-September 1978).

The new Professor of Pure Mathematics is David Brannan. *For those of you who do not already know him here are some biographical details. He graduated from Glasgow University in* 1964 *and obtained his PhD in Complex Analysis at Imperial College London under the supervision of Prof J Clunie. Since then he has held teaching appointments in Maryland, Glasgow, and Queen Elizabeth College London, and has held summer teaching posts in Maryland, New York State, Illinois, and Ottawa. He has been a course tutor on* M201, M231 *and* M332 (*five years*)*, and has taught at* M201 *Summer Schools. He is an External Examiner for* M331. *He has been secretary of the London Mathematical Society since* 1971.

Eddie Kent