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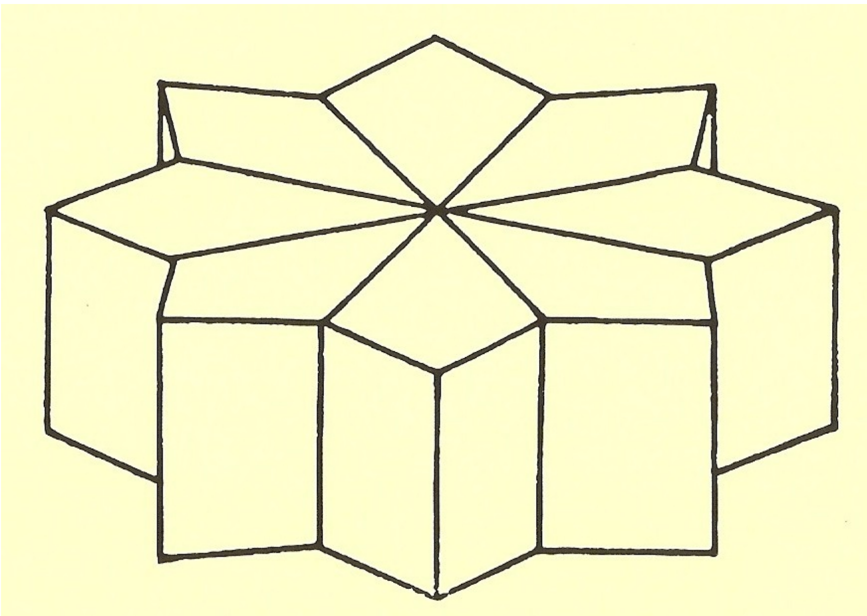

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Zero sum Pascal Triangle

Sebastian Hayes

Pascal's Triangle has now been going for almost a thousand years and arouses as much interest as ever. Omar Khayyam, the eleventh century poet whose verses in praise of wine, women and song were immortalised by Edward Fitzgerald in his free translation, wrote about it centuries before Pascal was born. The thirteenth century Chinese mathematician Yang Hui mentions it and states that it is discussed by an eleventh century Chinese, Chia Hsien, whose works are now lost. (See Joseph's fascinating book *The Crest of the Peacock, Non-European Roots of Mathematics.*)

It is well-known that if we take a row (or diagonal according to the presentation) and make the signs alternate, the sum is zero. This is obvious if we have a row with an even number of coefficients since the row is symmetrical about the mid-point so plus and minus terms cancel out; e.g.

$$1 \quad -3 \quad 3 \quad -1$$

It makes no difference whether we start with a positive or negative digit.

When we have an odd number of digits, a row still sums to zero; e.g.

$$1 \quad -4 \quad 6 \quad -4 \quad -1$$

Why? Because, according to the rule of formation, every row is the sum of the one above and the row above shifted one place along with sign changed. Thus we have

$$\begin{array}{ccccc} 1 & -3 & 3 & -1 & \\ & -1 & 3 & -3 & 1 \\ 1 & -4 & 6 & -4 & 1 \end{array}$$

But the columns don't indicate powers—it's just a matter of adding all the positive and negative digits together, and the sum of two rows that individually sum to zero is zero however the terms are added up.

We can see this by noting that entries in Pascal's Triangle can be viewed as the coefficients of the expansion of $(1 + x)^n$. Setting $x = -1$, the result follows.

This is nothing new. But what I have only just found out is that if you multiply any row of Pascal's Triangle with alternating signs by the first n natural numbers you still get zero. Try it and see. For example,

$$\begin{array}{rrrrrl} 1 & -4 & 6 & -4 & 1 & \text{multiplied by} \\ 1 & 2 & 3 & 4 & 5 & \text{term by term gives zero.} \end{array}$$

But the natural numbers appear as column 1 in Pascal's Triangle (if we make the first column the zeroth). So I wondered what would happen if I used the first triangular numbers, column 2, namely 1, 3, 6, 10, ... to multiply 1 -4 6 -4 1 or any subsequent row. The result—zero.

What about the next column—1, 4, 10, 20, ...? The result—zero. And so it goes on provided you make sure the column number $r = 0, 1, 2, 3, \ldots$ is less than the row number. Thus 1 3 6 10 (column 2) multiplying 1 -3 3 -1 gives zero, but not multiplying 1 -2 1.

What about shifting the column coefficients along one space and starting with 0? That is, multiplying by 0 1 3 6 10 .... What happens then? You've guessed it—zero again. Try it with any column and any (sign-alternating) row remembering the proviso.

In fact for column r you get exactly $r + 1$ zero sums if you slide the digits across. For example, since 1, 4, 10, 20, ... is the third column of Pascal's Triangle, you start off with four zeros if you slide it across any row with $n > 3$.

The r variable is being overworked since it is being used in ${}^nC_r$ and then as column number. Keeping r for the terms in the row, I will henceforth use k for the multiplying column where there is risk of confusion.

THEOREM. *If a column k from Pascal's Triangle is multiplied term by term by a row from Pascal's Triangle with alternating sign, $\pm{}^nC_r (n > k)$, and then slid across, the result is $k + 1$ zero sums and then the coefficients of ${}^{n-k}C_r$.*

An example will make this clear.

$$
\begin{array}{rrrrrcrl}
-1 & 4 & -6 & 4 & -1 & = & {}^4C_r & \qquad r = 0, 1, 2, 3, 4 \\
1 & 2 & 3 & 4 & 5 & = & 0 & \\
 & 1 & 2 & 3 & 4 & = & 0 & \\
 & & 1 & 2 & 3 & = & -1 & \\
 & & & 1 & 2 & = & 2 & \\
 & & & & 1 & = & -1 & \\
\cline{6-7}
 & & & & & = & 0 & \\
1 & 3 & 6 & 10 & 15 & = & 0 & \qquad k = 2
\end{array}
$$

Every row is being multiplied by the top row term for term.

Why does this happen? Proof is by induction and depends on the way in which Pascal's Arithmetic Triangle is built up. In my original article I presented the 'triangle' in square form, but since the triangular version is better known I revert to this. We have in effect a so-called *lower triangular matrix*, i.e. all entries above the leading diagonal are zero and entries ${}^nX_r$

with n, $r = 0, 1, 2, 3, \ldots$ give the coefficients ${}^nC_r$ in the expansion of $(1+x)^n$. I always find it confusing, incidentally, that the r in ${}^nC_r$ indicates the column and not the row.

column	0	1	2	3	4	5	...
row							
0	1	0	0	0	0	0	
1	1	1	0	0	0	0	
2	1	2	1	0	0	0	
3	1	3	3	1	0	0	
4	1	4	6	4	1	0	
5	1	5	10	10	5	1	

In any column the first non-zero entry is on the main diagonal with $n = r$. Normally one obtains entry ${}^nC_r$ by using

$$ {}^{n-1}C_{r-1} + {}^{n-1}C_r \;=\; {}^nC_r, \tag{i}$$

but in this case we are also interested in the relation

$$ \sum_{m=0}^{n} {}^mC_r \;=\; {}^{n+1}C_{r+1}; \tag{ii}$$

i.e. summing down a column and jumping a space across diagonally.

We start with $k = 0$, i.e. the zeroth column: $[1\ 1\ 1\ 1\ \ldots\ 1]$. Multiplying this by

$$ -{}^nC_0 + {}^nC_1 - {}^nC_2 + {}^nC_3 - \cdots + (-1)^{n-1}\,{}^nC_n \;=\; {}^nC_r,$$

the result, as we already know, is zero i.e. $0 + 1$ zeros.

Shifting across we get $\pm{}^nC_r$ minus the first entry, then $\pm{}^nC_r$ minus the first two entries and so on. Since the full line sums to zero, we obtain $-(-{}^nC_0) = {}^nC_0$, then ${}^nC_0 - {}^nC_1$. However, ${}^nC_0 = {}^{n-1}C_0$ and ${}^nC_0 - {}^nC_1 = -{}^{n-1}C_1$. Next time we will have the full line minus the first three entries and, using what we have already obtained, we get $-{}^{n-1}C_1 + -(-{}^nC_2)$ or, ${}^nC_2 - {}^{n-1}C_1 = {}^{n-1}C_2$ using (i).

Proceeding in this way we find that for column $k = 0$ the coefficients of ${}^{n-1}C_r$ appear as desired. This can be proved by induction.

Moreover, if we sum the vertical columns (as in the example given above) we get the next column since this is how Pascal's Triangle is built up. And since $\pm{}^nC_r$ sums to zero for any n we start off the next multiplication, column against $\pm{}^nC_r$, with a zero sum.

Now suppose that for all column values up to $k-1$ this relation has been true. We have thus reached column k which when multiplied by

$$-{}^nC_0 + {}^nC_1 - {}^nC_2 + {}^nC_3 - \cdots \pm {}^nC_n$$

gives zero because this is the sum of the previous triangle and the coefficients ${}^{n-(k-1)}C_r$ with alternating signs.

To take a specific example, if $k = 3$ we have $[1\ 4\ 10\ 20\ 35\ \ldots]$; multiply by $\pm{}^nC_r$ and this sums to zero. If we take away the previous column, i.e. l, 3, 6, 10, ..., we get 0, 1, 4, 10, 20 and if we repeatedly subtract the previous column, this, by the rule of formation of Pascal's Triangle, is equivalent to sliding l, 4,10, 20, ... across one space at a time. Now, by hypothesis, l, 3, 6, 10, ... multiplying

$$-{}^nC_0 + {}^nC_1 - {}^nC_2 + {}^nC_3 - \cdots + (-1)^{n-1}\,{}^nC_n$$

has given zero.

Zero $-$ zero $=$ zero, so the sum of the second line of $k = 3$ is zero. For the third row we subtract 0, 1, 3, 6, 10, ... from the second giving 0, 0, 1, 4, 10, ... But, since l, 3, 6, l0, ... is column $k-1$ and the relation is true by hypothesis up to this point, the product of 0, l, 3, 6, 10, ... with $\pm{}^nC_r$ is again zero.

This explains the appearance of $k+1$ zeros when we slide across column k—provided the relation is true for columns 0, l, ..., $k-1$. If we continue subtracting the previous column we eventually get non-zero sums but they will be those obtained by the previous column when multiplying by $\pm{}^nC_r$ or rather they will be those sums with sign reversed since we are subtracting all the time.

The above can easily be made into a rigorous induction argument.

How did I arrive at this (to me at any rate) surprising and satisfying result? In an extremely devious way. Mathematicians like to pretend that their results arrive in a logical manner but in fact they rarely do—they often come about by just playing around and stumbling across a pattern by chance.

Barry Lewis, in an unpublished article he showed me, came up with the delightful set of zero sums—which I have never seen before—formed by multiplying

$$-{}^nC_0 + {}^nC_1 - {}^nC_2 + {}^nC_3 - \cdots + (-1)^{n-1}\,{}^nC_n$$

by successive powers of the natural numbers, i.e. by

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & \ldots \\ 1 & 2^2 & 3^2 & 4^2 & 5^2 & \\ 1 & 2^n & 3^n & 4^n & 5^n & \end{array}$$

He proved this by using calculus methods but I decided I wanted to find an elementary proof. I found it tough going even proving power one, i.e. the the natural numbers, let alone all possible powers! I did, however, hit upon the method of setting out the multiplication in the form of a triangle, e.g.

$$\begin{array}{ccccc} 1 & 1 & 1 & \ldots & 1 \\ & 1 & 1 & \ldots & 1 \\ & & 1 & \ldots & 1 \\ & & & \ldots & \\ & & & \ldots & 1 \end{array}$$

and this proved useful at a later stage.

Getting nowhere to start with, I decided to investigate the general case of a set of functions with coefficients which sum to zero, i.e.

$$A_0 f_0(n) + A_1 f_1(n) + \cdots + A_n f_n(n)$$

or polynomials where we only allow integral values for x and, for the moment, integral coefficients as well. This proved a bit too general but led me to consider symmetric and anti-symmetric functions which sum to zero— $\pm^n C_r$ with n even is a symmetric function since $f_r(n) = f_{n-r}(n)$ while $\pm^n C_r$ with n odd gives an anti-symmetric function since $f_r(n) = -f_{n-r}(n)$. Examples are -1, 2, -1 and -1, 3, -3, 1.

Now, if we set the coefficients at l, 2, 3, ..., we can lay the whole thing out as a full first line and then a triangle.

$$\begin{array}{lllll} f(n) & f_1(n) & \ldots & f_n(n) & \text{first line} \\ f_1(n) & f_2(n) & \ldots & f_n(n) & \text{triangle} \\ & f_2(n) & \ldots & f_n(n) & \\ & & \ldots & & \\ & & & f_n(n) & \end{array}$$

Each line of the triangle can be replaced by (sum first line $-f_0(n)$), (sum first line $-(f_0(n) + f_1(n))$) and so on.

So, apart from the recurring 'sum first line' we obtain a second triangle which replaces the first, call it triangle^c. And if the set is symmetric we obtain the original triangle upside down with sign reversed, or triangle^c = $-$ triangle.

$$\begin{aligned}\text{total} &= \text{sum first line} + \text{sum triangle}\\ &= (n+1)(\text{sum first line}) + \text{triangle}^c,\\ 2\ \text{total} &= (n+2)(\text{sum first line}) + (\text{triangle} + \text{triangle}^c).\end{aligned}$$

The above is true whether the set is symmetric or not and whatever the first line sums to. However if the first line sums to zero and the set is *symmetric* we obtain 2 total = 0.

Thus, rather laboriously, I managed to establish the case for l, 2, 3, ... and $\pm{}^nC_r$ with n even. The case n odd can be derived from this.

This seemed a paltry return for so much effort. However, realising that l, 1, l, 1, ... and 1, 3, 6, 10, ... were the first columns of the Pascal matrix, I wondered what would happen if I tried the next column and the next. Hey presto! Zero all the way down.

All this shows (1) what can be obtained by looking for an alternative proof when there's already a perfectly good one, and (2) how not getting the result you want can lead to something equally interesting.

In fact establishing the case for any column of the Pascal matrix proves Barry Lewis's set of zero sums as a special case. For the columns are l, n, $n(n+1)/2$ with general formula $n(n+1)\dots(n+r-1)/r!$.

Each column is thus a polynomial one degree higher than the previous column. The relationship between powers and formulae for columns is complicated and is in part the subject of Barry Lewis's article 'Hip, hip, array!' (M500 **162**) which started this particular ball rolling. But it suffices that this relation exists. Also, if the result is zero when we multiply a Pascal column by $\pm{}^nC_r$, the result will still be zero if the whole thing is multiplied by any integer (or real), and we can add and subtract as many of these multiples as we like. Thus, the powers are covered and in fact absolutely anything that can be worked up as sums and differences of any multiples of any column from Pascal's Triangle—a large group of functions.

This whole question resembles a crossroads with fascinating vistas in all directions. The real pay off would be passing to fractional and negative powers—an avenue which takes us towards the Riemann zeta function. As a humble start, I wondered what would happen if I used the Harmonic Series as multiplying set and stumbled across the pleasing relation

$$\sum \pm \frac{{}^nC_r}{r+1} = \frac{1}{n+1},$$

i.e.

$${}^nC_0 - \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 - \frac{1}{4}\,{}^nC_3 + \cdots + \frac{1}{n+1}(-1)^n\,{}^nC_n \;=\; \frac{1}{n+1}.$$

This doesn't mean that if you excise the last term (as it were) you always get zero. In fact, laying it out as a matrix we obtain

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 1 & 0 & 0 & 0 & 0 & 0 & \dots \\ 1 & 2 & 0 & 0 & 0 & 0 & \dots \\ 1 & 3 & 3 & 0 & 0 & 0 & \dots \\ 1 & 4 & 6 & 4 & 0 & 0 & \dots \\ 1 & 5 & 10 & 10 & 5 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} 1 \\ -1/2 \\ 1/3 \\ -1/4 \\ 1/5 \\ -1/6 \\ \dots \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1/2 \\ 0 \\ 1/3 \\ \dots \end{bmatrix}.$$

Returning to $\pm^n C_r/(r+1)$, if we don't actually take the sum but build up a sort of eccentric Pascal Triangle, we get

$$\begin{array}{llllllll} 1 \\ 1 & 1/2 \\ 1 & 1 & 1/3 \\ 1 & 3/2 & 1 & 1/4 \\ 1 & 2 & 2 & 1 & 1/5 \\ 1 & 5/2 & 10/3 & 5/2 & 1 & 1/6 \\ 1 & 3 & 5 & 5 & 3 & 1 & 1/7 \\ 1 & 7/12 & 7 & 35/4 & 7 & 7/2 & 1 & 1/8 \end{array}$$

I wondered what was the condition to obtain a line of integers (except for the last entry). This is left as a (simple) problem for the reader.

Problem 169.1 – Three people

EK

There are three people. One tells lies. One tells truths. One alternates. They get paid for each word. So find the cheapest and most elegant set of questions to identify them all correctly.

Explorer Leif Eirikson returned from his voyage to the New World to find that his name had been removed from the town register. He complained at the town meeting, viewing it as a slight. The town official immediately apologized, saying he must have taken Leif off his census. —**JRH**

Fermat's Last Theorem

A simple proof based on irrational numbers.

Peter L. Griffiths.

Rational and Irrational Non-Integers

There are two kinds of non-integers:

1. There are those non-integers which give integers when multiplied by an integer. We can call these rational non-integers.

2. There are those non-integers with an infinite number of non-recurring decimal places which cannot give integers when multiplied by an integer but always give results which have an infinite number of decimal places. We can call these irrational non-integers.

Basically rational numbers can be expressed as ratios, whereas irrational numbers cannot.

Examples of irrational numbers include π, e and the integer (> 1) roots of integers adjacent to integers raised to the power of integer (> 1) n, viz. $(t^n \pm 1)^{1/n}$, where t is an integer > 1.

The integer (> 1) root of any prime number is always irrational. In particular, 2 is a prime number. Hence $2^{1/n}$ must be irrational. It can be seen that $(1^n + 1^n)^{1/n}$ is not only irrational but it is also a special case of Fermat's Last Theorem $(a^n + b^n)^{1/n}$ where $a = b$.

Fermat's Last Theorem is usually stated in the following form: *Given that a, b, c, and n are integers with $n > 2$, $(a^n + b^n)$ never equals c^n.* In fact there is no need for a, b, and c, to be integers. They must however be rational numbers. Also n must be an integer otherwise as a power, n could convert a rational number into an irrational one. If Fermat's Last Theorem is correct then $(a^n + b^n)^{1/n}$ can be neither an integer nor a rational non-integer, but must be an irrational non-integer. A special case of $(a^n+b^n)^{1/n}$ is $(1^n + 1^n)^{1/n}$.

We need some results which are straightforward to prove:

If t and n are integers, $(t^n - 1)^{1/n}$ is a non-integer $< t$ but $> t - 1$.

If t and n are integers,$(t^n + 1)^{1/n}$is a non-integer $> t$ but $< t + 1$.

$(t^n - 1)^{1/n}$ is always irrational when t and n are integers > 1.

$(t^n + 1)^{1/n}$ is always irrational when t and n are integers > 1.

Since we have that both $(t^n - 1)^{1/n}$ and $(t^n + 1)^{1/n}$ are irrational, this means that $(t^n - 1)^{1/n}$ cannot equal d/f where d and f are integers. Also $(t^n+1)^{1/n}$ cannot equal j/k where j and k are integers. It therefore follows that $f^n t^n - f^n$ cannot equal d^n. Also $k^n t^n - k^n$ cannot equal j^n.

Fermat's Last Theorem therefore applies when the two smallest terms viz f^n and k^n are factors of at least one of the larger terms, viz. $f^n t^n$ and $k^n t^n$. The fact that t is an integer, and that, after dividing through by f^n and k^n, either a or b is unity means that the Diophantine ratios are not inserted so that these inequalities are valid even when $n = 2$.

To prove that $(a^n + b^n)^{1/n}$ is always irrational if $n \geq a > b > 1$ (all being integers), we proceed as follows.

The nth root of each of the prime factors of $a^n + b^n$ will be irrational. The only rational numbers will be the *integers* resulting from the same irrational number being multiplied by itself n times. Therefore $(a^n + b^n)^{1/n}$ will either be an integer or it will be irrational, it will not be a rational non-integer. Any integer to the power of n (an integer > 1) must be another integer. Any ratio to the power of n (an integer > 1) must be another ratio. Because the denominator for the integer ratio is always 1, the nth root of any integer must be either another integer or it may be irrational; it cannot be a ratio. The nth root of any ratio must be either another ratio or it may be irrational; it cannot be an integer.

Given $c^n = a^n + b^n$ where c is also an integer, we have $c = (a^n + b^n)^{1/n}$; hence $c < (2a^n)^{1/n}$, since $a > b$. Hence $c < 2^{1/n} a$.

For 2 substitute $e = 2.7182818$. *A fortiori*, $c < e^{1/n} a$; hence $c < (1 + 1/n)^a$ and c is also given as $> a$. Furthermore, c could encounter an integer $< a + a/n$ but $> a$, if $a > n$, and could encounter other integers if a further exceeded n. Hence for an irrational result, $a \leq n$. This is not a method which Fermat himself could have known since e tending to $(1 + 1/n)^n$ was not discovered until Abraham de Moivre's *Miscellanea Analytica* published in 1730.

Conversely for a rational result, $a > n$. But apart from the special conditions of the Diophantine algebraic identity, where a may or may not be greater than b, the chances of achieving a rational result when $a > n$ are infinitesimal for the following reasons. Assuming an infinite number of irrational amounts between integers, we can deduce that:

1. There is nil chance of encountering an integer from just above one integer to just below the next integer higher

2. There is 1 chance in $2 \times \infty$ of encountering an integer from just above one integer to just below the second integer higher.

3. There are 2 chances in $3 \times \infty$ of encountering an integer from just above one integer to just below the third integer higher. Etc.

The Diophantine Algebraic Identity when $n = 2$.

Mathematicians' attempts to solve Fermat's Last Theorem have invariably underestimated the importance of arriving at complete conclusions for the power of 2, largely because the power of 2 seems to be outside the scope of the problem set by Fermat. If the power n is 2 then it is possible to construct the Diophantine algebraic identity $(r^2 \mp q^2)^2 \pm (2rq)^2 = (r^2 \pm q^2)^2$, where r and q can be any integers.

Although this algebraic identity appears to be a special case of $a^n + b^n = c^n$, the Diophantine algebraic identity differs in that whereas we can consistently assume that $b < a$ in $a^n + b^n = c^n$, we cannot make such an assumption in $(r^2 \mp q^2)^2 \pm (2rq)^2 = (r^2 \pm q^2)^2$; $r^2 - q^2$ may or may not be higher than $2rq$, depending on the values given to r and q. The Diophantine Algebraic Identity consistently gives rational results when r and q are integers.

Where for the power of 2 the Diophantine Algebraic Identity does not occur, the assumption of $b < a$ applying to the higher powers can continue for the power of 2, as also can the infinitesimal chances of a rational result if $n < a$.

The general algebraic identity $(d/f)^n + (f/d)^n \pm 2 = ((d/f)^{n/2} \pm (f/d)^{n/2})^2$ can be expressed as $((d/f)^n) + (f/d)^n \pm 2)^{1/2} = (d/f)^{n/2} \pm (f/d)^{n/2}$. Let $n = 2$, then we have the specific algebraic identity $((d/f)^2 + (f/d)^2 \pm 2)^{1/2} = d/f \pm f/d$.

Square both sides, $(d/f)^2 + (f/d)^2 \pm 2 = (d/f \pm f/d)^2$. Complete the square, $(d/f)^2 \mp 2 + (f/d)^2 \pm 4 = (d/f \pm f/d)^2$, $(d/f \mp f/d)^2 \pm 4 = (d/f \pm f/d)^2$. Multiply both sides by $(df)^2$, $(d^2 \mp f^2)^2 \pm (2df)^2 = (d^2 \pm f^2)^2$, which is the Diophantine Algebraic Identity.

The Diophantine Algebraic Identity is therefore the specific case of the general algebraic identity, $((d/f)^n + (f/d)^n \pm 2)^{1/2} = (d/f)^{n/2} \pm (f/d)^{n/2}$, with $n = 2$, and is not the specific case of $a^n + b^n = c^n$ where $n = 2$, because $(d^2 \mp f^2)^n \pm (2df)^n$ cannot equal $(d^2 \pm f^2)^n$, where $n > 2$.

Continuous rational square roots can be obtained from the following algebraic identities (possibly similar to Fermat's method of infinite descent apparently inspired by Diophantus' *Arithmetica*, Book III, proposition XIX).

$$\begin{aligned}
((d/f)^n + (f/d)^n \pm (2^{1/n})^n)^{1/2} &= (d/f)^{n/2} \pm (f/d)^{n/2}, \\
((d/f)^{n/2} + (f/d)^{n/2} \pm (2^{2/n})^{n/2})^{1/2} &= (d/f)^{n/4} \pm (f/d)^{n/4}, \\
((d/f)^{n/4} + (f/d)^{n/4} \pm (2^{4/n})^{n/4})^{1/2} &= (d/f)^{n/8} \pm (f/d)^{n/8}, \quad \text{etc.}
\end{aligned}$$

At each stage of the continuous square rooting, the addition or subtraction of 2 is required. The source of Fermat's expression infinite descent appears to be the Greek word ἀπειραχῶς meaning 'in an infinity number of ways' which is mentioned in Diophantus' *Arithmetica*, Book III, proposition XIX.

Multiple powers

An equation of a multiple power mn being $a^{mn}+b^{mn}=c^{mn}$ can be analysed as $(a^m)^n + (b^m)^n = (c^m)^n$. Each term is the same but expressed in a different way. It will be noted that the root can change from $1/mn$ to $1/n$; $(a^{mn}+b^{mn})^{1/mn} = c$, $((a^m)^n + (b^m)^n)^{1/n} = c^m$.

If it can be assumed that the individual prime factor of the root has already produced an irrational result, then the multiple root will also produce an irrational result. This is still the basis of much of the research into Fermat's Last Theorem. The argument is that because $((a^m)^n + (b^m)^n)^{1/n}$ is irrational then so must $(a^{mn}+b^{mn})^{1/mn}$ be irrational.

For a contrary approach, see L. J. Mordell on page 4 of his *Three Lectures on Fermat's Last Theorem*. It should however be noted that Mordell has no comprehension of the beneficial use of irrationals. It should be noted that imaginary numbers have no application in the above proof of Fermat's Last Theorem. Gross misuse of $\sqrt{-1} = i$ has been one of the peculiar features of recent attempts to prove Fermat's Last Theorem.

Cotes's Formula but usually known as de Moivre's Formula being $e^{2\pi i} = \cos 2\pi + i \sin 2\pi$ appears to equal $1 + 0$, so that a further prime root p, giving $e^{2\pi i/p}$ seems to give the pth root of 1, whereas the p should be regarded as dividing 2π not $i = \sqrt{-1}$. In this way neither $\cos 2\pi/p$ nor $\sin 2\pi/p$ will be 1 nor 0.

Complex numbers are a mixture of imaginary and rational numbers, and as factors may help to indicate the real factors of an equation, but in other respects should play little or no part in rigorous proof. The proof of Fermat's Last Theorem should not need imaginary and complex numbers.

It should automatically follow from the context that the definition of irrational number in this paper excludes imaginary and complex numbers.

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P. ver Eecke, *Diophante d'Alexandrie*, Albert Blanchard, 1959, pages 108–109.

Countdown

Ledger White

A couple of minor points on the rules (Ralph Hancock M500 **165**). The smaller numbers are from one to ten, not one to nine; the target is 101 to 999, not 100 to 999 and all calculations must produce a positive integer.

Ralph is right when he says that subtlety is likely to take longer than a battering-ram approach on a such a small set of numbers. The various pieces of code I have demonstrate this. I found that enumerating all the possible combinations of calculations is not as easy as it looks. My first try was to say that there are 6! ways of ordering six numbers, with $4 \times 4 \times 4 \times 4 \times 4$ ways of writing the operations between them. But then what? How to count all the permutations of brackets?

So I tried again and this alternative method of evaluation suggests a recursive algorithm for programming. Starting with six numbers, there are 30 ways of selecting two operands and there are thus 120 calculations (30 each of addition, multiplication, subtraction and division) you can make. This leaves you with five numbers (four left over and the new result) from which there are 20 ways of selecting two operands and thus there are 80 calculations you can make for each of the 120 with six numbers. Continuing like this you get $120 \times 80 \times 48 \times 24 \times 8$ ways of making a calculation involving all six numbers. Then there are $120 \times 80 \times 48 \times 24$ ways using just five numbers, and so on. Brackets are not needed.

This adds up to 100,003,320 possible calculations. This is reduced considerably by not continuing with any calculation where the first operand is less than the second—to as little as 3,516,060 if all six numbers are different. Moreover, stopping the recursion when the second operand is less than 2 and the operation is division and stopping when a result is equal to an operand (for example, $8 - 4 = 4$; $8 \times 1 = 8$) both make a big difference. There are other techniques such as ensuring that in the set 1, 4, 4, 7, 8, 50, for example, you don't calculate (n op 4) or (4 op n) more than once—but these begin to cost as much as they save. (I am assuming that my reasoning and my arithmetic are correct!)

There was a piece of code published for an Acorn Archimedes (son of BBC microcomputer) about five years ago written in RISC assembler. Disassembling this code was easy enough, but determining the underlying algorithm was something else. But, on reading Ralph's article I had a go and it is very like my reasoning above. I have since produced routines in interpreted BBC BASIC V and RISC OS Assembler for Acorn's StrongArm series (grandson of the BBC microcomputer). I have also implemented it in Microsoft's C++ for the rest of the world! The sternest type of test is to use the set 9, 9, 9, 9, 9, 9 and 999 as data (not allowed in Countdown but it ensures lots of calculations). This takes about 300 seconds in the interpreted BASIC, about 70 seconds in C++ with a Pentium Pro PC and only about five seconds on the Acorn Assembler.

Typical sets of numbers without a solution are less than half of these times. Using random selections, most of the time there is a solution and it appears very quickly. So, you might take on the interpreted BASIC, even score against the C++ occasionally, but you haven't a cat in hell's chance against the Acorn Assembler which solves it more often than not in less than a tenth of a second!

Incidentally, the Countdown finalist in December 1997 was an M500 subscriber. Christine White lost by just a few points through offering a spelling acceptable to *Chambers* but not, alas, to the *OED*. Her number game solving was outstanding—finding obscure solutions that eclipsed even Carol Vorderman! She says that with the lights and the cameras the 'best' solution is the first one you get before the time is up.

Re: Sums of odd numbers

Sebastian Hayes

There are unfortunately a couple of errors in my article 'Sums of odd integers', M500 **166** 22–23. I say that 'N_G, the number of ways a positive odd integer p^m can be expressed as a sum of consecutive (positive) odd numbers is $(m+1)/2$ for m odd and $m/2$ for m even.' This should read '$(m+2)/2$ for m even.' For example, $3^4 = 81 = 25 + 27 + 29 = 1 + 3 + 5 + \ldots + 17$. (The number itself counts as a 'sum'.)

If $p = 2$, we lose a pair of factors and the number of ways for 2^m reduces to $(m-1)/2$ for m odd and $m/2$ for m even.

For 'If $N = 2aq^br^c$ is even …', a little further down, read 'If we have $N = 2^aq^br^c$ ….'

The total number of ways for N is given by the product of the number of ways for each prime power—thus the double of an odd number, $2^1q^br^c\ldots$, cannot be expressed as a sum of consecutive odd numbers.

Schools dominate exam result list

Banner headline in *Bromsgrove Advertiser*. Spotted by **JRH**.

Exploring Chaos

Barry Lewis

You are given a blank piece of paper, on which is drawn an equilateral triangle. You choose a starting point – anywhere, on or off the page – and then one of the vertices of the triangle is randomly chosen, and the mid-point of the starting point and the vertex is marked. The mid-point now becomes the new starting point and the procedure is repeated again, and again ... Such a process is called iteration. What emerges is a random pattern, made up of the marks left by successive iterates. Here's a program that generates it.

```
10   SCREEN 12
20   WINDOW (0,0)-(2,2)
30   INPUT " Choose the starting value for x ",x0
40   INPUT " Choose the starting value for y ",y0
50   CLS
60   PSET (x0,y0)
70   v%=INT(3*RND)
80   IF v%=0 THEN GOTO 150
90   IF v%=1 THEN GOTO 160
100  IF v%=2 THEN GOTO 170
150  xv=0:yv=0
151  GOTO 200
160  xv=2:yv=0
161  GOTO 200
170  xv =1:yv = SQR(3)
171  GOTO 200
200  x1=(x0+xv)/2:y1=(y0+yv)/2
210  PSET (x1,y1)
220  x0=x1:y0=y1
230  GOTO 70
```

For those new to QBASIC.

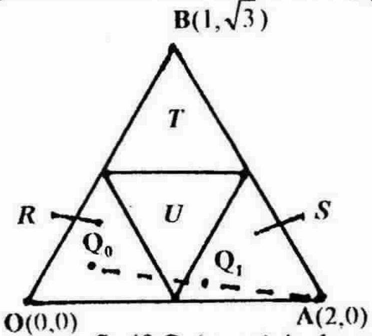
- QBASIC is a programming language that comes as part of MSDOS and WINDOWS (all current versions) – *not* all manufacturers install it on new systems so if you can't find it, search on your system disk(s). You should find two files QBASIC.EXE (double click on this to run it) and QBASIC.HLP.
- Just type in the program as shown, click on the RUN menu and select START to run a program.
- To exit from a running program hold down simultaneously the keys labelled Ctrl and Pause/Break.
- To learn more about QBASIC use the help menu.

Were you surprised at the outcome? Now experiment with different starting points – inside, outside and on the boundary of the triangle. Let's add some colour. Change/add the following lines.

```
51  C=7
60  PSET (x0,y0),C
209 C=POINT(x1,y1)
210 PSET (x1,y1),C+1
```

Many, many questions suggest themselves, but before exploring them, let's look at the mathematics to see why this outcome is inevitable.

Consider an equilateral triangle **OAB** subdivided into four equal regions, ***R, S, T*** and ***U*** as shown. Suppose that $\mathbf{Q}_0$ is a starting point *inside* ***R*** with coordinates (x_0, y_0). Then we have



$$\left.\begin{array}{ll} y_0 & < \sqrt{3}x_0 \\ y_0 + \sqrt{3}x_0 & < \sqrt{3} \\ y_0 & > 0 \end{array}\right\} \quad \ldots (1).$$

These equations position $\mathbf{Q}_0$ *within* the three boundaries that define the region ***R***.

So if $\mathbf{Q}_1(x_1, y_1)$ is the next iterate when the vertex **A** (2,0) is randomly chosen, then it is the mid-point of the line joining $\mathbf{Q}_0$ and **A.** So we have

$$x_1 = \frac{(x_0+2)}{2} \quad \Rightarrow \quad x_0 = 2x_1 - 2 = 2(x_1 - 1)$$
$$y_1 = \frac{(y_0+0)}{2} \quad \Rightarrow \quad y_0 = 2y_1.$$

Substituting these new values into equations (1) we obtain

$$\begin{array}{lll} 2y_1 < 2\sqrt{3}(x_1-1) & \Rightarrow & y_1 < \sqrt{3}x_1 - \sqrt{3} \\ 2y_1 + 2\sqrt{3}(x_1-1) < \sqrt{3} & \Rightarrow & y_1 < -\sqrt{3}x_1 + \frac{3}{2}\sqrt{3} \\ 2y_1 > 0 & \Rightarrow & y_1 > 0 \end{array}$$

But this is the boundary of a region inside ***S***, and so $\mathbf{Q}_1$ lies within ***S***. By symmetry, if the randomly chosen vertex had been **B** then $\mathbf{Q}_1$ would similarly lie inside the region ***T***.

What happens if **O** is the randomly chosen vertex? It is easy to prove that $\mathbf{Q}_1$ then lies in ***R*** alongside $\mathbf{Q}_0$. If $\mathbf{Q}_0$ is in the region ***U*** then it is easy to see (and prove) that $\mathbf{Q}_1$ is in the region corresponding to the randomly chosen vertex – ***R*** for **O**, etc. So the region *U*, apart from some initial iterates, can never be revisited! It is a geometric black hole - literally.

Now each of the regions ***R***, ***S***, ***T*** and ***U*** are themselves equilateral triangles and behave in the same way as their parent equilateral triangle – and so the fractal nature of the emergent pattern.

Exactly the same pattern emerges if $\mathbf{Q}_0$ is initially outside the triangle. Suppose that $\mathbf{Q}_0$ is outside the triangle and closest to the side **AB** so that it is furthest from the vertex **O**. If the chosen random vertex is either **B** or **A**, then $\mathbf{Q}_1$ is still outside the triangle, but now closer to it. If, however, the chosen random vertex is **O**, then $\mathbf{Q}_1$ has x and y coordinates one half of the corresponding coordinates of $\mathbf{Q}_0$. Ultimately, therefore, $\mathbf{Q}_1$ (or rather, its successor after many iterates) will end up inside the triangle.

Moreover, this is not just a property of the equilateral triangle. Any given triangle is the image of an equilateral triangle, under an *affine* transformation. Such a transformation preserves properties such as *mid-point* and *inside* so that starting with any triangle results in a similar pattern, realised for that particular triangle.

And now to explore further. Here are some suggestions and tips.

- You can enlarge the viewing `WINDOW` by changing its coordinates. **Type all lines (ie that following a line number) as a single line.**

- To investigate Squares, Pentagons, ... You need to randomly select one of the 4, 5, ... vertices and to have available their coordinates for the mid-point calculation. These additional lines are for a pentagon:

```
5   a=3.1416/180
20  WINDOW (-1,0)-(1.5,2)
70  v%=INT(5*RND)
110 IF v%=3 THEN GOTO 180
120 IF v%=4 THEN GOTO 190
```

```
160 xv=1:yv=0
161 GOTO 200
170 xv=1 + COS(72*a):yv=SIN(72*a)
171 GOTO 200
180 xv =.5:yv=SIN(72*a)+ SIN(36*a)
181 GOTO 200
190 xv=-COS(72*a):yv=SIN(72*a)
191 GOTO 200
```

Try to predict what will happen for a square and test it out.

- But the most interesting experiment is to adapt the mid-point algorithm. What about taking points closer to the starting point $\mathbf{Q}_0$ than the randomly chosen vertex? This produces fascinating results. In what follows, I have incorporated another choice – the weighting between the two iteration points. I have scaled it by a factor of 1000 so that an entry of 500 reproduces the mid-point procedure. Other choices produce ... well, that's for you to discover; suffice to say there are some real surprises in store! Remember to change the `WINDOW` coordinates if necessary.

```
45 INPUT " Choose the weighting factor ",r
200 x1=(r*x0+(1000-r)*xv)/1000:
y1=(r*y0+(1000-r)*yv)/1000
```

Try this for the pentagon with a ratio of 350. Now experiment.

- We might adapt the algorithm further:

```
200 x1=r*(x0+xv)/1000:y1=r*(y0+yv)/1000
```

Try this for the pentagon with a ratio of 350, 850. What has happened ? Expand the viewing area and draw in the pentagon itself.

```
20 WINDOW (-1,0)-(8,8)
61 LINE(0,0)-(1,0)
62 LINE(1,0)-(1 + COS(72*a),SIN(72*a))
63 LINE (1 + COS(72*a),SIN(72*a))-
   (.5,SIN(72*a) + SIN(36*a))
64 LINE(.5,SIN(72*a) + SIN(36*a))-
   (-COS(72*a),SIN(72*a))
65 LINE (-COS(72*a),SIN(72*a))-(0,0)
```

The possibilities seem endless.

Cannon balls, etc.

Chris Pile

Since $1^2 + 2^2 + \ldots + 24^2 = 70^2$, the square pyramid of base side 24 can be knocked down to form the square of side 70. This can be placed under the square pyramid of base side 69 to make the square pyramid of base side 70. That is

$$\mathrm{SP}_{24} + \mathrm{SP}_{69} = \mathrm{SP}_{70},$$

where SP_n indicates the square pyramid of base side n. This is not the smallest solution for square pyramids, because we also have

$$\mathrm{SP}_{42} + \mathrm{SP}_{45} = \mathrm{SP}_{55}.$$

You ask in M500 **166** if we can construct a tetrahedral pyramid from two others of the same size. I believe the only solution is

$$\mathrm{TP}_3 + \mathrm{TP}_3 = \mathrm{TP}_4.$$

The number of balls is $10+10 = 20$. For two different sized tetrahedral pyramids, the smallest number of balls would be $120+560 = 680$, corresponding to

$$\mathrm{TP}_8 + \mathrm{TP}_{14} = \mathrm{TP}_{15}.$$

Other possible combinations are $\mathrm{TP}_{20}+\mathrm{TP}_{54} = \mathrm{TP}_{55}$, $\mathrm{TP}_{30}+\mathrm{TP}_{55} = \mathrm{TP}_{58}$ and $\mathrm{TP}_{39} + \mathrm{TP}_{70} = \mathrm{TP}_{74}$. The number of balls in a tetrahedral pyramid can be found from the fourth diagonal of Pascal's triangle.

It is interesting to note the repetitive pattern generated by the final digit of these numbers. Two tetrahedral pyramids can make a square pyramid, in the somewhat trivial relations $\mathrm{TP}_1 + \mathrm{TP}_2 = \mathrm{SP}_2$, $\mathrm{TP}_2 + \mathrm{TP}_3 = \mathrm{SP}_3$, $\mathrm{TP}_3 + \mathrm{TP}_4 = \mathrm{SP}_4$, etc.

There are also many ways that two square pyramids can be reconstructed as a tetrahedral pyramid. A couple of examples, where the edge lengths combine to give the edge of the tetrahedron, are $\mathrm{SP}_5+\mathrm{SP}_{20} = \mathrm{TP}_{25}$, and $\mathrm{SP}_{20} + \mathrm{SP}_{76} = \mathrm{TP}_{96}$.

As a moderate cannon ball enthusiast, I am pleased to note that a tetrahedral pyramid of side 23 has 2300 balls. Also, since four square pyramids of size n can be reconstructed as a tetrahedral pyramid of base $2n$, we have $4\mathrm{SP}_{38} = \mathrm{TP}_{76}$, or, in balls, $4 \times 19019 = 76076$.

Finally, we go back to the original SP_{24}, with 4900 balls. Four of these give a tetrahedral pyramid of side 48, with 19,600 balls, which is the only tetrahedral pyramid (apart from the trivial TP_2) to have a square number of balls.

Solution 167.2 – Cows

> Fritz and Helmut sold a herd of cows and obtained for each cow as many pounds as there were cows in the herd. They decided that they would each take £10 in turn from the sum obtained. Fritz got the last £10 and there was less than £10 left for Helmut, so Fritz gave Helmut his pocket knife in compensation. How much was the knife worth?

John Halsall

The number of cows can be denoted by $10a + b$ (making $0 < b \leq 9$). The sum obtained will then be $100a^2 + 20ab + b^2$. The part of this sum represented by $100a^2 + 20ab$ was distributed without any problems. In the remaining amount (b^2) the number of tens must be odd, because less than £10 remained when Fritz had taken his last turn.

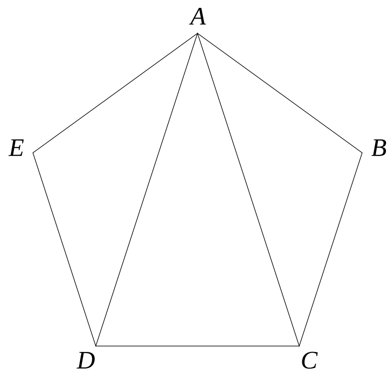
Therefore b^2 must be l, 4, 9, **16**, 25, **36**, 49, 64 or 81, and the only ones with an odd number of tens end in 6.

So when Fritz took his last £10, only £6 was left for Helmut. I surely don't have to explain that the knife was worth £2.

Nicht wahr?

Problem 169.2 – Chords

Sebastian Hayes



If we have a regular pentagon inscribed in a circle with unit radius, show that the product of the chords from any vertex to each of the others is equal to 5. That is

$$(AB)(AC)(AD)(AE) = 5.$$

Then show that a similar relation holds for any regular n-gon.

'99.9999-ish per cent reliability, give or take a per cent ...'—Internet expert on *You and Yours*, R4, reassuring listeners that it's safe to buy on-line with a credit card. Spotted by **JRH**.

Problem 169.3 – Squares in arithmetic progression

John Reade

What is the maximum number of squares you can have in arithmetic progression? For example, can you find integers a, b, c, d such that

$$b^2 - a^2 = c^2 - b^2 = d^2 - c^2?$$

Etc., etc.

Eds—In case you find John's problem too easy, try this one instead . . .

What is the maximum number of squares you can have in geometric progression?

Problem 169.4 – Functional inequality

John Bull

The following problem was posted on the Internet last year, but a solution was never offered, neither by the proposer, nor anyone else:

> The function f takes a positive integer, n, as an operand, and must produce a positive integer result; that is, the function is undefined unless both n and $f(n)$ are positive integers. If, for any positive integer, n, it is always true that $f(n+1) > f(f(n))$, prove that $f(n) = n$ must follow as a consequence.

The problem was originally expressed in more succinct mathematical notation, but I have spelt it out in full to ensure that everyone understands it and can appreciate the subtleties. For the moment it can only be taken as a conjecture, although some experimentation suggests intuitively that the result does follow. It's a sort of Internet version of Fermat's Last Theorem—the email containing the proof being lost in cyberspace!

Can anyone offer a watertight proof or refutation?

'So the Universe began as a tiny particle. Would Professor Hawking explain . . . how this particle got there?'—Letter to *The Times*.

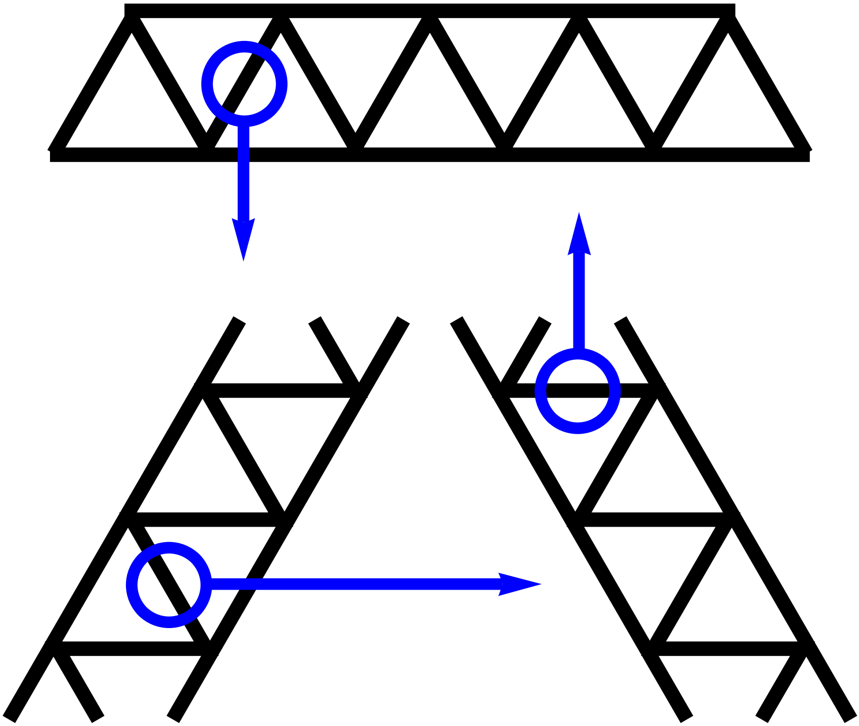
'Got where?'—Subsequent letter to *The Times*.

[Quoted in the *Weekend Times*. Spotted by **EK**.]

Problem 169.5 – A fractal bridge beam

Ken Greatrix

Does this bridge have any strength?



ADF—In case it isn't obvious from the diagram, there is a self-similarity situation here. What we have is one of the spanning supports of a typical railway bridge. It is constructed from two long bits of iron braced with short pieces set at angles of 60 and 120 degrees. Furthermore, each element itself has a similar structure, being made up out of smaller elements joined together in the same manner. Furthermore, each smaller element itself has a similar structure, being made up out of smaller elements joined together in the same manner. And so on.

The infinite truth: $\infty = \infty$. True or false? — **Ken Greatrix**.

OU maths courses comparison

David Ireland

M101: A logger sells a truckload of timber for \$100. His cost of production is 4/5 of this price. What is his profit?

M203: A logger exchanges a set L of lumber for a set M of money. The cardinality of set M is 100 and each element is worth a dollar. Make a square array of 100 dots to represent the the elements of set M. The set P of profits can be put in 1 to 5 correspondence with set M. What is the cardinality of set P?

MDST242: A logger sells a truckload of lumber for \$100. His cost of production is \$80 and his profit is \$20. Underline the number 20.

MU120: By cutting down beautiful forest trees, an environmentally ignorant logger makes a profit of \$20. What do you think of this way of making a living? In your group, use role-playing to determine how birds and squirrels in the forest feel.

(Adapted from 'Mathematical education through the years', *The Ottawa Citizen*. See also 'Mathematical examinations through the years', M500 **156**, page 13.)

Groceries

Peter Fletcher

Like Eddie Kent (M500 **167** p. 28), I have not thrown away any money on the National Lottery, and I have no intention to throw away any. The numbers on the balls are just labels, and I do wonder how many people actually realize this. What if the these labels were instead, say, APPLE, PEAR, BANANA, POTATO, CARROT, CHERRY, etc.? The numbers themselves, like my suggested names, have no significance whatsoever. What would Eddie Kent's fax 'LOTTERY SECRETS REVEALED' have to say about how to choose a winning basket of fresh groceries?

Complex complex complex

JRH

An architect designed a shopping *complex*. Then he made several intricate additions so he had a *complex complex*. Then he got worried about it so he had a *complex complex complex*.

Now it's your turn. The sillier the better.

Grazing oxen

John Bull

I am honoured to have been attributed a mathematical model by Stephen Sparrow in M500 **167**, although Bull's model of oxen grazing is hardly in the same league as Fourier series, Euler numbers, Hermite polynomials, etc. Unfortunately, I thought I had made it clear that I was offering alternatives, and that I felt neither model to be satisfactory.

The first model assumes linear grass growth. This cannot be true otherwise grass would grow without limit. The alternative model allows nonlinear grass growth, but assumes that a steady state is never reached; that is, that all the grass will be consumed sometime, however small the number of oxen per acre. This will not be true either.

Both models fit the given data, but neither are 'correct', although in either case it might be easy to jump to this conclusion. At minimum, a model for grass growth needs to be introduced before one could have any confidence in the results.

The trivial objection that it would not be possible to have an infinite number of oxen per acre applies to both models. Both assume that time will approach zero as numbers increase. The more interesting case is what happens as numbers fall. With grass always growing, 'time to consume it all' is a doubtful concept anyway, whatever the model.

Observation of a steady state from farming practice may be useful for validating a model, but not for creating one. The birth and death process for fish in the North Sea is unlikely to be the same as the growth model for grass, except that we could safely assume that neither are linear. If linearity is assumed over some range, then this limits the propriety of the model, and it may not be valid to extrapolate to a steady state beyond the range of the input observations.

As I said in the conclusion to the article there is insufficient to derive an acceptable model: too few observations; missing variables (such as to represent grass growth); no axiomatic principles (such as those of probability); and no reference to physical laws (such as conservation of energy).

During the Manhattan project, Edward Teller gave a talk in which he used various 'ball park' estimates to show that setting off an atomic bomb would, in all probability, ignite the atmosphere of the earth. A hush fell over the audience. Then Robert Oppenheimer cleared his throat and asked Teller if his 'ball park' estimates included setting the speed of light equal to 1 cm/s.

Twenty-five years ago

A collection of paragraphs from M500 15

Leslie Naylor—You can make [Latin, Graeco-Latin and magic] squares by several methods, depending on how many Credits you have, e.g.

(a) Trial, Error and More Trial (M100 students)

(b) Modular Arithmetic (M100 unit on remainder classes)

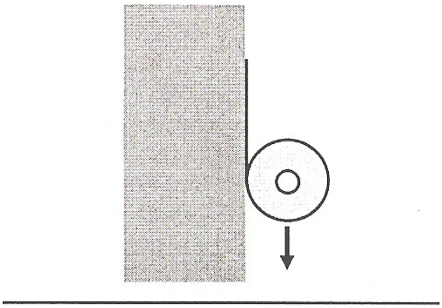
(c) Solving Simultaneous Equations (M201)

(d) Permutations of a Group (M202)

(e) By Computer (M251)

(f) By looking in a book on Randomized Experiments for Statistical Analysis (MDT241).

Bob Margolis—A lunatic student at QMC (London) once taped the end of a roll of toilet paper to the side of the tower block there. He then allowed the thing to unroll itself down the wall. At what point is the total energy the same as if the whole roll had simply been dropped from the same starting point?



Yvonne Kedge—I rise, Sinbad. Not to the bait of the correlation between unfemininity and mathematical ability but to any aspersion cast against the wit and humour of mathematicians which has been sustaining the delight in my excursions with the OU and not least M500.

The Mathematics Faculty itself admittedly failed to sustain such dizzy virtues upon certain auspicious occasions—Minsky!—but I happily need not dwell on the past. How sad if the Mathematicians' flights of fancy were not to be tempered with humour.

Datta Gumaste—There is a disease exclusively found in mathematicians. Those who enjoy playing the game of mathematics—as opposed to those who find joy in using it—are more likely to catch this disease.

Morphasia was first introduced openly in the UK by the OU, and those unfortunate ones who decided to study M100 were the first victims.

Unit 3 of M100 is responsible for inflicting it, and M202 for its continuous dangerous spread. It is becoming clear that no previous history of mathematical sophistication is necessary for a person to get afflicted by this mental disorder (although there is no sufficient evidence to support this statement).

Some of the observable symptoms are:

(a) Blindness—i.e. inability to see things as they are. (b) Sleepless nights and restless days. (c) Tendency to use poetry as a normal medium of communication. (d) Obsession to draw triangles and rectangles most of the time.

Bill Shannon—It is hard luck on those who do not like the limited choice of 3rd level maths courses, but I cannot see that an honours degree, mostly in maths, would be appreciably degraded by the inclusion of one 3rd level credit in another faculty's courses. There may well be eccentric mathematicians who actually enjoy partial differential equations!

Winter Week-end

Norma Rosier

This is an annual residential Weekend to dispel the withdrawal symptoms due to courses finishing in October and not starting again until February. It is an opportunity to get together with friends, old and new, and do some interesting mathematics.

The nineteenth M500 Society WINTER WEEK-END will be held at Nottingham University from **Friday 7th to Sunday 9th January, 2000**. Ian Harrison is running it and the theme will be announced later. It promises to be as much fun as ever!

Cost: approximately £120 for M500 members, £125 for non-members (but not yet fixed). This includes accommodation and all meals from dinner on Friday to lunch on Sunday. Please send a stamped, addressed envelope for booking form to **Norma Rosier**, after September 12th, when all details should be known.