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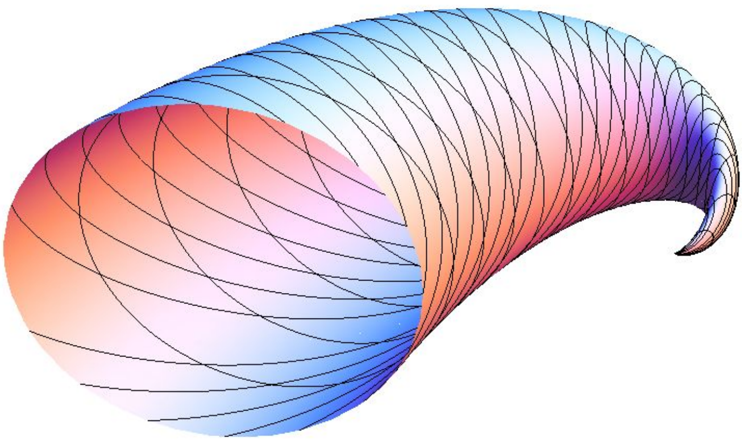

The M500 Society and Officers

The M500 Society is a mathematical society for students, staff and friends of the Open University. By publishing M500 and 'MOUTHS', and by organizing residential weekends, the Society aims to promote a better understanding of mathematics, its applications and its teaching.

The magazine M500 is published by the M500 Society six times a year. It provides a forum for its readers' mathematical interests. Neither the editors nor the Open University necessarily agree with the contents.

MOUTHS is 'Mathematics Open University Telephone Help Scheme', a directory of M500 members who are willing to provide mathematical assistance to other members.

The September Weekend is a residential Friday to Sunday event held each September for revision and exam preparation. Details available from March onwards. Send SAE to Jeremy Humphries, below.

The Winter Weekend is a residential Friday to Sunday event held each January for mathematical recreation. Send SAE for details to Norma Rosier, below.

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Developing a fractal program for a TI83 calculator

Patrick Meehan

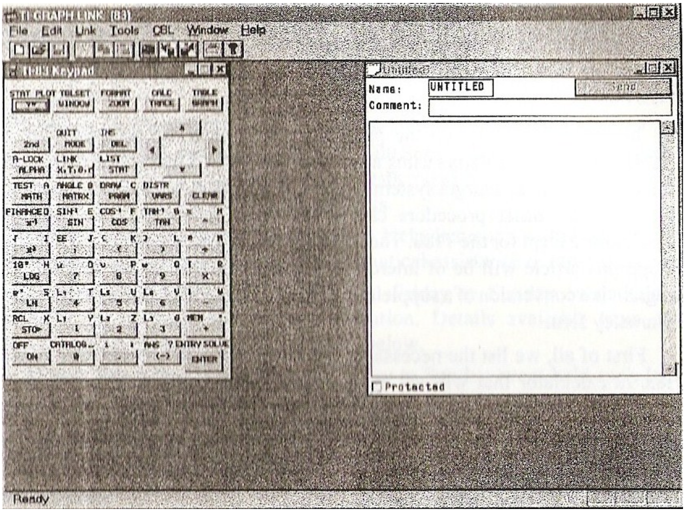
This article will explain the procedure for developing programs for Texas Instruments calculators using a personal computer. Thus, a user's TI calculator effectively becomes a target system. Although the target system used in this article is a TI83, a similar procedure can be followed for other Texas Instruments calculators, except for the TI80. The TI80 does not possess a communications port. I hope this article will be of interest to MS221 students, as the example fractal program is a conversion of a supplementary MS221 MathCAD file, which produces a Barnsley fern.

First of all, we list the necessary equipment that will be needed: obviously a TI8x/92 calculator that will act as the target system; also a copy of the Texas Instruments TI-Graph Link program, plus a serial communications lead in order to connect the calculator to a PC; a copy of Virtual TI; and finally an assembler shell specific to a particular model of calculator. What follows is a brief explanation of the above items and the role they play in program development.

TI-Graph Link handles the communications between a PC (or Macintosh) and a TI calculator. With this you can upload and download calculator programs between a PC and a TI calculator. Also TI-Graph Link allows a user to enter calculator programs from within the program editing window. To the left of the editing window is a copy of a TI calculator keypad. This allows a user to enter single key-press commands such as *store* and other menu based Basic commands. There is a different version of TI-Graph Link for each TI calculator model. A copy appropriate for a specific calculator can be downloaded via the Internet at www.ti.com/calc/docs/calchome.html. A view of the TI-Graph Link 83 screen is shown at the top of the next page.

You can purchase a computer kit called Graphlink 99 from Oxford Educational Supplies. It includes a black communication cable. Their web address is www.oxford-educational.co.uk. Supplied with Graphlink 99 is a CD ROM, which contains the TI-Graph Link software for all models of TI calculators.

Virtual TI is a TI calculator emulator that runs under Windows on a PC. This software is freeware and can be downloaded from www.ticalc.org as a 'zip' file. Finally, an assembler shell will be needed which has to be uploaded on to the target TI calculator. For the TI83 this assembler shell is called Ashell.83g. You can obtain an appropriate assembler shell from www.ticalc.org or www.ti-files.org (download the file called TI83ashell.zip or Ashell.zip and then extract using WinZip). There are different assembler shells for each model of calculator.



An assembler shell allows a TI calculator to run machine code programs (the language that the processor directly executes, unlike the inbuilt BASIC, which is interpreted). This enhancement is necessary so that the onboard ROM of the TI calculator can be downloaded on to a PC. Please note that this code is copyright material, so do not hand out the ROM code of your calculator to other people. When installing Virtual TI a machine code program called *dumprom* is sent to the TI calculator, which is then executed in order to dump the onboard ROM code to a PC. This firmware is used by Virtual TI in order to emulate a TI calculator.

Installing the necessary software

To install TI-Graph Link either execute the downloaded executable that was obtained from the TI web site or run the setup program from the Graphlink 99 CD. Execute TI-Graph Link and from the *Link* menu option select the appropriate cable type and COM port.

The next step is to upload Ashell.83g to a TI83 calculator. First a word about the file structures used by TI-Graph Link. BASIC programs for a TI83 have the file extension of 83p. However, for large software projects it is better to split your programming code between a main program block and several subprograms and have these subprograms called as subroutines. Such a collection of files can be grouped together by TI-Graph Link. The file extension of such grouped files is denoted by 83g (or 89g for a TI89 calculator etc.). Hence Ashell.83g in fact contains several programs; a BASIC program called Ashell and three machine code programs, two of which are used by Ashell.

So, connect the black link communications lead between the TI83 and a serial port of a PC. From the *Link* menu option of TI-Graph Link select *Send*, and then from the file selection menu navigate to and choose Ashell.83g. Select the OK button and Ashell.83g will be sent to your calculator. (Make sure your calculator is turned on. Also, with the latest version of TI-Graph Link there is no need to put your calculator into communications mode as the necessary commands are sent in silent mode to the TI calculator by TI-Graph Link.)

Your calculator's program selection screen should look something like this.

```
EXEC EDIT NEW
1:ANGLES
2:ARITH
3:ASHELL
4:CLRARGS
5:COSFOR1
6:COSFOR2
7↓DUMPROM
```

As you can see, Ashell on my TI83 is the third option within the program menu.

Next install Virtual TI. This is done by running vti.exe after expanding VirtualTIvti.zip (depending on where you download Virtual TI from the file might be called something else). Several menu guided steps need to be followed. Make sure the TI calculator is still connected to the PC serial port and is turned on. Select the appropriate calculator model and assembler shell combination from the Virtual TI menu options, then select the correct communications cable and the COM port. (These menu options automatically appear during the VTI installation procedure.) When prompted place the TI calculator into receive mode and send the *dumprom* program to the calculator. As can be seen from the captured calculator screen on the right, *dumprom* should now appear in the calculator's program selection menu.

Now you will need to execute Ashell to continue. After Ashell is executed the calculator screen should look similar to the screen capture, right. (The bitmap image is in fact a screen capture of Virtual TI.)

What you should be looking at on your calculator screen is the Ashell equivalent to the program selection menu. This menu only displays machine code programs and BASIC programs that start with '::' at the beginning of the program. As well as the TBOUT2 program selection DUMPROM should also appear. When Virtual TI is ready execute *dumprom* on the TI calculator by highlighting it using the cursor keys and pressing enter. The onboard ROM of the calculator will be downloaded to your PC. To interrupt *dumprom* if anything goes wrong press the ON key. To exit Ashell press the Graph key. (A word of warning—run TBOUT2 at your peril. This is a program that emulates the old arcade style breakout game. I cannot find any key press that will actually interrupt execution. The only way to exit is to instantaneously lose each program run until the program automatically exits.) Once the ROM dump is complete you should find a ti83_1.rom file in the same directory as Virtual TI.

Virtual TI allows you to switch between ROMs from different TI calculator models. Virtual TI uses various *skins*, which are the bitmap images of the various TI calculator models.

Fractal program development

Students of MS221 will come across a MathCAD file, called 221B4-03.MCD, as part of their course materials, which is only mentioned in passing as being relevant to the study of iterations and fixed points with regards to chaotic behaviour. The recurrence equations within this file produce a Barnsley fern. The program listing on the next page, called Fern.83p, is a conversion of 221B4-03.MCD into a TI83 BASIC program. (Note: the symbols `->` represent the *store* command.) This program listing can be entered via TI-Graph Link and then saved to disk as Fern.83p. At this stage do not upload this program to the TI calculator.

Fractal program description

(I do not have any in-depth expertise in the use of complex numbers or fractals. My own knowledge in such matters goes as far as MST121, MS221, plus whatever I have managed to pick up from various other sources. However, a basic description of how the fractal program works will now follow.)

The first four lines of Fern.83p set the values of the X and Y axes for the TI83 graphics screen. The AxesOff command removes the X and Y scales from view. Next, the values of ten constants are set. These values are taken from the MathCAD file 221B4-03.MCD. Two lists, L_4 and L_5, have their first values set to zero. The graphics screen is then cleared via the ClrDraw command. List L_6 has its first value initialized to a random real number between 0 and 1 via the *rand* command; strictly speaking this step is not necessary but it does follow the method used in the MathCAD file. The program then enters a *For* loop which uses variable Q to repeat the contained code 700 times. Hence, 700 values are placed in list L_4, in list L_5, and in list L_6. The values contained in list L_4 represent the X values, the values contained in the list L_5 represent the Y values, and the values contained in the list L_6 represent the Z values. The Z values are all generated via the *rand* command.

```
-4->Xmin
4->Xmax
0->Ymin
10->Ymax
AxesOff
0.75->A
0.85->B
0.95->C
0.85->E
0.05->F
-0.15->G
0.25->H
0.18->J
-0.25->K
0.18->L
1.5->M
0->L4(1)
0->L5(1)
ClrDraw
rand->L6(1)
For(Q,1,700,1)
rand->L6(Q)
If L6(Q)<<A
Then
E*L4(Q)+F*L5(Q)->L4(Q+1)
Else
If L6(Q)<<B
Then
G*L4(Q)+H*L5(Q)->L4(Q+1)
Else
If L6(Q)<C
Then
J*L4(Q)+K*L5(Q)->L4(Q+1)
Else
0->L4(Q+1)
End
End
End
If L6(Q)<A
Then
-F*L4(Q)+E*L5(Q)+M->L5(Q+1)
Else
If L6(Q)<B
Then
H*L4(Q)+-G*L5(Q)+(M/2)->L5(Q+1)
Else
If L6(Q)<C
Then
-K*L4(Q)+J*L5(Q)+M->L5(Q+1)
Else
L*L5(Q)->L5(Q+1)
End
End
End
Pt-On(L4(Q),L5(Q))
End
ClrList L4,L5,L6
```

Thus, the use of the *rand* command is what gives the program its 'chaotic' behaviour. The Z values contained in L_6 are never actually plotted. They are used within the 'If Then' conditionals for each of the X and Y coordinates. Therefore the Z coordinates in L_6 are tested against the constants A, B and C to decide which calculation is used to create the necessary X and Y coordinates contained in lists L_4 and L_5 respectively. The variable Q is used as the index to each of the lists and is incremented by one for each pass of the *For* loop. The other constants are used within the calculations for deciding the values for the X and Y coordinates. Previously calculated values for X and Y are used to produce the next coordinate pair. This represents the necessary feedback in order to achieve the bifurcation of the plotted points. The basic equations for such points are of the form

$$X_{n+1} \;=\; aX_n + bY_n + c, \qquad Y_{n+1} \;=\; dX_n + eY_n + f.$$

The last command within the *For* loop, `Pt-On(L4(Q), L5(Q))`, plots the last calculated values for the X and Y coordinates. Once the *For* loop has completed the lists, L_4, L_5, L_6 are cleared in order to recover valuable memory space.

The program on a TI83 takes about four minutes to complete. However, during development the program might have been typed in incorrectly or perhaps the design of the algorithm is faulty. Therefore, it would be far better to test and debug the program on an emulator, i.e. Virtual TI, rather than repeatedly crash the program on a TI calculator. So the next task is to upload Fern.83p to Virtual TI.

Loading programs into Virtual TI

Execute vti.exe, and then move the mouse cursor over to the *On* key and click the left mouse button. (All of the VTI keys operate in the same way as a TI calculator.) You should see a blinking block cursor on the VTI screen. Next, move the mouse cursor over to the LCD screen and click the right mouse button. A menu will appear. At the top of the menu will be *Send File to VTI*. Click on this selection and then navigate to the correct directory, where Fern.83p resides, and open the file. Fern.83p will be loaded into the program selection menu of VTI.

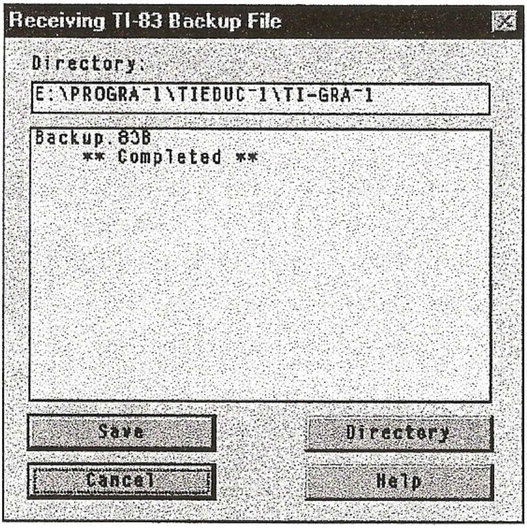
You can now enter the program selection screen of VTI and execute Fern.83p. (Use the same key presses as for your TI calculator.) You can increase the number of times that the *For* loop is executed in order to create a better looking fern structure. Increasing the limit to 800 for the Q variable should suffice. (Note: the maximum number of values that a TI83 list can contain is 999. However, this value is dependent on the amount of free memory available.) The value of Q has been set to 700 in order to cater for other programs that have been stored within my TI83. Therefore, check the amount of RAM available in your TI calculator before setting Q to a higher value and uploading Fern.83p. As VTI is being used as a development system there should always be plenty of memory available. This is because there is less need to store known working programs within Virtual TI.

During the development process several program edits, uploads, and reruns may be needed in order to achieve a stable working program. When this has been realized the next step is to upload the program to a TI calculator.

Opposite is a screen capture of Virtual TI showing the final output of Fern.83p. We can now load the working program on to a TI calculator.

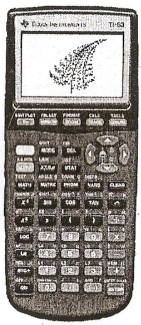
Uploading Fern.83p to a TI calculator

As an added precautionary measure, before uploading Fern.83p, a complete backup of a TI calculator can be downloaded to a PC. In the event of a major catastrophe you can restore all the settings of a TI calculator from the saved backup file. With the TI calculator switched on and connected to a PC start up TI-Graph Link. Select the *Link* menu option and then click on *Get Backup.* Once the download has completed save the file Backup.83B by clicking on the*Save* button. (See below.)

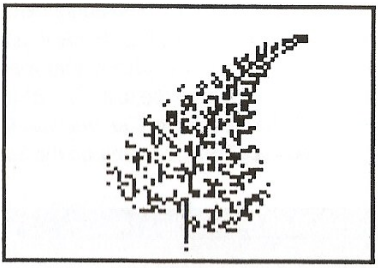


Now upload Fern.83p. From the *Link* menu option click on *Send* and then navigate to Fern.83p. Highlight Fern.83p and then click on the OK button. The selected file will be uploaded to the calculator. Fern.83p should now appear within the program selection menu of the TI calculator, as shown below.

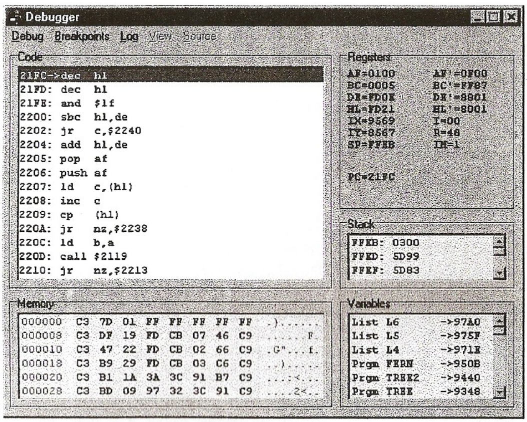
```
EXEC EDIT NEW
6↑COSFOR2
7:DUMPROM
8:FERN
9:FOURIER
0:GCIRCLE
 :GEOM
 ↓ITERATE
```



Fern.83p is now ready to execute. A screen dump of the final graphics output of Fern.83p from my TI83 is shown below. The fern has been produced with the Q variable set to increment to 750, the maximum that my TI83 would accept without crashing. If you have only a few programs stored in your calculator you could leave the value within the *For* loop to 800, the same value that was used on VTI.



It is highly unlikely that you could in fact seriously crash a TI8x/92 calculator by trying to run a BASIC program with errors in its code. This is certainly not the case with machine code programs. Hence, for such programs VTI becomes invaluable as a test bed. VTI contains an inbuilt debugger so that you can trace the execution of machine code programs. (See the image below.)



As mentioned earlier, the assembler shell Ashell allows a user to run machine code programs on a TI83 calculator. The TI series of calculators, up to the TI86, use the Zilog Z80 microprocessor. There are plenty of shareware and freeware assemblers available from the Internet that you can use for creating machine code programs. Ashell.zip contains an utility called Devpac83.com, which will convert Z80 assembler output files into a suitable format for Z80 based TI calculators. The TI89 and TI92 both use the

Motorola 68000 microprocessor (a more powerful processor than the Z80). Hence, a Motorola 68000 Assembler will be needed for these calculators.

The benefit of BASIC programs is their portability between TI calculator models. The benefit of machine code programs is their greater speed of execution.

Hints and tips

When connecting the black serial communications lead between a PC and a TI calculator I found that TI-Graph Link defaulted to the COM1: serial port. If, like my PC, a serial mouse is connected to COM1: then you might have difficulty getting TI-Graph Link to switch to a different COM port. This problem occurs despite there being the option, within the *Link* menu selection, of changing the default COM port. However, I have only used this software on Windows NT4.0 operating system, and therefore this problem may be specific to NT. Fortunately, NT is not fussy about which COM port is used by the serial mouse.

Under the NT4.0 operating system you have the option of setting up user accounts within particular inbuilt 'user groups'. Each of these user groups has a different level of access privileges under NT. From a security point of view, particularly with regard to virus programs, I have always preferred to use a personal account within the 'Power User' group. Virtual TI worked perfectly with this account under NT Service Pack 5. However, when I installed Service Pack 6a a 'Failed to set data for LastEmuVersion' error message appeared when I tried to run VTI. Since then VTI will only run under an Administrator account.

A mechanical engineer, a chemical engineer and a software engineer were in a car when it suddenly stopped.

Mechanical engineer – "I think it must be the timing chain. I'll strip down the engine and fix it."

Chemical Engineer – "No, I think it's the petrol. I'll take a sample and analyse it."

Software Engineer – "I've no idea what's wrong, but let's try closing the windows, turning everything off, turning everything back on again and then re-opening the windows."

[Sent by **Tony Huntington**.]

A computer is like an air-conditioner. It stops working properly if you open windows.

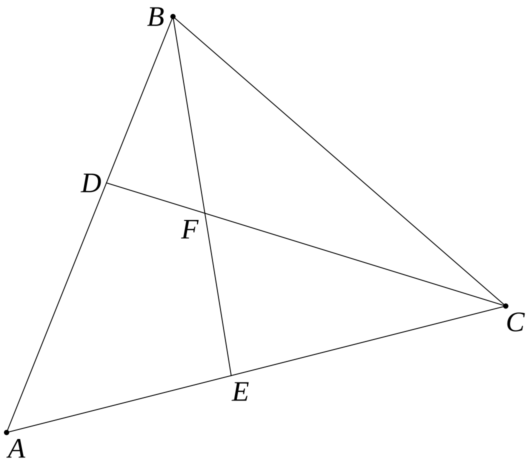
Problem 180.1 – Two questions about triangles

John Hulbert

Consider a triangle ABC, with point D on AB, and point E on AC, such that $CD = BE$. Let F be the point of intersection of CD and BE.

(i) What is the locus of point F as points D and E move along AB and AC?

(ii) If points D and E are fixed, and line AF extended meets BC at right angles, is triangle ABC necessarily isosceles?



One of the questions in a physics degree exam at the University of Copenhagen was, 'Describe how to determine the height of a skyscraper with a barometer.'

One student wrote 'Tie a long piece of string to the barometer, then lower it from the roof to the ground. The length of the string plus the length of the barometer will equal the height of the building.'

Given that Copenhagen is in Denmark and the student won the Nobel Prize for Physics, who was he? [**EK**]

'You need two per cent salt for every kilo of meat . . . '—Hugh Fearnley Whittingstall on making salami, *Woman's Hour*, BBC R4, 7 Feb 2001. [Spotted by **JRH**.]

Solution 178.7 – Series

Show that $\dfrac{x}{1-x} - \dfrac{x^3}{1-x^3} + \dfrac{x^5}{1-x^5} - \ldots \; = \; \dfrac{x}{1+x^2} + \dfrac{x^2}{1+x^4} + \dfrac{x^3}{1+x^6} + \ldots.$

John Bull

This cannot be true for all x. A simple inspection shows it to be undefined for $x = 1$. So we proceed on the assumption that $-1 < x < 1$.

Call the LHS series S, and expand each term by the binomial theorem, which is permissible if and only if $-1 < x < 1$. We then have

$$
\begin{aligned}
S \;=\;& x + x^2 + x^3 + x^4 + \ldots \\
& - x^3 - x^6 - x^9 - x^{12} - \ldots \\
& + x^5 + x^{10} + x^{15} + x^{20} - \ldots \\
& - x^7 - x^{14} - x^{21} - x^{28} - \ldots \\
& + \ldots,
\end{aligned}
$$

where each term is expressed as a separate row. Now rearrange this so that each column is expressed as a row, then

$$
\begin{aligned}
S \;=\;& x - x^3 + x^5 - x^7 + \ldots \\
& + x^2 - x^6 + x^{10} - x^{14} + \ldots \\
& + x^3 - x^9 + x^{15} - x^{21} + \ldots \\
& + x^4 - x^{12} + x^{20} - x^{28} + \ldots \\
& + \ldots.
\end{aligned}
$$

Each row can now be seen to be a binomial series expansion of each successive term on the RHS. Thus it is shown that LHS = RHS, as required.

Solved in a similar manner by **Peter Fletcher**.

Problem 180.2 – Unlimited prize

Paul Richards

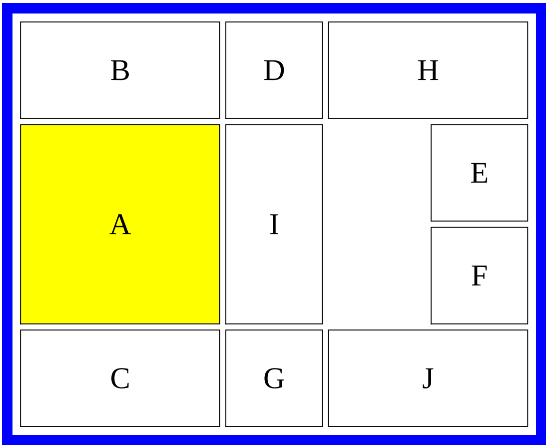
A casino owner devises a gambling game for which he can advertise an unlimited prize, but which also has a predictable distribution of payouts.

A gambler pays a stake, and the croupier throws a six-sided die repeatedly until all the numbers from 1 to 6 inclusive have appeared, irrespective of repetition. The gambler receives winnings proportional to the number of throws of the dice.

What is the correct stake to make the game fair?

Solution 178.6 – Ten blocks

Ten pieces of various sizes slide round a board in the obvious manner. Move the large square from the middle-left of the array to the middle-right.



Dick Boardman

Denote the pieces by letters, as above, and divide the board into 20 numbered squares, opposite. Piece `A` originally covers squares 1, 2, 5, 6 and the object of the puzzle is to move it to the position that occupies squares 13, 14, 17, 18.

0	4	8	12	16
1	5	9	13	17
2	6	10	14	18
3	7	11	15	19

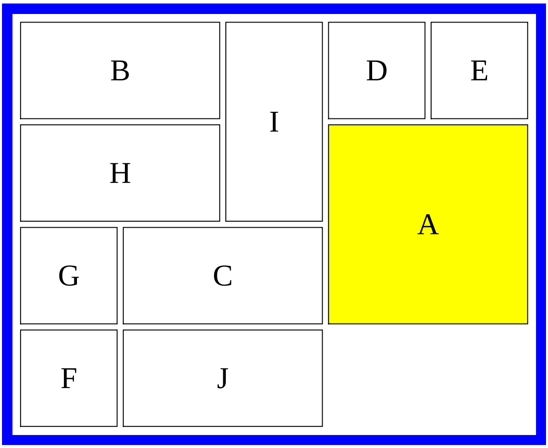
Initially, the method used was a breadth-first search in which all the new positions resulting from a given position were placed on a stack. Positions which occurred before, positions which are symmetrical to previous positions and positions in which pieces of similar shape are swapped were rejected. This method found a solution in 106 moves after processing about 12000 positions.

Later versions of this program use a 'best-first' search method in which the stack is searched for the node with the highest value and the next positions are generated from that. Positions with the same value are processed sequentially. This reduces the number of processed positions to about 5000. The value of a position is taken as the number of the first column containing piece `A`.

The solution is given as a series of moves in the form `X`n, where `X` is the name of the piece being moved and n is the number of the square to move it to. Where a piece occupies several squares, the number used is the lowest, i.e. the top left square.

```
I13, D 9, H 8, E16, G10, J11, F19, I17, D13, D14, H 9, E12,
I16, F18, J15, G11, D10, F14, I17, E16, H 8, D 9, D13, G10,
J11, G 9, I18, D17, F13, J10, C 7, C11, A 2, G 5, F 9, D13,
E17, G 1, F 5, D 9, E13, I17, C15, I16, J14, A 6, G 2, F 1,
D 5, G 3, F 2, D 1, A 5, C11, C 7, J15, E14, E18, A 9, D 5,
D 6, B 1, H 4, H 0, A 8, D10, D14, F 6, G 2, C 3, J11, E19,
D18, A 9, H 4, H 8, B 0, F 5, G 1, C 2, J 7, J 3, A10, H 9,
B 4, G 0, F 1, H 5, I12, D17, D16, E18, E17, A14, C 6, J 7,
F 2, F 3, G 1, B 0, G 2, H 1, I 8, D12, E16, A13
```

This is the final position:



Problem 180.3 – Fence

Ron Potkin

A farmer has an L-shaped barn which measures $80' \times 80'$ with a $30' \times 40'$ rectangle cut out of one corner. He also has a $50'$ roll of fencing which he wants to use to enclose a grazing area next to the barn. What is his largest possible grazing area?

Dec 25 = Oct 31. Decimal 25 is 31 in octal notation; $2 \cdot 10 + 5 = 3 \cdot 8 + 1$.

Solution 178.2 – Construct another square

> Given a unit circle and its centre, construct a square of side one unit, using only a ruler.

Dick Boardman

My solution uses the following construction: Given a line segment AB, its midpoint O and a point P, draw a line through P parallel to AB.

Draw line AP and extend it to an arbitrary point C. Draw lines OC, BC and BP. Let BP and OC intersect at E. Draw AE and extend it to meet BC at D. Then PD is parallel to AOB.

Proof. Area of triangle ACO = area of triangle BCO. Area of triangle AEO = area of triangle BEO. By subtraction,

$$\text{area of triangle } AEC \;=\; \text{area of triangle } BEC. \tag{1}$$

Let $CP/AP = m$. Area of triangle $CBP = m$ times area of triangle ABP. Area of triangle $CEP = m$ times area of triangle AEP. By subtraction,

$$\text{area of triangle } CEB \;=\; m \text{ times area of triangle } AEB. \tag{2}$$

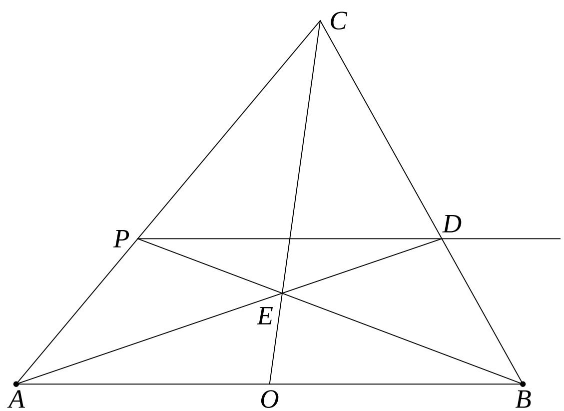
Using (1) and (2),

$$\text{area of triangle } AEC \;=\; m \text{ times area of triangle } AEB. \tag{3}$$

Let $CD/DB = n$. Then similarly

$$\text{area of triangle } AEC \;=\; n \text{ times area of triangle } AEB. \tag{4}$$

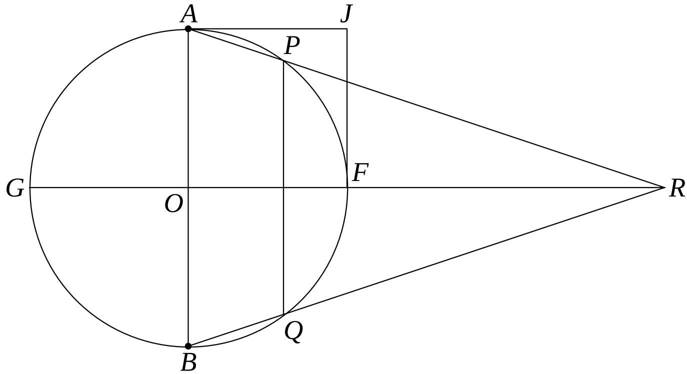
From (3) and (4) we conclude that $m = n$ and so PD is parallel to AB.



The full construction is as follows.

Draw a diameter of the circle AB and name the centre O. Choose a point P on the circle. Draw a line parallel to AOB through P using the given construction. Let this meet the circle the second time at Q. Draw AP and BQ to meet at R. Draw and extend OR to meet the circle at F and G. By symmetry, OR is perpendicular to AB.

Construct a line through F parallel to AOB and a line through A parallel to FOG crossing at J. Then $JFOA$ is the required square.



ADF writes—Observe that the ruler is used only to draw straight lines through well-defined points. As a couple of readers demonstrated, the construction becomes much easier if it is permitted to use the ruler in an imaginative manner. Notice also that significant use is made of the curved part of the circle-and-its-centre. This is expected. A unit line segment by itself is not sufficient for constructing the irrational length $\sqrt{2}$, the diagonal of the square.

Problem 178.2 and Problem 176.5 (Given a unit line segment, construct a unit square using only a pair of compasses—see M500 **176** 28 and p. 25 of this issue) are special cases of a more general result: None of the traditional 'Euclidean' ruler-and-compasses constructions actually require both instruments.

As suggested above, it seems that we can dispense with the compasses if there is a unit circle already on the page. This is indeed the case, as was proved by Jakob Steiner in *Die geometrischen Konstruktionen ausgeführt mittels der gareden Linie und Eines festen Kreises* (Berlin 1833). On the other hand, L. Mascheroni (*La geometria del compasso*, Pavia 1797) proved that Euclidean constructions do not require the ruler if one has access to a fixed unit line segment. Heinrich Dorrie discusses these two theorems (as well as 98 others) in *100 Great Problems of Elementary Mathematics* (Dover, New York 1965).

The irascible genius

Eddie Kent

We use the word 'bit' quite a bit these days, and we are not talking of horses. Everyone except my mum knows bit is short for binary digit, but someone must have thought of it first. That person was Claude Shannon, who died on February 24. He was born in Michigan in 1916, thus he made it to 84, though his last few years were spent in a nursing home with Alzheimer's disease.

He graduated from Michigan University in electrical engineering and mathematics, doing less than brilliantly in the latter. This was unfortunate as much of his work in later life needed mathematics so he had to invent it as he went along. He was fascinated by George Boole's *Laws of Thought*, where logical expressions were formalized in the way that we now know as Boolean algebra.

At MIT he was something of a loner, partly because his imagination was always leaping far beyond the intellects of his colleagues, and not least because of his practice of travelling though the corridors of the university on a unicycle. People grew used to warning one another of his unsteady advance.

For his masters degree he worked under Vannevar Bush, the inventor of the differential analyser. This was the heart of the analog computer. Contemporary thinking saw the message as inextricably bound up with the waveform that carried it: the medium was the message. Shannon realised that Boole's precision could be used to convert messages into strings of binary digits (which he called bits) and thus free them from the medium that carried them.

His thesis, published in 1940, was called 'A symbolic analysis of relay and switching circuits.' It showed how to treat Boole's symbols as switches: Yes, No; True, False; On, Off. This would work on an electric circuit and could use a very simple notation, strings of 0s and 1s. It was immediately apparent that this was the springboard from analogue to digital computers, and Bell snapped him up.

His next paper, 1948, 'A mathematical theory of communication,' contained his ideas on separating message and medium. It provided a basis for information theory, showing how to measure the efficiency of a transmission. He called this a measure of entropy, taking the word from another discipline.

In the fifties he was programming a computer to play chess, and managed to arrange a meeting with Mikhail Botvinnick (the world champion). Unfortunately neither spoke the other's language, and their interpreters knew nothing about either chess or computers. The discussion, though interesting, was degraded by being carried through a 'noisy channel' as he called it. However, he was around in 1980 when Bell's computer Belle won the International Computer Chess Championship.

In 1958 he was back in MIT, and on his unicycle, made more perilous now because he insisted on juggling as he proceeded. He also worked on a motorised pogo stick and a computer program called THROBAC-1, which calculated in Roman numerals. Another of his interests was a mechanical mouse that he developed. This was trapped in a maze and had copper whiskers that rang a bell when the mouse reached its goal of an electrical terminal he called cheese. He was able to teach the mouse to learn and it thus became an early example of AI.

Though he worked alone he was known as a sympathetic listener to anyone who had a problem, and he had the ability to grasp any argument without the need of lengthy explanations. But on the other hand he was less than patient with anyone who couldn't keep up with him—hence he was known as a genius, but irascible (like Babbage). He was awarded laurels from all over the world and it is said he modified a dry cleaning rack to hang his gowns on.

He abandoned his invention of a two-seater unicycle when no one would ride on it with him, but later modified his conventional one by making the hub off-centre, so he bobbed up and down like a duck. He also worked in cryptography and devised the method of adding extra bits to a message as an error correction device. His name was attached to many ideas he developed: the Shannon Capacity, for instance, and the Shannon Limit. In 1993 his *Collected Papers* came out, running to a thousand pages.

His design for a rocket-powered frisbee remains unpublished.

What do they mean?

Jeremy Humphries

BBC news again (R4 7/3/2001). 'The number of nurses struck off has gone up by half. The number of complaints against nurses has gone up by 16 per cent.'

This sort of stuff is very irritating. Do they mean that in Queen Victoria's time two nurses were struck off, and now they have been joined by a third one? Or 1000 nurses were struck off yesterday and 1500 were struck off today? Or what?

The BBC sometimes tell me that a certain stock, or index, has fallen or risen by so many pence, or points; which information on its own is meaningless. They tell me that the number of deaths from some cause or other has gone up. Wow! Somebody died of something. Hold the front page! Occasionally they tell me that the number of deaths from some cause or other has gone down. I am consequently afraid to go near my local churchyard, lest I meet any of the people whose deaths have been cancelled climbing out of their graves.

Problem 180.4 – Favourite lottery numbers

Chris Pile

You have eight favourite numbers. What is the minimum number of National Lottery tickets you need to purchase to guarantee that you win at least a 5-number prize if six of your favourite numbers are drawn?

In other words, what is the minimum size of L, where L is a set of 6-element subsets of $F = a, b, c, d, e, f, g, h$ such that each 6-element subset of F contains at least five elements of least one member of L?

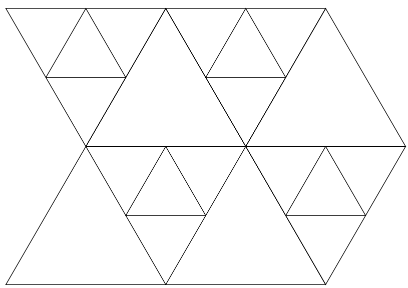
What about seven favourite numbers? Or nine, ten, eleven, ...?

ADF—Chris also sent an advertisement which suggests that the answer is at most 7 tickets for 9 favourite numbers, 14 for 10 numbers and 22 for 11 numbers. The ad even goes on to say that if you are willing to part with some money, you can acquire the actual plans that tell you to how to buy the lottery tickets.

Problem 180.5 – Triangular tiles

Barbara Lee

A cardboard square of side 2 cm is thrown at random on to a floor tiled with a triangular pattern, right. The side of the large triangle is 20 cm and the side of each small triangle is 10 cm. What is the probability that the square will not land on any of the lines where the tiles join? Assume this pattern repeats over a large area of floor and all the tiles are equilateral triangles.



Two mathematicians were having a discussion in a pub about the general awareness and level of knowledge of mathematics amongst the general public. Bill thought that this awareness was pretty good but Ben thought otherwise. When Ben went to the bathroom, Bill called the barmaid over and said to her, "When my friend comes back, I'll call you over and ask you a question. No matter what I say, you reply 'one third of x cubed'." The barmaid repeated this a couple of times to make sure she got it right, then she went about her business. When Ben returned, Bill offered to test his theory and he called the barmaid over. He said to her, "What is the integral of x squared?" "One third of x cubed," came the reply, and after a short pause she added, "plus a constant." [Sent by **Peter Gibbs**.]

Fractal music

Eddie Kent

Everyone knows what a fractal is. My on-board dictionary says it is 'A geometric pattern that is repeated at ever smaller scales to produce irregular shapes and surfaces that cannot be represented by classical geometry' which sounds more boring than it really is. Briefly, in the dying years of the nineteenth century an Italian gentleman called Guiseppe Peano began experimenting with so-called 'pathological' curves. He was a man who delighted in finding counterexamples to other people's definitions. For instance when some poor fool defined a line as having length but no breadth, so it is entirely one-dimensional, Peano found a way of bending a line in such a way that it completely filled a two-dimensional space. This gave him the idea of a non-integer dimension. He went on to design the 'snowflake' curve. Take an equilateral triangle and make the middle third of each line the base for another equilateral triangle, then do the same again for each line in the resulting figure, and again, and so on. The curve you end up with 'at the limit' has infinite length but encloses a finite area. It has dimension $(\log 2)/(\log 3) = 0.6309$.

Peano had to work theoretically. Any attempt to draw one of his creations very soon becomes hopelessly entangled. However, a computer is ideal for this sort of job, and in fact there is a program freely available that will do it for you. It is called FRACTINT and will work on any PC. No knowledge of mathematics is required. The program is based on work done by Benoit Mandelbrot, who noticed, among other things, that in Peano's diagrams each iteration is similar to the one before. He experimented with algebraic rather than geometric objects. He would take a function, like $y = f(z)$; insert a value for z, find the corresponding value of y and feed it back into z, and keep on doing this until he could see what was happening to it. Some initial values would remain stable, some would vanish and others would go off to infinity (at various 'speeds'). He then coloured the initial points (or pixels) accordingly. Surprisingly this produced very strong patterns, the most famous being the well known 'apple', from $f(z) = z \tan z + c$.

All of those well-known pretty psychedelic computer art pictures are generated in this way. Now the BBC has come up with the idea of setting fractals to music. This is not an absolutely new concept: someone once wrote a set of variations on the New York skyline and Mozart was not above playing games. But the nature of fractals is that they contain their own variations. At each magnification the same picture appears, but subtly altered. To make music a pitch is assigned to each pixel in the same way as a colour is given in the traditional model. This is called data sonification. The theme is chosen, then variations are added. Once started the procedure continues mathematically, but the result is imprecise because of the underlying chaotic nature of the material. We are born and live surrounded by fractals in nature, and thus we inevitably respond at a deep level to fractal music. To hear some examples, download

http://www.vanderbilt.edu/VUCC/Misc/Art1/Sonify/Mandi.html.

A formal mathematical definition

Nick Pollock

This article attempts to express a definition of informal mathematics, the continuity of a function, in the formal notation of first-order logic. This is useful when trying to prove the negation of a property whose definition is complicated.

Informal definition of continuity

Let $f : \mathbb{R} \to \mathbb{R}$ be a function and $\delta, \epsilon \in \mathbb{R}$.

> f is continuous at $a \in \mathbb{R}$ if, for any $\epsilon > 0$, there is a $\delta > 0$ such that $|f(x) - f(a)| < \epsilon$ for all x satisfying $|x - a| < \delta$.

The negation of this definition is

> f is not continuous at $a \in \mathbb{R}$ if there is an $\epsilon > 0$ for which there is no $\delta > 0$ such that $|f(x) - f(a)| < \epsilon$ for all x satisfying $|x - a| < \delta$.

or, equivalently,

> f is not continuous at $a \in \mathbb{R}$ if there is an $\epsilon > 0$ such that for any $\delta > 0$ there is an x satisfying $|x - a| < \delta$ for which $|f(x) - f(a)| \geq \epsilon$.

Formal definitions

Consider the theorem

> If x is greater than 4, then x is greater than 3.

If no universe of discourse is given this statement is meaningless. What if x is 'a bowl of petunias' for example? This is usually solved informally by reducing the domain of x:

> If x is an integer greater than 4, then x is greater than 3.

Now what about

> If x is an integer greater than 4, then $\sqrt{x}$ is greater than 2?

Most people are quite happy with this statement, but the implicit universe of discourse introduced in the first part of the theorem, the integers, does not always include $\sqrt{x}$, and the relation 'greater than' must be defined on more than the integers for the theorem to make sense. This problem is particularly acute in theorems like the ϵ/N definition of convergence, where some variables are real numbers and some are integers.

The next question is how to express this sort of statement in first-order logic. Given that the universe of discourse is $\mathbb{R}$ and that the relation $>$ is defined on $\mathbb{R}$, there seems to be a couple of possibilities,

$$T_1 \;=\; \forall x((x > 4) \wedge (x > 3)), \qquad T_2 \;=\; \forall x((x > 4) \Rightarrow (x > 3)).$$

The problem with T_1 appears when you try to negate it:

$$\neg T_1 \;=\; \exists x \neg((x > 4) \wedge (x > 3)) \;=\; \exists x(\neg(x > 4) \vee \neg(x > 3)).$$

If we call $x > 4$ the condition of the theorem and $x > 3$ the implication, then $\neg T_1$ is true if the implication is false, or if the condition is false, which is not very helpful.

Statement T_2 seems to work much better, as can be seen from its negation

$$\neg T_2 \;=\; \exists x \neg((x > 4) \Rightarrow (x > 3)) \;=\; \exists x((x > 4) \wedge \neg(x > 3))$$

and the following truth table.

$x > 4$	$x > 3$	T_2	$\neg T_2$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

The fact that T_2 is true when the condition is false is a bit peculiar but doesn't now interfere with the use of the formula.

Formal definition of continuity

Let the universe of discourse be $\mathbb{R}$; then $|x - y|$ is defined for all x, y, and the formal definition of continuity of the function $f : \mathbb{R} \to \mathbb{R}$ at $a \in \mathbb{R}$ is

$$C(f, a) \;=\; \forall \epsilon(\epsilon > 0 \Rightarrow (\exists \delta(\delta > 0 \wedge (\forall x(|x - a| < \delta \Rightarrow (|f(x) - f(a)| < \epsilon)))))).$$

'Automatic' negation now yields

$$\begin{aligned}\neg C(f, a) &= \exists \epsilon(\epsilon > 0 \wedge \neg(\exists \delta(\delta > 0 \wedge (\forall x(|x - a| < \delta \Rightarrow (|f(x) - f(a)| < \epsilon)))))) \\ &= \exists \epsilon(\epsilon > 0 \wedge (\forall \delta(\delta > 0 \Rightarrow \neg(\forall x(|x - a| < \delta \Rightarrow (|f(x) - f(a)| < \epsilon)))))) \\ &= \exists \epsilon(\epsilon > 0 \wedge (\forall \delta(\delta > 0 \Rightarrow (\exists x(|x - a| < \delta \wedge \neg(|f(x) - f(a)| < \epsilon)))))) \\ &= \exists \epsilon(\epsilon > 0 \wedge (\forall \delta(\delta > 0 \Rightarrow (\exists x(|x - a| < \delta \wedge (|f(x) - f(a)| \geq \epsilon)))))),\end{aligned}$$

which reads

f is not continuous at a if there is an $\epsilon > 0$ such that for all $\delta > 0$ there is an x satisfying $|x - a| < \delta$ such that $|f(x) - f(a)| \geq \epsilon$,

which I think is the right answer!

Conclusion

The key to this use of first order logic seems to be, if $C(x)$ is the condition on x and $I(x)$ is the implication of x satisfying the conditions, then write the universal formula

$$U = \forall x(C(x) \Rightarrow I(x))$$

if the implication is true, and the existential formula

$$E = \exists x(C(x) \wedge \neg I(x))$$

if the implication is false. Then $U = \neg E$, as required, and $C(x)$ always appears in a positive sense to limit the bound variable to its correct domain.

In the Hilbert Hotel laundrette

Martin Cooke

In the Hilbert Hotel laundrette
They can't keep coloureds separate:
They try to divide all the Reds
From all the not-Reds, in their heads,
By slicing lines through the rainbow
Between which all Red things would go,
But whilst these lines' sides are then tinged
Indistinctly, they'd be unhinged
If they thence take, as Red, yellows
Pinks and their spectral bedfellows,
Or if they too readily let
Elusively captioned scarlet
Evict crimson. Naturally
Wholesome numbers like 1,2,3, ...
Aren't totally collectable,
Even on an endless table:
If they reach something infinite
Then that's one more than some finite,
But if they always stay finite
They'll never have reached each finite ...
So this Hotel is a building
That they must always be building;
Not because it's time-and-spatial,
But, like $\mathbb{N}$*, since it's sequential.*

Distances from the centroid

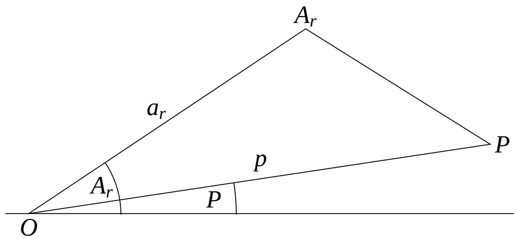
Sebastian Hayes

In M500 **175**, Barry Lewis uses complex numbers to derive a most interesting result:

> Given any set of points in the plane whose centroid is G and any other point P then, if P describes a circle whose centre is G, the sum of the squares of the distances of P from each of the points is constant (M500 **175** 10.)

This result can also be obtained by more elementary methods. On p. 22 of the same issue I give the general formula for the sum of the squares of the distances as

$$\sum_{r=0}^{n-1}(A_rP)^2 \;=\; np^2+\sum_{r=0}^{n-1}a_r^2-2p\sum_{r=0}^{n-1}a_r^2\cos(A_r-P).$$



We simply apply the cosine rule—the accompanying diagram is self-explanatory.

Now let P describe a circle around the origin. The distance p in the formula does not change and the various distances a_r do not change either. We need only examine the last term. We have

$$\sum_{r=0}^{n-1}a_r^2\cos(A_r-P) \;=\; \cos P\sum_{r=0}^{n-1}a_r^2\cos A_r+\sin P\sum_{r=0}^{n-1}a_r^2\sin A_r.$$

However, if the origin is the centroid of the points $A_0, A_1, \ldots, A_{n-1}$, then, by definition, $\sum_{r=0}^{n-1}a_r^2\cos A_r=0$ and, likewise, $\sum_{r=0}^{n-1}a_r^2\sin A_r=0$.

So the position of the point P is irrelevant as long as it remains the same distance from the centroid. Hence Barry's result.

One would expect such a striking relation to have been put to use in some nineteenth century mechanical linkage but as far as I know this is not the case.

A 'solution' to the St Petersburg paradox

Martin Cooke

Some mathematicians 'solved' this paradox (see my reference [1] of M500 **178** 35) by pointing out that the problem is impossible (because there must be some practical limit to the amount that could be paid out, whereas there is no limit to the delay before the appearance of a head, and hence to the amount which could be owed to B). But if the infinite cash were to be accounted for separately, that is, if it were only there to enable unlimited dues to be met after all, then the problem would only be impossible physically; it would still be a mathematically valid problem. However, the idea does illuminate the paradoxical nature of the problem. [The St Petersburg paradox: A coin is tossed repeatedly. Whenever a head appears A and B exchange money. If B pays A £2^t, where t is the length of the preceding run of tails, then fairness requires that A should pay B £∞.—ADF]

For example, if A could lose only a million pounds, at the very most, then (since 10^6 is a little less than 2^{20}) the expected loss is only

$$\frac{1}{2} + \frac{2}{4} + \cdots + \frac{2^{19}}{2^{20}} + \left(\frac{1}{2^{21}} + \frac{1}{2^{22}} + \dots\right) \cdot 10^6,$$

which is less than (replacing 10^6 by 2^{20}) $1/2 \cdot 20 + 1 = 11$. In the eighteenth century, an empirical test of this game found that, over 2,084 games, the actual mean was less than 5, which is of the order that might be expected (this test used real coin-tosses!) since one-in-a-million events are unlikely. If those games had been played for £10 then they would have netted A over £10,000.

For the actual problem, imagine that A could be made to pay unlimited finite amounts (so A does not actually have infinite capital already, and imagine it's monopoly money too). A might still play for a million pounds, even though the mathematical expectation of infinite loss is accurate for this original problem. That would probably be because A would be very unlikely to lose much of that million. Basically it is the difference between mode and mean types of expectation. There is also a median, which would be as infinite as the mean expectation, despite the fact that only 'much' smaller finite amounts could ever be lost, because the potential infinity of the expectation is not a potential for infinity but unlimited potential. In many ways the vast but finite elements of this latter are harder to imagine than the structural properties of the former.

There is still the paradoxical nature of this game to be accounted for. It is paradoxical because it challenges reasonable assumptions. If you were to toss a thousand tails in a row you might well assume that this was excellent evidence for a biased coin, and if a million then how could you doubt it? This would appear to have all the force of scientific evidence producing a rational fact. Logic might then imply that if you had an unbiased coin you would not get a billion tails in a row, playing the game once. You might think it to be irrational not to play this game, once, for a trillion pounds.

However, if you were to advise others to be as rational, or thought this a good enough reason to play again, or others thought the same way independently, then terrible things could happen. It is as if the introduction of the mean concept into assumptions based on the mode concept of expectation make the reasonable expectations for a whole set of games less than the sum of the reasonable expectation of any one game. This can't be true (any more than the outcome of a fair toss can depend upon prior tosses) so perhaps (to propose a 'solution' similar to the one I proposed in the case of the surprise exam paradox) it is not, after all, rational to play this game for a trillion (monopoly) pounds.

Problem 180.6 – Two pedestrians

Simon Rolph

Two pedestrians, P_1 and P_2, are travelling at right angles to each other, but are obscured by the corner of a building. Each is travelling parallel to the wall nearest to them. Pedestrian P_i is travelling at a constant speed of V_i and the perpendicular distance between P_i and the building is L_i, $i = 1, 2$.

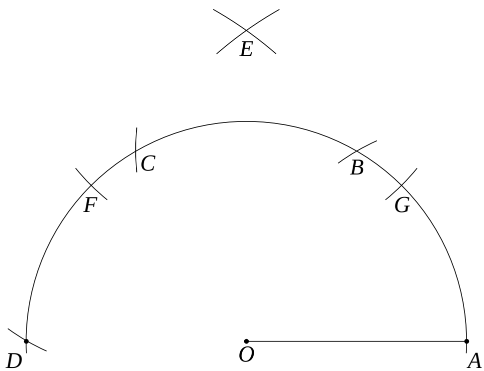
If the time they meet occurs at zero, find the time before impact at which they can both see each other across the corner of the building.

Solution 176.5 – Construct a square

> Given a unit line segment, construct a unit square using compasses only.

Gordon Alabaster

Given unit line segment OA, scribe a circle with centre O and radius 1. With the compasses set at 1, mark off from A to B, from B to C and from C to D, round the circle. By Pythagoras' theorem, AC has length $\sqrt{3}$. Set the compasses to $\sqrt{3}$. Scribe arcs from A and from D to cross at E. By Pythagoras' theorem, $OE = \sqrt{2}$.



Set the compasses to 1. Scribe arcs from E to intersect the circle with centre O at F and G. Then the quadrilateral $OFEG$ is a unit square since $OF = FE = EG = GO = 1$ and $OE = \sqrt{2}$.

Letters to the Editors

Re: Problem 178.1 – Lottery guarantee.

Dear Tony,

With regard to the argument that 18424/20 tickets are required, this is stated in the article in M500 **161** to give *twenty* prizes of £10, thereby implying that to guarantee *one* £10 prize a minimum of 922/20, say 47, tickets are required. I think that 47 tickets is too small.

The probability of winning (only) the £10 prize is

$$20\left(\frac{6}{49}\cdot\frac{5}{48}\cdot\frac{4}{47}\right)\left(\frac{43}{46}\cdot\frac{42}{45}\cdot\frac{41}{44}\right) = \frac{1}{56.6}.$$

This implies a minimum of 57 tickets would be required. However, the probability of winning *any* prize (i.e. at least £10) is 1/53.6, so that 54 tickets are necessary. The great difficulty is how you actually select the number in the correct combinations for the minimum number of tickets. I have not been able to find a systematic way of covering all 49 numbers with only fifty-something and I would be very interested to see how this can be achieved whatever the minimum number of tickets is needed to guarantee a win of at least £10.

The 'expected value' return per ticket is about 45p, so for every ticket purchased at £1 there is an 'expected loss' of 55p. Even with a 'clever strategy' which guarantees a £10 win it is difficult to break even.

Yours sincerely,

Chris Pile

ADF—Problem 178.1 appears to be the subject of much research and, as far as I am aware, the answer is not known. The current record holders are Dragan Stojiljkovic and Rade Belic, who show that 163 National Lottery tickets are sufficient to guarantee a £10 prize. Details of their plan can be found in http://lottery.merseyworld.com/Wheel/Wheel.html.

Coincidence

Dear Jeremy,

The population of the country is about 50/60 million. Ten of them are officers of the M500 committee. Three of them are my children. What is the probability of one of my children selling his/her house to an officer on the M500 committee?

The probability presumably depends a great deal on the 'relation' between myself and the committee members. That is we are/were all involved with the OU.

A few years ago, my son, Jim, was living at 32 Lucas Place!

Colin Davies

$1 + 1 = 2$

Dear Tony,

Maybe I can clarify Malcolm Fowler's question [M500 **178** 36] concerning the relationship between Russell and Whitehead's proof that $1 + 1 = 2$ and Gödel's Incompleteness Theorem. It is indeed the case that Russell and Whitehead's aim in *Principia Mathematica* was to show that mathematics can be rigorously constructed from logic, namely their Theory of Types. Their proof that $1 + 1 = 2$ is part of that grand design.

Gödel's rightly celebrated Incompleteness Theorem states that any consistent axiomatic system rich enough to encompass the arithmetic of the positive integers is incomplete. This means that there are true statements about the positive integers that cannot be proved within the given axiomatic system. However, Gödel's result does not say that we are unable to prove anything within such an axiomatic system. Clearly, $1 + 1 = 2$ is one of the 'facts' that the Russell–Whitehead system can prove but Gödel tells us that there will be others that it cannot prove.

With best wishes,

John Taylor

Calendar

Dear Eddie,

Thanks a lot for M500 **178**. I read Singmaster's enormous article with great interest. His entry for *c*.1555 is interesting. Long ago I saw a rhyme in Latin hexa-meters:

Julius, Aprilis Septemque Novemque tricenos:
Unum plus reliqui; Februs tenet octo vicenos,
At si bissextilis fuerit, superadditur unus.

(July, April, September and November thirty: One more for the rest; February has twenty-eight, But should it be a bissextile year, one more is added.)

This is pretty degenerate Latin and looks medieval, though the hexameters seem fairly metrically kosher (which medieval attempts at hexameters are usually not). By the way, the Roman month is called Quintilis, not Quintilius. Singmaster mentions that Quintilis was renamed Julius by a sycophantic Senate, but not that when Augustus reorganized the Julian calendar he renamed Sextilis after himself, and furthermore stole a day from February (which had previously had 29/30 days) so that his month would have 31 days like Julius.

Tom Dale's and Max Bramer's question: surely thousands of readers are going to see this one. Can hardly be bothered to say that the next term is T and the last one I. Or am I sticking my head into some diabolical trap?

And as for Jeremy Humphries's limerick:

Said the lad to his dad, "Thank you, father,
But what's all this silly palaver?
A twelve-sided hat
Will never lie flat
Unless it's a full balaclava."

Best wishes,

Ralph Hancock

Problem 180.7 – Interesting series

John Bull

Find the sum of this infinite series for $x \neq -1$:

$$\frac{1}{1+x} + \frac{x}{(1+x)(1+x^2)} + \cdots + \frac{x^{2^n-1}}{(1+x)(1+x^2)\ldots(1+x^{2^n})} + \cdots.$$

Films likely to be of interest to mathematicians

The Matrix
The Prime of Miss Jean Brodie
The Eigenvector Sanction
Casablanca
Twistor
A Night at the Operator
The Texas Chain-rule Massacre
American Pi
Lord of the Rings
Affine Madness
Homomorphism Alone
Angles with Dirty Faces
Gunfight at the OK Corollary
Raiders of the Lost Arc

That's all we have so far. Now it's your turn.

Casablanca? We must have been thinking of Humphrey Bogart's words, 'Here's lookin' at Euclid.'

Christmas Day = Hallowe'en? Explain. [**Colin Davies**]

Twenty-five years ago

From M500 33 & 34

John Parker—A goat is tethered to a point on the perimeter of a circular silo. The length of the rope just permits him to graze as far as the diametrically opposite side of the silo. What is his grazing area?

Max Bramer—Given $n+1$ positive integers all less than or equal to $2n$, prove that at least two must be coprime.

Datta Gumaste—If B is some branch of mathematics, it is essentially a collection of theorems and their proofs, where each theorem is implied by a statement assumed or proved to be true. Let $C = \{t : t \text{ is a proof of a theorem}\}$; $U = \{p : p \text{ is an axiom or definition}\}$; R is implication ($\Rightarrow$). Then (C, R, U) satisfies the definition of art. But this shows that B, which is any branch of mathematics, is an art, and hence *mathematics is an art.*

But when you are in love you do not need a proof. You simply love, and in loving you are love.

'Imagine the perplexity of a man outside time and space, who has lost his watch, his measuring rod and his tuning fork.'—A. Jarry.

M500 Mathematics Revision Week-end 2001

JRH

The **27th M500 Society Mathematics Revision Week-end** will be held at **Aston University**, **Birmingham** over **14 – 16 September 2001**.

Tutorial sessions start at 19.30 on the Friday and finish at 17.00 on the Sunday. The Week-end is designed to help with revision and exam preparation, and is open to all OU students. We intend to present most OU maths courses. On the Saturday evening we have mathematical lecture by our guest speaker, **Professor Roger Webster** of Sheffield University.

After the lecture **Charles Alder** is running a disco. In parallel and for the less energetic we plan to organize a *ceilidh*. If you play a musical instrument, please do remember to bring it with you.

See http://freespace.virgin.net/jeremy.humphries/sept.htm for full details and an application form, or send a stamped, addressed envelope to

Jeremy Humphries, M500 Week-end 2001.

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