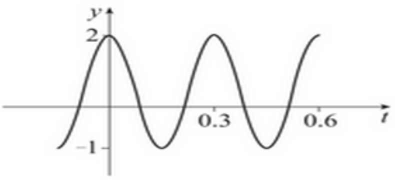
Mechanics 2 (Units 9, 10, 11)

Note there are two representations of springs, the coiled version as now favoured by MST210 and the representation used in these questions – the mathematics remains the same!

(88)

The position-time graph of a particle oscillating vertically at the end of a small spring is



Which option gives the amplitude and period of the motion?

Options

A amplitude = 1.5, period = 0.3 **B** amplitude = 1.5, period = 0.3π

C amplitude = 1.5, period = 0.6 **D** amplitude = 2, period = 0.3

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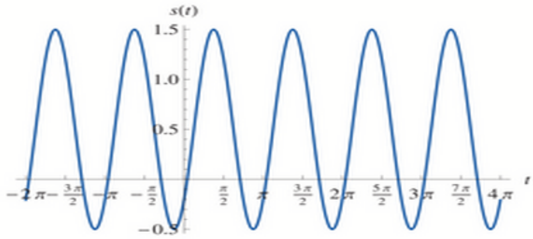
(89)

The position–time graph of a particle oscillating vertically at the end of a small spring is shown below.



Which option gives the amplitude and period of the motion?

Options

A amplitude = 1.5, period = π B amplitude = 1.5, period = $\pi/2$ **C** amplitude = 1, period = $\pi/2$ **D** amplitude = 1, period = : π

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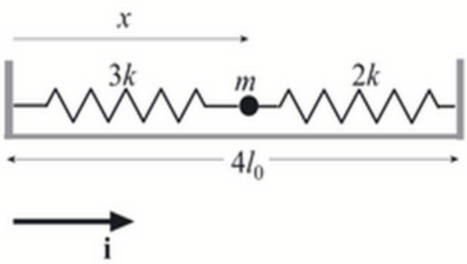
(90)

A particle of mass m is attached to two model springs of stiffness $3k$ and $2k$, as shown below. Both springs have natural length l_0, and their other ends are attached to fixed points a distance $4l_0$ apart. The distance of the particle from the left-hand fixed point is x.



Which option gives the value of x when the particle is in equilibrium?

Options

A $\frac{11}{5}l_0$ **B** $\frac{9}{5}l_0$ **C** $\frac{5}{9}l_0$ **D** $\frac{2}{3}l_0$

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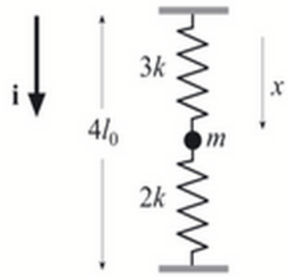
(91)

A particle of mass m is attached to two model springs of stiffness $3k$ and $2k$, respectively. Both springs have natural length l_0, and their other ends are attached to fixed points a vertical distance $4l_0$ apart. The distance of the mass from the upper fixed point at time t after release is x.



Which option gives the period of small oscillations when the particle is displaced from equilibrium and released?

Options

A $2\pi\sqrt{\frac{5k}{m}}$ **B** $\sqrt{\frac{k}{m}}$ **C** $2\pi\sqrt{\frac{k}{m}}$ **D** $2\pi\sqrt{\frac{m}{5k}}$

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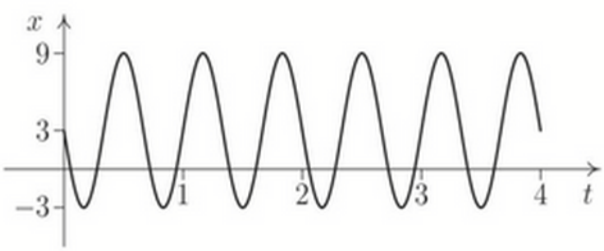
(92)

The position–time graph of a particle oscillating horizontally between two springs is shown below.



Which option gives the most appropriate equation for the displacement as a function of time?

Options

A $x = 3 - 6\sin(2\pi t)$ **B** $x = 3 - 3\sin(2\pi t)$

C $x = 3 - 6\sin\left(\frac{3}{2}\pi t\right)$ **D** $x = 3 - 6\sin(3\pi t)$

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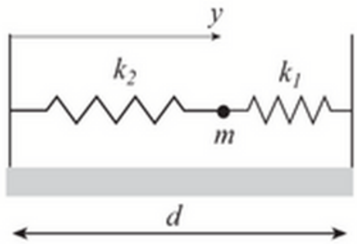
(93)

A particle of mass m is attached to two model springs of stiffness k_1 and k_2 and natural lengths l_1 and l_2, respectively. The other ends of the springs are attached to fixed points a horizontal distance d apart. Let y be the position of the particle relative to the end of the left-hand spring, as shown. The particle is displaced from its equilibrium position and moves horizontally on a smooth plane.



Select the option that gives the equation of motion of the particle.

Options

A $m\ddot{y} + (k_1 + k_2)y = k_2(l_2 - d) + k_1 l_1$

B $m\ddot{y} + (k_1 + k_2)y = k_1 l_1 - k_2 l_2 + k_1 d$

C $m\ddot{y} + (k_1 + k_2)y = k_2(d - l_2) + k_1 l_1$

D $m\ddot{y} + (k_1 + k_2)y = k_2 l_2 - k_1 l_1 + k_1 d$

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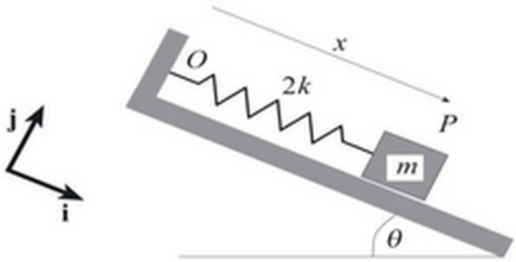
(94)

A particle P of mass m is attached to a model spring of natural length l_0 and stiffness $2k$. The other end of the spring is attached to a fixed point O. The particle rests on a smooth plane inclined at an angle θ to the horizontal. The distance OP is x.



(a) Draw a force diagram showing all the forces acting on the particle.

(b) Find the distance x_{eq} when the particle is in equilibrium.

(c) Determine the equation of motion of the particle in terms of m, k, x, θ, l_0 and g, the magnitude of the acceleration due to gravity.

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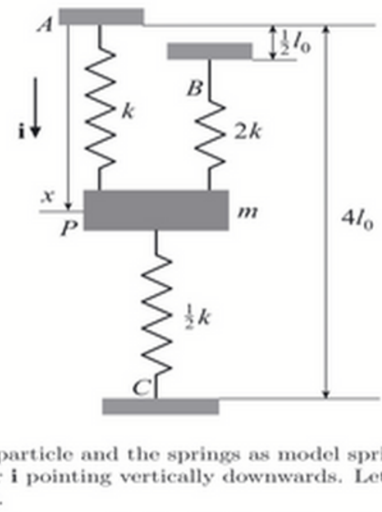
(95)

A block P of mass m is attached to three springs whose other ends are attached to fixed points A, B and C. The stiffnesses of the three springs are k, $2k$ and $\frac{1}{2}k$, and their natural lengths are l_0, $\frac{1}{2}l_0$ and $2l_0$, respectively. The point B is a distance $\frac{1}{2}l_0$ below A and the point C is a distance $4l_0$ below A, as shown in the diagram.



Model the block as a particle and the springs as model springs. Take the origin at A, and the unit vector $\mathbf{i}$ pointing vertically downwards. Let x be the displacement of P from A at time t.

You may ignore air resistance and other frictional forces in this question.

95 continued

Model the block as a particle and the springs as model springs. Take the origin at A, and the unit vector $\mathbf{i}$ pointing vertically downwards. Let x be the displacement of P from A at time t.

You may ignore air resistance and other frictional forces in this question.

(a) Draw a force diagram indicating all the forces acting on the particle.

(b) Express all the forces in terms of the given unit vector, explaining how you derive the expressions for forces due to the springs.

(c) Determine the equilibrium position of the particle.

(d) Derive the equation of motion of the particle when it is not in equilibrium.

(e) Find the general solution of the equation of motion.

(f) The particle is initially released from rest at a distance $\frac{8}{7}l_0$ below A. Determine the solution of the equation of motion that satisfies these initial conditions.

(g) Write down the period and the amplitude of the oscillations of the particle during its subsequent motion in terms of m, g, l_0 and k.

(h) Draw a sketch of the graph of x against t for $t \geq 0$, clearly indicating the amplitude, period, starting position and the average position.

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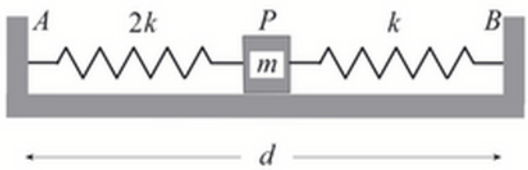
(96)

A particle P of mass m lies on a smooth plane and is connected to two springs, both of natural length l_0. The left-hand spring has stiffness $2k$ and its other end is attached to a fixed point A. The right-hand spring has stiffness k and its other end is attached to the point B, a distance d from A. The distance of the particle from A is x.



(a) Draw a force diagram showing all the forces acting on the particle.

(b) Find the distance x_{eq} when the particle is in equilibrium.

(c) Determine the equation of motion of the particle.

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(97)

A particle of mass 2 kg is held on a smooth slope by a spring that lies parallel to the slope and whose top end is fixed. The spring has stiffness $25\,\mathrm{N\,m^{-1}}$ and natural length 0.5 m. The slope is inclined at an angle of $\pi/8$ to the horizontal. Find how far along the slope from the fixed end of the spring the particle rests in equilibrium.

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(97)(a)

In this question you should define any variables or forces that you introduce.

A particle of mass 4 kg lies on a vertical spring whose base is on the ground. The spring has stiffness $100\,\mathrm{N\,m^{-1}}$ and natural length 0.5 m.

(a) Find the differential equation of motion for the particle.

(b) Find the equilibrium position of the particle above the ground in terms of g, the magnitude of the acceleration due to gravity.

(98)

A particle of mass 4 kg is attached to an elastic model string. (An elastic model string is similar to a model spring, but it cannot be in compression — it just goes slack when the vertical distance between the particle and fixed point is less than the natural length.) The other end of the string is attached to a fixed point, and the particle is initially at the same height as this fixed point. The string has stiffness $100\,\mathrm{N\,m^{-1}}$ and natural length 0.5 m.

The string is initially slack when the particle is released, and the particle drops under gravity so that eventually the string extends and slows the particle down. Air resistance is negligible.

(a) Find an expression for the total mechanical energy when the string is extended.

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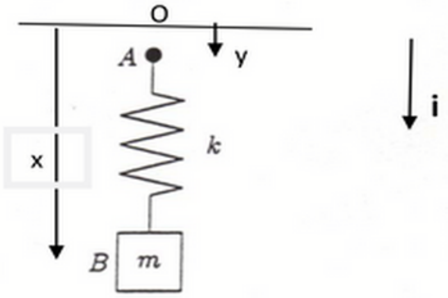
(99)

A particle of mass m is attached to one end B of a perfect spring AB of stiffness k and natural length l. The system is free to move in a vertical line with B below A, as shown in the diagram. This question is concerned with how the particle at B moves when the other end A of the spring is moved vertically in a specified manner. The positions of A and B will be measured relative to a fixed origin O vertically above A.



The vertical displacement at time t of the particle at B from the fixed point O is denoted by $x(t)$, whereas the vertical displacement from O of the other end A of the spring is denoted by $y(t)$, a known function of time. In the appropriate SI units the constants have the values $m = 1$, $k = 4$ and $l = 5$, and the magnitude g of the acceleration due to gravity may be assumed to have the value 10. Throughout the question you should neglect any effects due to damping and air resistance.

(i) By using a force diagram, or otherwise, specify the directions and magnitudes of the forces acting on the particle. You should explicitly state any assumptions you have made in addition to those given in the question.

(ii) Hence show that the motion of the particle is given by the differential equation

$$\ddot{x} + 4x = 30 + 4y.$$

Initially the system is at rest in its equilibrium position with

$$y(t) = 0 \qquad (t \leq 0).$$

After the instant $t = 0$, the point A is forced to oscillate vertically with downwards displacement

$$y(t) = 1 - \cos t \qquad (t > 0).$$

(iii) For the initial period $t \leq 0$ when the point A is stationary, find the equilibrium length of the spring. What are the values of x and $\dot{x}$ at $t = 0$?

(iv) Find the general solution of the equation of motion for the particle at B for $t > 0$. By using the initial conditions at $t = 0$ which you obtained in part (iii), derive an expression for the displacement $x(t)$ of the particle for $t > 0$.

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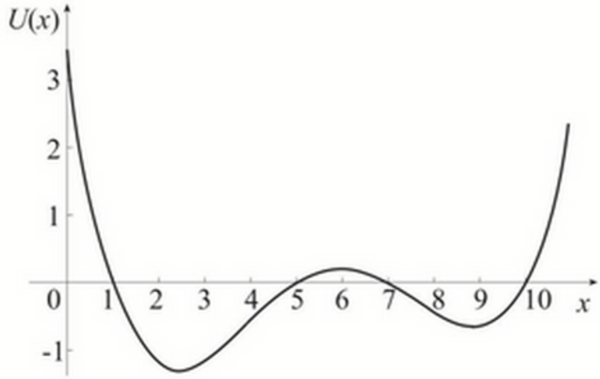
(100) The figure below shows the potential energy function $U(x)$ of a system.



Given that the total energy in the system is zero, which option represents the largest possible range of x for which motion is possible?

Options

A No motion is possible **B** $1 \le x \le 10$ **C** $1 \le x \le 5$ or $7 \le x \le 10$ **D** $5 \le x \le 7$

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(101) A particle of mass m moves under the influence of a force $\mathbf{F}(x) = (x + \cos x)\mathbf{i}$ along the x-axis. Select the option that could represent the total mechanical energy of the particle at position x.

Options

A $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}x^2 - \sin x$ **B** $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}x^2 + \sin x$

C $\frac{1}{2}m\dot{x}^2 - \frac{1}{2}x^2 + \sin x$ **D** $\frac{1}{2}m\dot{x}^2 - \frac{1}{2}x^2 - \sin x$

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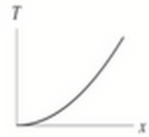
(101)(a) An object is dropped from rest at the top of a high building and falls under the force due to gravity alone. Taking the ground as the datum for gravitational potential energy, which option gives the graph of kinetic energy T of the object as a function of the distance x through which it has fallen?

Options

A


B


C


D


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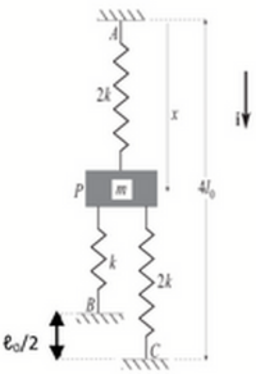
(102) A block P of mass m is attached to three springs whose other ends are attached to fixed points A, B and C. The stiffnesses of the three springs are $2k$, k and $2k$, and their natural lengths are l_0, $\frac{1}{2}l_0$ and $2l_0$, respectively. The point C is a distance $4l_0$ below A, and the point B is a distance $\frac{1}{2}l_0$ above C, as shown in the diagram.



(a) Determine an expression for the total mechanical energy of the system.

(b) By differentiating this expression with respect to time show that the equation of motion of the block is given by

$$m\ddot{x} + 5kx = mg + 9k\ell_0.$$

(103) A particle of mass m is attached to one end B of a perfect spring AB of natural length l_0 and stiffness $4\,mg/l_0$ where g is the magnitude of the acceleration due to gravity. The spring is hung vertically with the end A fixed and B below A. The particle is released from rest when the length of the spring is $\frac{1}{2}\,l_0$.

Neglecting frictional forces such as air resistance, show that the particle will come instantaneously to rest when the length of the spring is $2l_0$.

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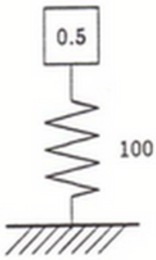
(104)



A particle of mass 0.5 kg is supported on a vertical perfect spring of natural length 0.4 metres and stiffness $100\,\mathrm{N\,m^{-1}}$. The lower end of the spring is fixed, as shown in the figure. Initially the spring has its natural length and the particle is moving downwards with a speed of $2\,\mathrm{m\,s^{-1}}$. You may assume that the magnitude of the acceleration due to gravity is $10\,\mathrm{m\,s^{-2}}$, and you may neglect the effects of friction and air resistance on the subsequent motion.

(i) Find the initial mechanical energy of the system, taking the lower end of the spring as the datum for the gravitational potential energy; remember to specify the units.

(ii) Use the law of conservation of energy to show that the speed v of the particle when the length of the spring is x is given by

$$v = \sqrt{20(-10x^2 + 7x - 1)}.$$

(iii) Find the minimum length of the spring during the motion.

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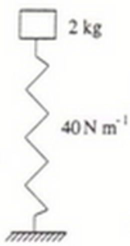
(105)



A particle of mass 2 kg is supported on a vertical perfect spring of natural length 2 metres and stiffness $40\,\mathrm{N\,m^{-1}}$. The lower end of the spring is fixed and the particle is oscillating vertically upwards and downwards. You may assume that the magnitude of the acceleration due to gravity is $10\,\mathrm{m\,s^{-2}}$.

(i) Find an expression for the total mechanical energy function for the system, carefully defining your coordinate system and the datum for the gravitational potential energy.

(ii) When the spring has its natural length, the speed of the particle is $\frac{5}{2}\,\mathrm{m\,s^{-1}}$. By using the conservation of mechanical energy, find the minimum and maximum lengths of the spring during the motion.

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(106) A particle of mass 0.2 kg is attached to one end of a perfect spring which is hanging vertically from a fixed point A. The spring has stiffness $50\,\mathrm{N\,m^{-1}}$ and a natural length of 0.1 metres. The system is oscillating in a vertical line with the particle below A. You may assume that the magnitude of the acceleration due to gravity is $10\,\mathrm{m\,s^{-2}}$.

(i) Find an expression for the total mechanical energy function for the system, carefully defining your coordinate system and the datum for the gravitational potential energy.

(ii) When the particle is 0.17 metres below A it has speed $2\,\mathrm{m\,s^{-1}}$. By using the conservation of mechanical energy, establish whether the spring is ever in compression during the motion.