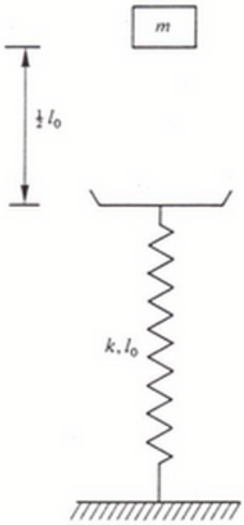
M500MST210 Revision Questions Mechanics 2(ii)

(107)



A particle of mass m is dropped a distance $\frac{1}{2}l_0$ onto a light scale pan supported by a light spring of stiffness $k = 15\,mg/l_0$ and natural length l_0, where g is the magnitude of the acceleration due to gravity. The spring is vertical and its lower end is fixed, as shown in the diagram. Use the conservation of energy to find the maximum compression of the spring during the subsequent motion.

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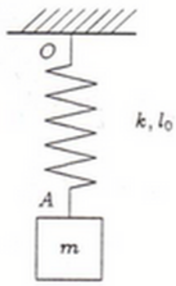
(108)

A particle of mass m is attached to one end A of a perfect spring OA of natural length l_0 and stiffness $k = 2mg/l_0$, where g is the magnitude of the acceleration due to gravity. The spring is hung vertically with the end O fixed and A below O, and the particle is oscillating vertically. It is observed that when the spring has its natural length the particle's speed is $\sqrt{(8gl_0/9)}$. By using the principle of conservation of energy, or otherwise, find the minimum and maximum lengths of the spring during the motion.



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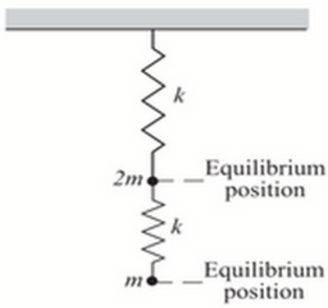
(109)

A particle of mass $2m$ is suspended from a fixed point by a spring of stiffness k and natural length l_0. A second, identical spring is attached to the particle, and a particle of mass m is fixed to its end. The system hangs vertically in equilibrium, as shown. Take the datum for potential energy in each spring to be the point of zero extension (i.e. the natural length) of that spring.



Select the option that corresponds to the energy stored in the two springs.

Options

A $\dfrac{3m^2g^2}{k}$ **B** $\dfrac{7m^2g^2}{2k}$ **C** $\dfrac{5m^2g^2}{k}$ **D** $\dfrac{9m^2g^2}{2k}$

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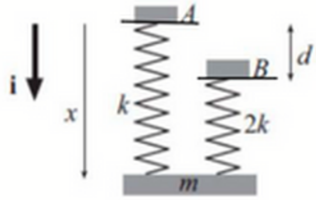
(110)

A particle of mass m is connected to two vertical model springs, each of natural length l_0. The left-hand spring has stiffness k and its other end is attached to a fixed point A. The right-hand spring has stiffness $2k$ and its other end is attached to a fixed point B, a vertical distance d below A. The particle is constrained to move in a vertical line, and there are no resistive forces.



(a) Draw a force diagram showing all the forces acting on the particle when it is a distance x from A. where x > d.

(b) Derive an expression for the total energy of the system in terms of x.

(c) Hence derive the equation of motion of the particle.

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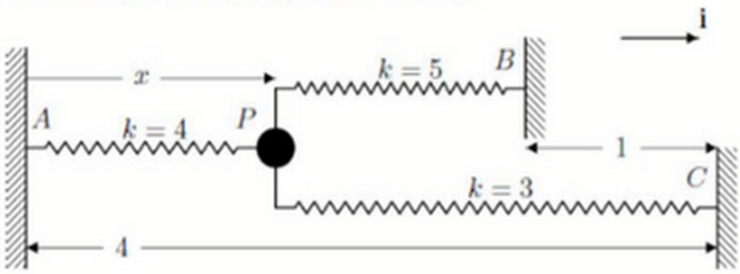
(111) (a) A particle P of mass 3 kg is constrained to move in a horizontal straight line along a smooth track. Attached to the particle are three springs whose parameters are given in the table below. The arrangement is shown in the diagram below. The unit vector, **i**, is defined to be horizontal as shown in the diagram. The distance AC is 4 m, and BC is 1 m.



Spring	stiffness	natural length
AP	$4\,\mathrm{N\,m^{-1}}$	3 m
PB	$5\,\mathrm{N\,m^{-1}}$	1 m
PC	$3\,\mathrm{N\,m^{-1}}$	2 m

What is the total potential energy in the three springs at time t when the displacement of P from A is $x(t)$? Hence write down an expression for the total mechanical energy of the spring system.

(b) If the particle is released from rest at the point x = 2, determine the distance from A when the particle next comes to rest. What is the amplitude of the oscillations?

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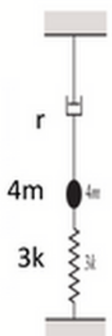
(112)

A particle of mass $4m$ is attached to a model damper of (positive) damping constant r and a model spring of natural length l_0 and stiffness $3k$. The other end of the damper is fixed, and the other end of the spring is also fixed vertically below it, as shown.

Select the option giving a condition that gives the widest possible rang values for r such that the system undergoes oscillations when displace from equilibrium.



Options

A $r^2 < 12mk$ **B** $r^2 > 12mk$

C $r^2 < 48mk$ **D** $r^2 > 48mk$

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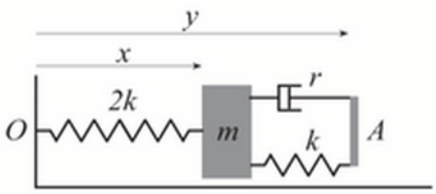
(113)

A particle of mass m is attached to a model spring of stiffness $2k$ and natural length l_0. The other end of the spring is fixed to a wall at O. The particle is also attached to a second model spring, of stiffness k and natural length l_0, and a model damper of damping constant r.



The other ends of the spring and damper are attached to a plate A that moves in such a way that its displacement from O at time t is given by $y(t)$. The displacement of the particle from O at time t is $x(t)$. Which option gives the equation of motion of the particle?

Options

A $m\ddot{x}+r\dot{x}+3kx=ky+r\dot{y}+kl_0$

B $m\ddot{x}+r\dot{x}+3kx=ky-r\dot{y}+kl_0$

C $m\ddot{x}+r\dot{x}+3kx=ky+r\dot{y}-kl_0$

D $m\ddot{x}+r\dot{x}+3kx=ky-r\dot{y}-kl_0$

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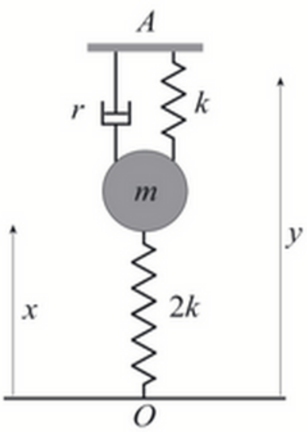
(114)

A particle of mass m is attached to a model spring of stiffness $2k$ and natural length l_0. The other end of the spring is fixed to a floor at O. The particle is also attached to a second model spring, of stiffness k and natural length l_0, and a model damper of damping constant r. The system lies in a vertical plane.



The other ends of the second spring and damper are attached to a plate A that moves in such a way that its displacement from O at time t is given by $y(t)$. The displacement of the particle from O at time t is $x(t)$. Which option gives the equation of motion of the particle?

Option

A $m\ddot{x}+r\dot{x}+3kx=ky+r\dot{y}+kl_0-mg$

B $m\ddot{x}+r\dot{x}+3kx=ky+r\dot{y}-kl_0-mg$

C $m\ddot{x}+r\dot{x}+3kx=ky-r\dot{y}+kl_0-mg$

D $m\ddot{x}+r\dot{x}+3kx=ky-r\dot{y}-kl_0-mg$

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(115) A particle of mass 5 kg is suspended from the ceiling by a spring of stiffness $100\,\mathrm{N\,m^{-1}}$ and natural length 0.5 metres. A damper whose damping constant is $50\,\mathrm{N\,m^{-1}\,s}$, is also attached to the particle; the other end of the damper is attached to the floor at a point directly below the point of suspension of the spring. The height of the ceiling above the floor is 2.5 metres. The particle moves in a vertical line. The magnitude g of the acceleration due to gravity can be taken as $10\,\mathrm{m\,s^{-2}}$.

(i) Draw a diagram of the system indicating clearly the forces acting on the particle during motion. Define your origin of coordinates, unit vector(s) and variable(s)

Draw a force diagram. Define each force in terms of the given parameters and your choice of variable and unit vector.

(ii) Derive the equation of motion of the particle.

(iii) Find the equilibrium position of the particle.

(iv) The particle is pulled down a short distance from its equilibrium position and then released from rest. Describe qualitatively its subsequent motion.

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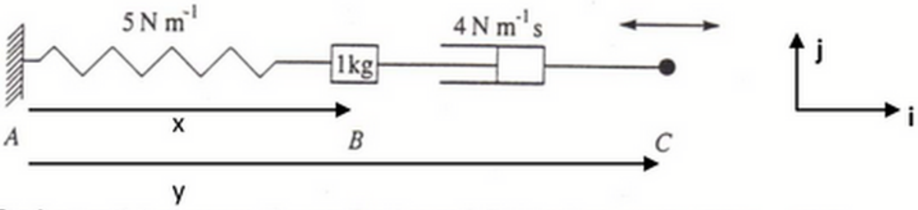
(116)



A particle B of mass 1 kg moves along a horizontal frictionless track. The particle is attached to a fixed point A on the track by a model spring and to a forcing point C on the track by a model damper. The spring has stiffness $5\,\mathrm{N\,m^{-1}}$ and natural length 1 metre, and damper constant is $4\,\mathrm{N\,m^{-1}\,s}$. The displacement of the forcing point C along the track from the fixed point A at time t is denoted by $y(t)$, a known function of time.

(i) Construct a force diagram with all the forces acting on the particle and express each force in terms of the unit vectors, **i** and **j**, as shown above.

(ii) Show that the displacement x of the particle B from the fixed point A satisfies the differential equation

$$\ddot{x} + 4\dot{x} + 5x = 4\dot{y} + 5.$$

(iii) If the displacement of the forcing point is given by

$$y(t) = 2 + \tfrac{1}{4}\sin t,$$

Explain why that after the elapse of a sufficiently long time, the motion of the particle B becomes approximately sinusoidal with the same frequency as the forcing point C. Find the amplitude and phase of this steady-state forced vibration of the particle.

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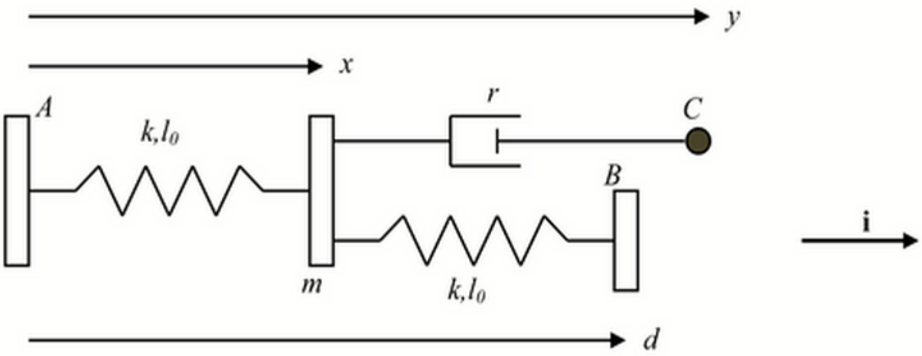
(117) A particle of mass m is attached to two model springs and a model damper as shown. Both springs have a natural length l_0 and stiffness k and their other ends are attached to fixed points A and B, a distance d apart. The damping constant of the damper is r and its other end is at the point C.



The point C is subject to forcing in such a way that its distance from A is given as the function of time $y(t)$. The distance of the particle from A is denoted as x.

(a) Draw a force diagram showing all the horizontal forces acting on the particle and write down expressions for each force in vector form.

(b) Find the equation of motion for the particle.

You are now given that $m = 1$ kg, $l_0 = 1$m, $d = 3$m, $k = 1$ Nm^{-1} and $r = 4$ Nsm^{-1}.

You are also given that $y(t) = y_0 + \sin(t)$.

(i) Show that the equation of motion can now be written as $\ddot{x} + 4\dot{x} + 2x = 3 + 4\cos(t)$.

(ii) What can you say about the damping of the system given by this equation of motion?

(iii) Find the particular integral for this equation of motion.

(iv) Find the amplitude of the steady state oscillations of the particle.

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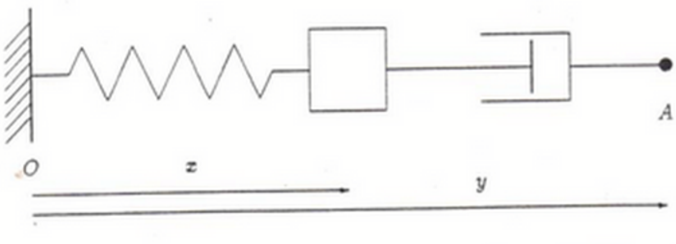
(118)



A particle of mass 1 kg moves along a horizontal frictionless track. The particle is attached to a fixed point O on the track by a perfect spring and to a forcing point A on the track by a perfect dashpot. The spring has stiffness 10 kg s^{-2} and natural length 1 metre. The constant of the dashpot is 3 kg s^{-1}. The displacement of the forcing point A along the track from the fixed point O is $y(t)$, which is a known function of time t.

(i) Show that the displacement z of the particle from the fixed point O satisfies the differential equation

$$\ddot{z} + 3\dot{z} + 10z = 10 + 3\dot{y}.$$

(118) continued

(ii) If the displacement of the forcing point A, in metres, is given by

$y(t) = 2 + 0.1\sin(\Omega t)$, show that the magnitude of the amplitude of the

steady-state oscillations is given by $A(\Omega) = \dfrac{0.3\Omega}{\sqrt{(10-\Omega^2)^2 + 9\Omega^2}}$.

(iii) Determine the values of: (a) $A(0)$, (b) $A(\Omega)$ as $\Omega \to \infty$.

Does this system exhibit resonance? Justify your conclusion.

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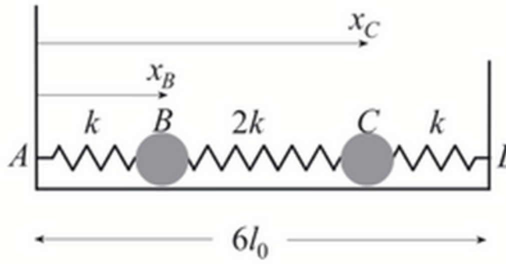
(119) Two particles, B and C, of unit mass are constrained to move in a straight line on a frictionless horizontal plane. They are connected as shown by three model springs and their equilibrium positions (x_B, x_C, respectively) are measured from the fixed point A. The distance AD is $6l_0$, and the springs AB and CD are of stiffness k and natural length l_0. Spring BC has stiffness $2k$ and natural length l_0



If the system is in equilibrium, which option gives the vector $\begin{bmatrix} x_B \\ x_C \end{bmatrix}$?

Options

A $\begin{bmatrix} \frac{3}{2}l_0 \\ 3l_0 \end{bmatrix}$ **B** $\begin{bmatrix} 2l_0 \\ 4l_0 \end{bmatrix}$ **C** $\begin{bmatrix} \frac{7}{4}l_0 \\ \frac{5}{2}l_0 \end{bmatrix}$ **D** $\begin{bmatrix} \frac{11}{5}l_0 \\ \frac{19}{5}l_0 \end{bmatrix}$

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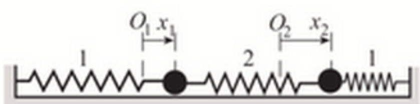
(120) Two particles of unit mass are constrained to move in a straight line on a frictionless horizontal plane. They are connected as shown by three model springs, and their positions x_1, x_2 are measured from their equilibrium positions. The outer two springs have unit stiffness, and the central spring has stiffness 2.



The equation of motion of the system is

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Select the option that gives the normal mode angular frequencies of this system.

Options **A** $1, \sqrt{5}$ **B** $1, -\sqrt{5}$ **C** $\sqrt{5}, \sqrt{2}$ **D** $\sqrt{2}, 1$

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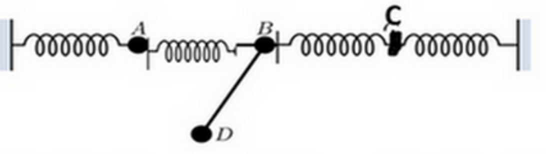
(120)(a)

Consider the mechanical system consisting of four particles labelled A, B, C and D, as shown in the figure below. The particles are connected by either springs or rigid rods (which are shown in the figure by thick lines). Particles A, B and C are constrained to move along a straight line between the connections to the rigid walls. Particle D is freely hinged below particle B to move in a vertical plane.



Select the option that gives the number of degrees of freedom of the mechanical system.

Options

A **1** B **2** C **3** D **4**

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(120)(b)

Consider the second-order differential equation

$$\frac{d^2x}{dt^2} + x^2\frac{dx}{dt} + x = t.$$

Select the option that gives an equivalent system of first-order differential equations.

Options

A
$$\frac{dy}{dx} = t,$$
$$\frac{dy}{dt} = t - x^2y - x$$

B
$$\frac{dy}{dt} = y,$$
$$\frac{dx}{dt} = t - x^2y - x$$

C
$$\frac{dx}{dt} = y,$$
$$\frac{dy}{dt} = t - x^2y - x$$

D
$$\frac{dy}{dt} = x,$$
$$\frac{dx}{dt} = t - x^2y - x$$

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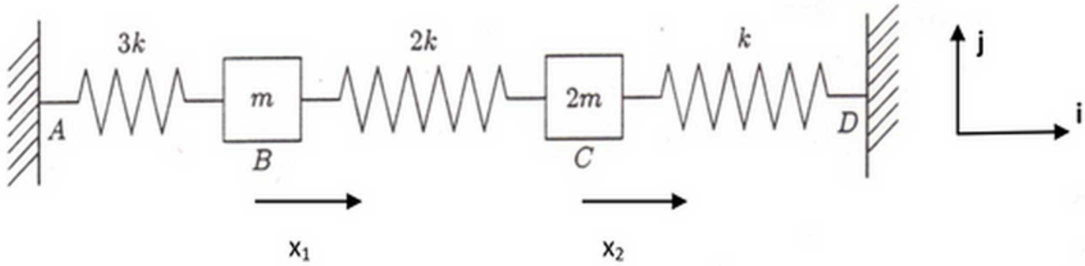
(121)



Three perfect springs AB, BC and CD have stiffnesses $3k$, $2k$ and k respectively, and equal natural lengths l_0. A particle of mass m is attached to the springs at B and a second particle of mass $2m$ is attached at C. The ends A and D of the springs are fixed to two points a horizontal distance $3l_0$ apart, as shown in the diagram, and the system is free to oscillate along the horizontal line AD. You may assume that the only forces acting on the particles in a horizontal direction are those due to the springs.

(i) Draw a force diagram for all the forces acting on each particle. If x_1 and x_2 are the displacements when the particles at B and C are displaced from their equilibrium positions, write down the *change in spring forces* acting on each particle.

(ii) Deduce that the equations of motion of the two particles are

$$m\ddot{x}_1 = -5kx_1 + 2kx_2,$$
$$m\ddot{x}_2 = kx_1 - \tfrac{3}{2}kx_2.$$

(iii) Determine the normal mode angular frequencies and corresponding eigenvalues. Write down the general solution of the system of differential equations.

(iv) The system is set in motion with particle B having an initial displacement from its equilibrium position of 4 cm towards the fixed point D, particle C having an initial displacement from its equilibrium position of 1 cm towards the fixed point A and both particles initially at rest. Sketch a graph of showing the motions of the motions of the two particles with time, indicating the most important features.

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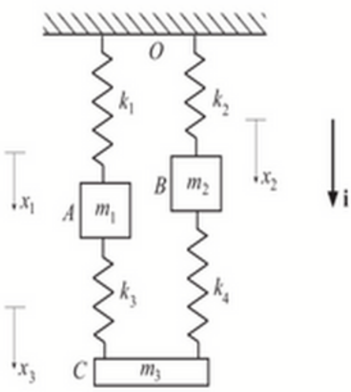
(122)

Three particles A, B, C of masses m_1, m_2 and m_3, respectively, are connected by four model springs as shown in the figure below. Two of the springs are suspended from a fixed support, and the particles are constrained to move in vertical lines. The stiffness of each model spring is shown on the diagram.



(122) continued

(a) For each particle, draw a force diagram showing all the forces acting on the particle, and define any forces that you introduce.

The masses are displaced from their respective equilibrium positions, and after t seconds have passed, their respective displacements from their equilibrium positions are x_1, x_2 and x_3.

(b) Now consider the change in the forces due to displacement from equilibrium. Copy and complete the following table to show the *change in each spring force* (due to the displacement from equilibrium) in terms of the displacements x_1, x_2 and x_3, and the unit vector $\mathbf{i}$.

Mass	Spring	Δl_i	k_i	$\widehat{\mathbf{s}}_i$	$\Delta \mathbf{H}_i$
A	OA		k_1		
	AC		k_3		
B	OB		k_2		
	BC		k_4		
C	AC		k_3		
	BC		k_4		

What are the changes due to displacement from equilibrium for the other forces? Briefly justify your answer.

(c) Hence derive the equation of motion for each particle and show that in matrix form, the equations may be expressed as

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} -\dfrac{k_1+k_3}{m_1} & 0 & \dfrac{k_3}{m_1} \\ 0 & -\dfrac{k_2+k_4}{m_2} & \dfrac{k_4}{m_2} \\ \dfrac{k_3}{m_3} & \dfrac{k_4}{m_3} & -\dfrac{k_3+k_4}{m_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

You are now given the following values.

m_1	m_2	m_3	k_1	k_2	k_3	k_4
1 kg	2 kg	4 kg	$4\,\mathrm{N\,m^{-1}}$	$1\,\mathrm{N\,m^{-1}}$	$1\,\mathrm{N\,m^{-1}}$	$2\,\mathrm{N\,m^{-1}}$

and the normal mode angular frequency are: $\omega_1 = 2.249$, $\omega_2 = 0.534$ and $\omega_3 = 1.380$,

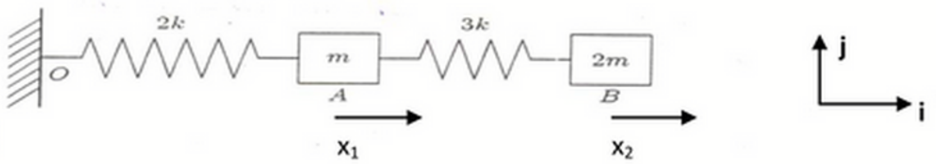
with corresponding eigenvectors $[-0.998 \quad -0.017 \quad 0.060]^T$, $[0.162 \quad 0.627 \quad 0.76$
$[0.120 \quad -0.920 \quad 0.372]^T$, respectively.

Write down the general solution of the system of differential equations.

(d) If the particles are initially released from rest, give a general displacement vector $\mathbf{x}_0$ (from the equilibrium position) that would lead to normal mode motion when all the particles move in phase.

(e) If the system is in normal mode motion at the highest normal mode angular frequency, state which of the particles are moving in phase. Briefly justify your answer.

(123)



Two perfect springs OA and AB have stiffnesses $2k$ and $3k$ respectively, and equal natural lengths l_0. A particle of mass m is attached to the springs at A, and another, of mass $2m$, at B. The end O of the first spring is fixed, and the system is free to oscillate along a horizontal line as shown in the diagram. You may assume that the only forces acting on the particles in a horizontal direction are those due to the springs.

(i) Draw a force diagram acting on each of the particles. If x_1, x_2 are displacements of A and B from their respective equilibrium position, determine and write down the change in spring forces acting on each particle.

(ii) Deduce that the equations of motion of the particles are

$$m\ddot{x}_1 = -5kx_1 + 3kx_2,$$
$$m\ddot{x}_2 = \tfrac{3}{2}kx_1 - \tfrac{3}{2}kx_2.$$

(iii) Find the normal mode angular frequencies for the system and the corresponding normal mode displacement ratios.

(iv) Give a set of initial conditions on $x_1(t)$ and $x_2(t)$ which would result in the particles executing *one* of the normal mode motions, and write down expressions for the displacements at time t of both of the particles for the motion determined by your initial conditions.

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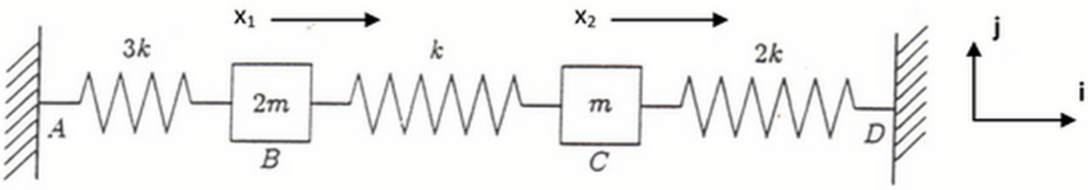
(124)



Three perfect springs AB, BC and CD, each of natural length l_0, have stiffnesses $3k$, k and $2k$ respectively. A particle of mass $2m$ is attached to the springs at B and another, of mass m, at C, as shown in the diagram. The ends A and D of the springs are fixed to two points which are a horizontal distance $3l_0$ apart, and the system is free to oscillate along the horizontal line AD.

x_1 and x_2 are the displacements of particles B and C from their respectively equilibrium positions at time t after release from rest.

(i) Write down the change in spring forces acting on each particle.

(ii) Determine, but do not solve, the equation of motion of the particles.

You must carefully define all symbols that you introduce in your solution.

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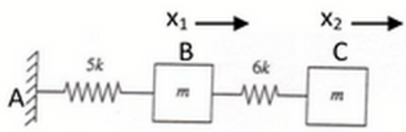
(125) The equation of motion of the particles in the spring system



are:
$$m\ddot{x}_1 = -11kx_1 + 6kx_2,$$
$$m\ddot{x}_2 = 6kx_1 - 6kx_2.$$

where x_1, x_2 are respective displacements from the equilibrium positions.

If the particles are released from rest with B having an initial displacement away from A of $3\ell_0/10$ from its equilibrium position and C an initial displacement towards A of $\ell_0/5$, find expressions for x_1 and x_2 at time t.
